CHAPTER 2

Polynomials

EXAM DRILL

SOLUTIONS

1. **(a) :** Since α, β are the zeroes of $2x^2 + 6x - 6$, we have

$\therefore \quad \alpha + \beta = \frac{-6}{2} = -3$ and $\alpha\beta = \frac{-6}{2} = -3$.

Hence, $\alpha + \beta = \alpha\beta$

2. **(b) :** Product of zeroes $= a/r \cdot a \cdot ar = -1$

$\left[\because \text{Product of zeroes in cubic polynomial } = \frac{-\text{Constant term}}{\text{Coefficient of } x^3}\right]$

$\Rightarrow \quad a^3 = -1 \quad \Rightarrow \quad a = -1$

3. **(b) :** Let $f(x) = px^3 + x^2 - 2x + q$

Since $(x + 1)$ and $(x - 1)$ are factors of $f(x) = px^3 + x^2 - 2x + q$

$\therefore \quad f(1) = 0$ and $f(-1) = 0$

Now, $f(1) = p + 1 - 2 + q = p + q - 1 = 0$

$\Rightarrow \quad p + q = 1 \qquad$ ...(i)

$f(-1) = 0 \Rightarrow -p + 1 + 2 + q = 0$

$\Rightarrow \quad -p + q = -3 \qquad$ ...(ii)

Solving (i) and (ii), we get $p = 2$ and $q = -1$

4. Let $p(x) = (k - 1)x^2 + kx + 1$

Given that, one of the zeroes is -3, then $p(-3) = 0$

$\Rightarrow \quad (k - 1)(-3)^2 + k(-3) + 1 = 0$

$\Rightarrow \quad 9(k - 1) - 3k + 1 = 0 \Rightarrow 6k - 8 = 0 \Rightarrow k = 4/3$.

5. On dividing $2x^2 - 18x - 45$ by $x - 16$, we have

$$\begin{array}{r|l} & 2x + 14 \\ \hline x - 16 & 2x^2 - 18x - 45 \\ & \underset{(-)}{2x^2} \underset{(+)}{- 32x} \\ \hline & 14x - 45 \\ & \underset{(-)}{14x} \underset{(+)}{- 224} \\ \hline & 179 \end{array}$$

$\therefore \quad 179$ should be subtracted from $2x^2 - 18x - 45$, so that 16 is zero of resulting polynomial.

6. Sum of zeroes $= \alpha + \beta = -\left(\frac{-6}{2}\right) = 3$

Product of zeroes $= \alpha\beta = 7/2$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (3)^2 - 2\left(\frac{7}{2}\right) = 9 - 7 = 2$

7. On dividing $2x^3 + 3x^2 - 8x - 12$ by $x^2 - 4$, we have

$$\begin{array}{r|l} & 2x + 3 \\ \hline x^2 - 4 & 2x^3 + 3x^2 - 8x - 12 \\ & \underset{(-)}{2x^3} \qquad \underset{(+)}{- 8x} \\ \hline & 3x^2 + 0 - 12 \\ & \underset{(-)}{3x^2} \qquad \underset{(+)}{- 12} \\ \hline & 0 \end{array}$$

Thus, the quotient is $2x + 3$ and remainder $= 0$.

8. Let $p(x) = 2ax^3 + 3x^2 + 5x + 2$

Let α, β and γ be the zeroes of $p(x)$, where $\alpha = 0$.

We know that, sum of product of zeroes taken two at a time $= \alpha\beta + \beta\gamma + \gamma\alpha = \frac{5}{2}$

$\Rightarrow \quad 0 \times \beta + \beta\gamma + \gamma \times 0 = \frac{5}{2} \Rightarrow \beta\gamma = \frac{5}{2}$

Hence, product of other two zeroes $= \frac{5}{2}$

9. Given, α and β are the zeroes of the polynomial, $f(x) = x^2 - 19x + k$

$\therefore \quad \alpha + \beta = 19 \qquad$ ...(i)

and $\alpha\beta = k \qquad$ ...(ii)

Also, $\alpha - \beta = 7$ (Given) $\qquad$ ...(iii)

Adding (i) and (iii), we get $2\alpha = 26 \Rightarrow \alpha = 13$

From (i), $\beta = 6$

Now, $\alpha\beta = k \Rightarrow 13 \times 6 = k \Rightarrow k = 78$

10. Given that, sum of zeroes $(S) = -\frac{3}{2\sqrt{5}}$

and product of zeroes $(P) = -\frac{1}{2}$

$\therefore \quad$ Required quadratic polynomial is

$$p(x) = x^2 - Sx + P = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2} = 2\sqrt{5}x^2 + 3x - \sqrt{5}$$

Using factorisation method,

$$p(x) = 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5}$$

$$= \sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = (2x + \sqrt{5})(\sqrt{5}x - 1)$$

Hence, the zeroes of $p(x)$ are $-\frac{\sqrt{5}}{2}$ and $\frac{1}{\sqrt{5}}$.

11. On dividing $x^3 - ax^2 + 6 - a$ by $x - a$, we have

$$\begin{array}{r|l} & x^2 \\ \hline x - a & x^3 - ax^2 + 6 - a \\ & \underset{(-)}{x^3} \underset{(+)}{- ax^2} \\ \hline & 6 - a \end{array}$$

$\therefore \quad$ Quotient $= x^2$ and remainder $= 6 - a$

12. Let $p(x)$ be the required polynomial.

$g(x) = -2x^2 + 3x - 2$ and $q(x) = x - 3$ and $r(x) = 4$

So, by division algorithm, we have $p(x) = g(x) \cdot q(x) + r(x)$

$\Rightarrow \quad p(x) = (-2x^2 + 3x - 2)(x - 3) + 4$

$= -2x^3 + 3x^2 - 2x + 6x^2 - 9x + 6 + 4$

$= -2x^3 + 9x^2 - 11x + 10$

13. (i) (b) : Graph of a quadratic polynomial is a parabolic in shape.

(ii) (c) : Since the graph of the polynomial cuts the x-axis at (–6, 0) and (6, 0). So, the zeroes of polynomial are –6 and 6.

∴ Required polynomial is

$p(x) = x^2 - (-6 + 6)x + (-6)(6) = x^2 - 36$

(iii) (c) : We have, $p(x) = x^2 - 36$

Now, $p(6) = 6^2 - 36 = 36 - 36 = 0$

(iv) (b) : Let $f(x) = x^2 + 2x - 3$. Then,

$$\text{Sum of zeroes} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{(2)}{1} = -2$$

(v) (d) : The given polynomial is $at^2 + 5t + 3a$

Given, sum of zeroes = product of zeroes

$$\Rightarrow \frac{-5}{a} = \frac{3a}{a} \quad \Rightarrow \quad a = \frac{-5}{3}$$

14. (i) (b) : The shape of the path of the soccer ball is a parabola.

(ii) (c) : The axis of symmetry of the given curve is a line parallel to y-axis.

(iii) (a) : The zeroes of the polynomial, represented in the given graph, are –2 and 7, since the curve cuts the x-axis at these points.

(iv) (d) : A polynomial having zeroes –2 and –3 is

$p(x) = x^2 - (-2 - 3)x + (-2)(-3) = x^2 + 5x + 6$

(v) (c) : We have, $f(x) = (x - 3)^2 + 9$

Now, $9 = (x - 3)^2 + 9$

$\Rightarrow (x - 3)^2 = 0 \Rightarrow x - 3 = 0 \Rightarrow x = 3$

15. (i) (a) : Since, the graph intersects the x-axis at two points, namely $x = 8, -2$.

So, 8, –2 are the zeroes of the given polynomial.

(ii) (b) : The expression of the polynomial given in diagram is $-x^2 + 6x + 16$.

(iii) (c) : Let $p(x) = -x^2 + 6x + 16$

When $x = 4$, $p(4) = -4^2 + 6 \times 4 + 16 = 24$

(iv) (d) : Let $f(x) = -x^2 + 3x - 2$

Now, consider $f(x) = 0 \Rightarrow -x^2 + 3x - 2 = 0$

$\Rightarrow x^2 - 3x + 2 = 0 \Rightarrow (x - 2)(x - 1) = 0$

$\Rightarrow x = 1, 2$ are its zeroes.

(v) (b) : Let α and β are the zeroes of the required polynomial.

Given, $\alpha + \beta = -3$

If $\alpha = 4$, then $\beta = -7$

∴ Representation of tunnel is $-x^2 - 3x + 28$.

16. (i) (c) : For finding α, β, γ, consider $p(x) = 0$

$\Rightarrow x^3 - 18x^2 + 95x - 150 = 0 \Rightarrow (x - 3)(x^2 - 15x + 50) = 0$

$\Rightarrow (x - 3)(x - 5)(x - 10) = 0 \Rightarrow x = 10$ or $x = 5$ or $x = 3$

Thus $\alpha = 10$, $\beta = 5$ and $\gamma = 3$

(ii) (d) : Here $\alpha = 10$, $\beta = 5$ and $\gamma = 3$

∴ Sum of product of zeroes taken two at a time

$= \alpha\beta + \beta\gamma + \gamma\alpha = (10)(5) + (5)(3) + (3)(10)$

$= 50 + 15 + 30 = 95$

(iii) (a) : Product of zeroes of polynomial $p(x) = \alpha\beta\gamma$

$= (10)(5)(3) = 150$

(iv) (b) : We have $p(x) = x^3 - 18x^2 + 95x - 150$

Now, $p(4) = 4^3 - 18(4)^2 + 95(4) - 150$

$= 64 - 288 + 380 - 150 = 6$

(v) (d) : $g(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$\Rightarrow g(x) = x^3 - 3x^2 - 16x - (-48) = x^3 - 3x^2 - 16x + 48$

17. Let $p(x) = x^2 - 12x + 35$

For zeroes, put $p(x) = 0$

$\Rightarrow x^2 - 12x + 35 = 0 \Rightarrow x^2 - 5x - 7x + 35 = 0$

$\Rightarrow x(x - 5) - 7(x - 5) = 0 \Rightarrow (x - 5)(x - 7) = 0$

$\Rightarrow x - 5 = 0$ or $x - 7 = 0 \Rightarrow x = 5$ or $x = 7$

Zeroes of $p(x)$ are 5 and 7.

$$\text{Sum of zeroes} = 5 + 7 = 12 = \frac{-(12)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = 5 \times 7 = 35 = \frac{35}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

18. Given quadratic polynomial is $x^2 - 8x + 7$.

Let α and β be its roots.

∴ $\alpha + \beta = 8$ ...(i)

$\alpha\beta = 7$ ...(ii)

∴ $\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{(8)^2 - 4 \times 7}$ [Using (i) and (ii)]

$= \sqrt{64 - 28} = \sqrt{36} = 6$

$\Rightarrow \alpha - \beta = 6$ ...(iii)

Adding (i) and (iii), we get $2\alpha = 14 \Rightarrow \alpha = 7$

From (i), we get $\beta = 8 - \alpha = 8 - 7 = 1$

OR

Given polynomial, $f(x) = 25P^2 - 15P + 2$

Since, α and β are roots of $f(x)$

$$\therefore \quad \alpha + \beta = \frac{15}{25} = \frac{3}{5} \text{ and } \alpha\beta = \frac{2}{25}$$

Sum of zeroes of the required polynomial

$$= \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{\alpha+\beta}{2\alpha\beta} = \frac{\left(\frac{3}{5}\right)}{2\left(\frac{2}{25}\right)} = \frac{\frac{3}{5}}{\frac{4}{25}} = \frac{3}{5} \times \frac{25}{4} = \frac{15}{4}$$

and product of zeroes of the required polynomial

$$= \left(\frac{1}{2\alpha}\right)\left(\frac{1}{2\beta}\right) = \frac{1}{4\alpha\beta} = \frac{1}{4\left(\frac{2}{25}\right)} = \frac{1}{\frac{8}{25}} = \frac{25}{8}$$

Hence, the required quadratic polynomial is

$$x^2 - \frac{15}{4}x + \frac{25}{8} \text{ or } \frac{1}{8}(8x^2 - 30x + 25).$$

19. Let α and β be the zeroes of the polynomial $f(x) = ax^2 + bx + c$. Then, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Let S and P denotes respectively the sum and product of the zeroes of a polynomial, whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Then, $S=\dfrac{1}{\alpha}+\dfrac{1}{\beta}=\dfrac{\alpha+\beta}{\alpha\beta}=\dfrac{-\frac{b}{a}}{\frac{c}{a}}=-\dfrac{b}{c}$

and $P=\dfrac{1}{\alpha}\times\dfrac{1}{\beta}=\dfrac{1}{\alpha\beta}=\dfrac{1}{\frac{c}{a}}=\dfrac{a}{c}$

Hence, the required polynomial $g(x)$ is given by

$g(x)=k(x^2-Sx+P)=k\left(x^2+\dfrac{bx}{c}+\dfrac{a}{c}\right)$, where k is any non-zero constant.

20. Let the cubic polynomial be ax^3+bx^2+cx+d and its zeroes are α, β and γ.

Then, $\alpha+\beta+\gamma=5=-\dfrac{b}{a}$, $\alpha\beta+\beta\gamma+\gamma\alpha=-11=\dfrac{c}{a}$

and $\alpha\beta\gamma=-23=-\dfrac{d}{a}$

If $a=1$, then $b=-5$, $c=-11$ and $d=23$

So, cubic polynomial is $x^3-5x^2-11x+23$.

OR

We have,

$$\begin{array}{r|l} & 7x^2+x+5 \\ \hline 2x-1 & 14x^3-5x^2+9x-1 \\ & 14x^3-7x^2 \\ & \underline{(-)\quad(+)} \\ & 2x^2+9x-1 \\ & 2x^2-x \\ & \underline{(-)\quad(+)} \\ & 10x-1 \\ & 10x-5 \\ & \underline{(-)\quad(+)} \\ & 4 \end{array}$$

$\therefore$ Quotient, $q(x)=7x^2+x+5$ and remainder, $r(x)=4$

21. On dividing $3x^3+x^2+2x+5$ by x^2+2x+1, we get

$$\begin{array}{r|l} & 3x-5 \\ \hline x^2+2x+1 & 3x^3+x^2+2x+5 \\ & 3x^3+6x^2+3x \\ & \underline{(-)\quad(-)\quad(-)} \\ & -5x^2-x+5 \\ & -5x^2-10x-5 \\ & \underline{(+)\quad(+)\quad(+)} \\ & 9x+10 \end{array}$$

$\therefore$ Quotient $=3x-5$ and remainder $=9x+10$

22. We have,

$$\begin{array}{r|l} & 2x^2-x-1 \\ \hline x^2-4x+1 & 2x^4-9x^3+5x^2+3x-9 \\ & 2x^4-8x^3+2x^2 \\ & \underline{(-)\quad(+)\quad(-)} \\ & -x^3+3x^2+3x-9 \\ & -x^3+4x^2-x \\ & \underline{(+)\quad(-)\quad(+)} \\ & -x^2+4x-9 \\ & -x^2+4x-1 \\ & \underline{(+)\quad(-)\quad(+)} \\ & -8 \end{array}$$

$\therefore$ Quotient $=2x^2-x-1$ and remainder $=-8$.

23. On dividing $6x^4+9x^3+17x^2+23x+10$ by $2x^2+3x+1$, we have

$$\begin{array}{r|l} & 3x^2+7 \\ \hline 2x^2+3x+1 & 6x^4+9x^3+17x^2+23x+10 \\ & 6x^4+9x^3+3x^2 \\ & \underline{(-)\quad(-)\quad(-)} \\ & +14x^2+23x+10 \\ & +14x^2+21x+7 \\ & \underline{(-)\quad(-)\quad(-)} \\ & 2x+3 \end{array}$$

But given remainder is $ax+b$

$\therefore\ ax+b=2x+3$

$\Rightarrow\ a=2$ and $b=3$ [On comparing like terms]

24. We have, $f(x)=abx^2+(b^2-ac)x-bc$

$=abx^2+b^2x-acx-bc=bx(ax+b)-c(ax+b)$

$=(ax+b)(bx-c)$

The zeroes of $f(x)$ are given by $f(x)=0$

$\Rightarrow\ (ax+b)(bx-c)=0$

$\Rightarrow\ ax+b=0$ or $bx-c=0$

$\Rightarrow\ x=-\dfrac{b}{a}$ or $x=\dfrac{c}{b}$

Thus, the zeroes of $f(x)$ are : $\alpha=-\dfrac{b}{a}$ and $\beta=\dfrac{c}{b}$

Now, $\alpha+\beta=-\dfrac{b}{a}+\dfrac{c}{b}=\dfrac{ac-b^2}{ab}$ and $\alpha\beta=-\dfrac{b}{a}\times\dfrac{c}{b}=-\dfrac{c}{a}$

Also, $-\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}=-\left(\dfrac{b^2-ac}{ab}\right)=\dfrac{ac-b^2}{ab}$

and, $\dfrac{\text{Constant term}}{\text{Coefficient of } x^2}=-\dfrac{bc}{ab}=-\dfrac{c}{a}$

Hence, sum of the zeroes $=-\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and, product of the zeroes $=\dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

25. Let α, β and γ be the zeros of polynomial $f(x)=x^3-5x^2-16x+80$, such that $\alpha+\beta=0$.

Then, sum of the zeroes $=-\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\Rightarrow\ \alpha+\beta+\gamma=-\left(-\dfrac{5}{1}\right)\Rightarrow\gamma=5$ $[\because\ \alpha+\beta=0]$

Also, product of the zeroes $=-\dfrac{\text{Constant term}}{\text{Coefficient of } x^3}$

$\Rightarrow\ \alpha\beta\gamma=-\dfrac{80}{1}$ $[\because\ \gamma=5]$

$\Rightarrow\ 5\alpha\beta=-80\ \Rightarrow\ \alpha\beta=-16$

$\Rightarrow\ -\alpha^2=-16$ $[\because\ \alpha+\beta=0\ \therefore\ \beta=-\alpha]$

$\Rightarrow\ \alpha=\pm 4$

Now, $\alpha+\beta=0$ and $\alpha=\pm 4\Rightarrow\beta=\mp 4$

Hence, the zeroes are 4, –4 and 5.

26. Since, α and β are the zeroes of the quadratic polynomial, $f(x)=kx^2+4x+4$

$\therefore\quad \alpha + \beta = -\dfrac{4}{k}$ and $\alpha\beta = \dfrac{4}{k}$

Now, $\alpha^2 + \beta^2 = 24 \Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 24$

$\Rightarrow\quad \left(-\dfrac{4}{k}\right)^2 - 2\times\dfrac{4}{k} = 24 \Rightarrow \dfrac{16}{k^2} - \dfrac{8}{k} = 24$

$\Rightarrow\quad 16 - 8k = 24k^2 \Rightarrow 3k^2 + k - 2 = 0$

$\Rightarrow\quad 3k^2 + 3k - 2k - 2 = 0 \Rightarrow 3k(k + 1) - 2(k + 1)$

$\Rightarrow\quad (k + 1)(3k - 2) = 0 \Rightarrow k + 1 = 0$ or $3k - 2 = 0$

$\Rightarrow\quad k = -1$ or $k = 2/3$

Hence, $k = -1$ or $k = 2/3$

OR

Let α and β be the zeroes of required quadratic polynomial.

Then, $\alpha + \beta = \dfrac{5+\sqrt{2}}{5-\sqrt{2}} + \dfrac{5-\sqrt{2}}{5+\sqrt{2}} = \dfrac{(5+\sqrt{2})^2 + (5-\sqrt{2})^2}{(5-\sqrt{2})(5+\sqrt{2})}$

$= \dfrac{25+2+10\sqrt{2}+25+2-10\sqrt{2}}{5^2-(\sqrt{2})^2} = \dfrac{54}{23}$

Also, $\alpha\beta = \dfrac{5+\sqrt{2}}{5-\sqrt{2}} \times \dfrac{5-\sqrt{2}}{5+\sqrt{2}} = 1$

So, required polynomial is given by

x^2 – (sum of zeroes)x + product of zeroes $= x^2 - \dfrac{54}{23}x + 1$

Since, zeroes of $x^2 - \dfrac{54}{23}x + 1$ is same as $23x^2 - 54x + 23$

$\therefore$ Required polynomial is $23x^2 - 54x + 23$.

27. It is given that α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$.

$\therefore\quad \alpha + \beta = -\left(-\dfrac{5}{2}\right) = \dfrac{5}{2}$ and $\alpha\beta = \dfrac{7}{2}$

Let S and P denotes respectively the sum and product of zeroes of the required polynomial.

Then, $S = (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = 5 \times \dfrac{5}{2} = \dfrac{25}{2}$

and, $P = (2\alpha + 3\beta)(3\alpha + 2\beta) = 6(\alpha^2 + \beta^2) + 13\alpha\beta$

$= 6\alpha^2 + 6\beta^2 + 12\alpha\beta + \alpha\beta = 6(\alpha^2 + \beta^2 + 2\alpha\beta) + \alpha\beta$

$= 6(\alpha + \beta)^2 + \alpha\beta = 6\left(\dfrac{5}{2}\right)^2 + \dfrac{7}{2} = \dfrac{75}{2} + \dfrac{7}{2} = 41$

Hence, the required polynomial $g(x)$ is given by

$g(x) = k(x^2 - Sx + P) = k\left(x^2 - \dfrac{25}{2}x + 41\right)$, where k is any non-zero real number.

28. Let $f(x) = kx^2 + 41x + 42$

Given, product of zeroes = 7

$\Rightarrow\quad 42/k = 7 \Rightarrow 42 = 7k$

$\Rightarrow\quad k = 6$

Putting $k = 6$ in polynomial

$p(x) = (k - 4)x^2 + (k + 1)x + 5$, we get

$p(x) = (6 - 4)x^2 + (6 + 1)x + 5$

$\Rightarrow\quad p(x) = 2x^2 + 7x + 5$

For zeroes of $p(x)$, put $2x^2 + 7x + 5 = 0$

$2x^2 + 5x + 2x + 5 = 0$

$\Rightarrow\quad x(2x + 5) + 1(2x + 5) = 0$

$\Rightarrow\quad (x + 1)(2x + 5) = 0$

$\Rightarrow\quad x = -1, x = -5/2$

$\therefore$ zeroes are -1 and $-5/2$.

29. Let α and β be the zeroes of the polynomial $p(x) = x^2 + 2kx + k$

$\therefore\quad \alpha + \beta = -2k/1 = -2k$ ...(i)

and $\alpha\beta = k/1 = k$ ...(ii)

Also, $\alpha = \beta$ (given)

From (i), $\alpha + \alpha = -2k$

$\Rightarrow\quad 2\alpha = -2k \Rightarrow \alpha = -k$ ...(iii)

From (ii), $\alpha \cdot \alpha = k$

$\Rightarrow\quad \alpha^2 = k \Rightarrow (-k)^2 = k$ [Using (iii)]

$\Rightarrow\quad k^2 - k = 0 \Rightarrow k(k - 1) = 0$

$\Rightarrow\quad k = 0$ or $k = 1$

So, the quadratic polynomial $p(x)$ will have equal zeroes at $k = 0$ and $k = 1$.

$\therefore$ $p(x)$ can have equal zeroes for some odd integer $k > 0$.

OR

Let $p(x) = 6x^4 + 8x^3 - 5x^2 + ax + b$ and $g(x) = 2x^2 - 5$

On dividing $p(x)$ by $g(x)$, we have

$$\begin{array}{r|l} & 3x^2 + 4x + 5 \\ \hline 2x^2 - 5 & 6x^4 + 8x^3 - 5x^2 + ax + b \\ & 6x^4 \qquad\quad - 15x^2 \\ & (-) \qquad\qquad (+) \\ \hline & \quad 8x^3 + 10x^2 + ax + b \\ & \quad 8x^3 \qquad\quad - 20x \\ & \quad (-) \qquad\quad (+) \\ \hline & \qquad 10x^2 + (20 + a)x + b \\ & \qquad 10x^2 \qquad\qquad - 25 \\ & \qquad (-) \qquad\qquad (+) \\ \hline & \qquad (20 + a)x + (b + 25) \end{array}$$

$\because$ $p(x)$ is completely divisible by $g(x)$.

$\therefore\quad (20 + a)x + (b + 25) = 0$

$\Rightarrow\quad 20 + a = 0$ and $b + 25 = 0$

$\Rightarrow\quad a = -20$ and $b = -25$.

30. Given, α and β are the zeroes of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$.

$\therefore\quad \alpha + \beta = -\dfrac{\text{Coefficient of } s}{\text{Coefficient of } s^2} = \dfrac{-(-6)}{3} = \dfrac{6}{3} = 2$

and $\alpha\beta = \dfrac{\text{Constant term}}{\text{Coefficient of } s^2} = \dfrac{4}{3}$

Now, $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} + 2\left(\dfrac{1}{\alpha} + \dfrac{1}{\beta}\right) + 3\alpha\beta$

$= \dfrac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\dfrac{\alpha+\beta}{\alpha\beta}\right) + 3\alpha\beta$

$= \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\dfrac{\alpha+\beta}{\alpha\beta}\right) + 3\alpha\beta$

[$\because a^2 + b^2 = (a + b)^2 - 2ab$]

$$= \frac{(2)^2 - 2(4/3)}{4/3} + 2\left(\frac{2}{4/3}\right) + 3 \times \frac{4}{3}$$

[$\because$ $\alpha + \beta = 2$ and $\alpha\beta = 4/3$]

$$= \frac{4 - \frac{8}{3}}{\frac{4}{3}} + 2 \times 2 \times \frac{3}{4} + 4 = \frac{12-8}{3} \times \frac{3}{4} + 3 + 4$$

$$= \frac{4}{3} \times \frac{3}{4} + 7 = 1 + 7 = 8$$

OR

Let the zeroes of the given polynomial $ax^2 + bx + b$ be $m\alpha$ and $n\alpha$.

$\therefore$ Sum of zeroes, $m\alpha + n\alpha = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-b}{a}$

$\Rightarrow$ $\alpha(m + n) = \frac{-b}{a} \Rightarrow \alpha = \frac{-b}{a(m+n)}$ ...(i)

and product of zeroes, $m\alpha \times n\alpha = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{b}{a}$

$\Rightarrow$ $mn\alpha^2 = \frac{b}{a} \Rightarrow mn\left[\frac{b^2}{a^2(m+n)^2}\right] = \frac{b}{a}$ [Using (i)]

$\Rightarrow$ $\frac{mnb}{a(m+n)^2} = 1 \Rightarrow \frac{mn}{(m+n)^2} = \frac{a}{b}$

$\Rightarrow$ $\frac{mn}{m^2 + 2mn + n^2} = \frac{a}{b}$

$\Rightarrow$ $\frac{1}{\frac{m^2}{mn} + \frac{2mn}{mn} + \frac{n^2}{mn}} = \frac{a}{b}$

[Dividing numerator and denominator of LHS by mn]

$\Rightarrow$ $\frac{1}{\frac{m}{n} + 2 + \frac{n}{m}} = \frac{a}{b} \Rightarrow \frac{m}{n} + 2 + \frac{n}{m} = \frac{b}{a}$

$\Rightarrow$ $\left(\sqrt{\frac{m}{n}}\right)^2 + 2\sqrt{\frac{m}{n}}\sqrt{\frac{n}{m}} + \left(\sqrt{\frac{n}{m}}\right)^2 = \frac{b}{a}$ $\left[\because 1 = \sqrt{\frac{m}{n}}\sqrt{\frac{n}{m}}\right]$

$\Rightarrow$ $\left(\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}}\right)^2 = \frac{b}{a}$

$\therefore$ $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \sqrt{\frac{b}{a}}$

[Here, we take a positive square root, because values of $\sqrt{\frac{m}{n}}$ and $\sqrt{\frac{n}{m}}$ are always positive.]

31. Let $p(x) = 4x^4 - 2x^3 - 6x^2 + x - 5$

If we subtract $ax + b$ from $p(x)$, then resulting polynomial $f(x)$ should be exactly divisible by $2x^2 + x - 2$

$\therefore$ $f(x) = 4x^4 - 2x^3 - 6x^2 + x - 5 - (ax + b)$

$= 4x^4 - 2x^3 - 6x^2 + (1 - a)x - (5 + b)$

Now, $f(x)$ must be completely divisible by $2x^2 + x - 2$.

$$\begin{array}{r|l} & 2x^2 - 2x \\ 2x^2 + x - 2 & 4x^4 - 2x^3 - 6x^2 + (1-a)x - (5+b) \\ & 4x^4 + 2x^3 - 4x^2 \\ & (-) \quad (-) \quad (+) \\ \hline & -4x^3 - 2x^2 + (1-a)x - (5+b) \\ & -4x^3 - 2x^2 + 4x \\ & (+) \quad (+) \quad (-) \\ \hline & (1 - a - 4)x - (5+b) \end{array}$$

Since, remainder must be equal to 0.

$\therefore$ $(1 - a - 4)x - (5 + b) = 0$

$\Rightarrow$ $-[(a + 3)x + (5 + b)] = 0$

$\Rightarrow$ $(a + 3)x + (5 + b) = 0$

$\Rightarrow$ $a + 3 = 0$ and $5 + b = 0 \Rightarrow a = -3$ and $b = -5$

$\therefore$ Required number to be subtracted $= ax + b = -3x - 5$.

32. Here, dividend, $p(x) = x^3 - 3x^2 - 3x - 3$,
quotient, $q(x) = x^2 - 4x - 2$ and remainder, $r(x) = 3x - 1$

We know that, $p(x) = g(x)\, q(x) + r(x)$

$\Rightarrow$ $x^3 - 3x^2 - 3x - 3 = g(x) \cdot (x^2 - 4x - 2) + (3x - 1)$

$\Rightarrow$ $x^3 - 3x^2 - 3x - 3 - (3x - 1) = g(x)\,(x^2 - 4x - 2)$

$\Rightarrow$ $g(x) = \frac{x^3 - 3x^2 - 3x - 3 - 3x + 1}{x^2 - 4x - 2} = \frac{x^3 - 3x^2 - 6x - 2}{x^2 - 4x - 2}$

Now, we divide $(x^3 - 3x^2 - 6x - 2)$ by $(x^2 - 4x - 2)$

$$\begin{array}{r|l} & x + 1 \\ x^2 - 4x - 2 & x^3 - 3x^2 - 6x - 2 \\ & x^3 - 4x^2 - 2x \\ & (-) \quad (+) \quad (+) \\ \hline & x^2 - 4x - 2 \\ & x^2 - 4x - 2 \\ & (-) \quad (+) \quad (+) \\ \hline & 0 \end{array}$$

So, $g(x) = x + 1$.

33. Let $p(x) = x^3 - 3\sqrt{5}\,x^2 - 5x + 15\sqrt{5}$

Given, $x - \sqrt{5}$ is a factor of $p(x)$.

For other factors, we divide

$x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5}$ by $x - \sqrt{5}$.

$$\begin{array}{r|l} & x^2 - 2\sqrt{5}\,x - 15 \\ x - \sqrt{5} & x^3 - 3\sqrt{5}\,x^2 - 5x + 15\sqrt{5} \\ & x^3 - \sqrt{5}\,x^2 \\ & (-) \quad (+) \\ \hline & -2\sqrt{5}\,x^2 - 5x + 15\sqrt{5} \\ & -2\sqrt{5}\,x^2 + 10x \\ & (+) \quad (-) \\ \hline & -15x + 15\sqrt{5} \\ & -15x + 15\sqrt{5} \\ & (+) \quad (-) \\ \hline & 0 \end{array}$$

For other zeroes, Put $x^2 - 2\sqrt{5}x - 15 = 0$

$\Rightarrow$ $x^2 - 3\sqrt{5}x + \sqrt{5}x - 15 = 0$

$\Rightarrow$ $x(x - 3\sqrt{5}) + \sqrt{5}(x - 3\sqrt{5}) = 0$

$\Rightarrow$ $(x - 3\sqrt{5})(x + \sqrt{5}) = 0 \Rightarrow x = 3\sqrt{5}, x = -\sqrt{5}$

$\therefore$ All zeroes of $p(x)$ are $\sqrt{5}, 3\sqrt{5}$ and $-\sqrt{5}$.