Triangles

SOLUTIONS

1. **(a) :** We have $\Delta ABC \sim \Delta DEF$

$\therefore \quad \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

Given, $\angle A = 47°, \angle E = 83°$

$\therefore \quad \angle B = 83°$

Now, in ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 47° - 83° = 50°$

2. **(a) :** We have, $\Delta ABC \sim \Delta XYZ$

$$\Rightarrow \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} \Rightarrow \frac{4}{x} = \frac{6}{7.2} = \frac{5}{6} \Rightarrow x = \frac{4 \times 7.2}{6}$$

$\Rightarrow \quad x = 4.8$ cm

3. **(d) :** In triangle CAB, if DE divides CA and CB in the same ratio, then $DE \parallel AB$.

$$\therefore \frac{CD}{DA} = \frac{CE}{EB} \Rightarrow \frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$\Rightarrow \quad 3x^2 + 4x + 9x + 12 = 3x^2 + 19x$

$\Rightarrow \quad 6x = 12 \Rightarrow x = 2$

4. **(b) :** We have $\Delta ABC \sim \Delta DEF$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2}{5}$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

5. **(c) :** Let ΔABC and ΔDEF be the similar triangles and AM and DN are their corresponding altitudes.

$\therefore \quad AM : DN = 4 : 9$

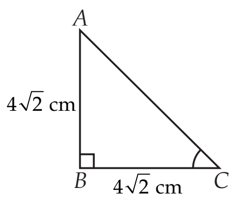
$$\text{Now, } \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AM^2}{DN^2}$$

[$\because$ The ratio of the area of two similar triangles is equal to the ratio of square of their corresponding altitudes]

$$= \left(\frac{AM}{DN}\right)^2 = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

6. **(b) :** Let ΔABC be the right isosceles triangle right angled at B.

$\therefore \quad BC = AB = 4\sqrt{2}$ cm



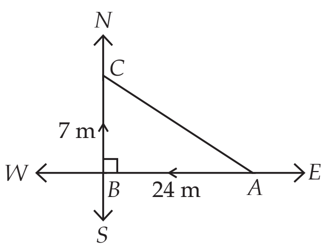
In ΔABC, by Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$= (4\sqrt{2})^2 + (4\sqrt{2})^2 = 32 + 32 = 64$

$\Rightarrow \quad AC = 8$ cm

7. **(c) :** Let A be the starting point and C be the final point.



In right ΔABC, by Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$= 24^2 + 7^2 = 576 + 49 = 62$

$\Rightarrow \quad AC = 25$ m

8. In triangle ABC, $DE \parallel BC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \qquad \text{[By B.P.T.]}$$

$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1} \Rightarrow x^2 - x = x^2 - 4 \Rightarrow x = 4$$

9. We have, $\Delta ABE \cong \Delta ACD$

$\therefore \quad AB = AC$ and $AD = AE$ [By C.P.C.T.] ...(i)

Now, in ΔADE and ΔABC,

$\angle A = \angle A$ [Common]

$\frac{AB}{AD} = \frac{AC}{AE}$ [Using (i)]

$\therefore \quad \Delta ADE \sim \Delta ABC$ [By SAS similarity criterion]

10. Let ΔABC and ΔDEF be two similar triangles such that $AB = 9$ cm.

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}$$

[$\because$ Ratio of corresponding sides of similar triangles is equal to the ratio of their perimeters]

$$\Rightarrow \frac{9}{DE} = \frac{36}{48} \Rightarrow DE = 12 \text{ cm}$$

11. Basic proportionality theorem : If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

12. SAS similarity criterion : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

13. Pythagoras theorem: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

14. In ΔABD and ΔASR, $RS \parallel DB$

$\therefore \quad \angle ABD = \angle ASR$ [Corresponding angles]

$\angle A = \angle A$ [Common]

$\therefore \quad \Delta ABD \sim \Delta ASR$ [By AA similarity criterion]

$$\Rightarrow \frac{AB}{AS} = \frac{AD}{AR} = \frac{BD}{RS} \Rightarrow \frac{3+3}{3} = \frac{x}{y} \Rightarrow x = 2y$$

15. **(i)** **(c) :** Since, $\angle B = \angle D = 90°$, $\angle AMB = \angle CMD$

($\because$ Angle of incident = Angle of reflection)

$\therefore \quad$ By AA similarity criterion, $\Delta ABM \sim \Delta CDM$

(ii) **(a)**

(iii) **(c) :** $\because \Delta ABM \sim \Delta CDM$

$$\therefore \frac{AB}{CD} = \frac{BM}{DM} \Rightarrow \frac{AB}{1.8} = \frac{2.5}{1.5}$$

$\Rightarrow \quad AB = \dfrac{2.5 \times 1.8}{1.5} = 3 \text{ m}$

(iv) (b) : Since, $\Delta ABM \sim \Delta CDM$

$\therefore \quad \angle A = \angle C = 30°$

[$\because$ Corresponding angles of similar triangles are also equal]

(v) (b) : Since, $\Delta ABM \sim \Delta CDM$

$\therefore \quad \dfrac{AB}{CD} = \dfrac{BM}{MD} \Rightarrow \dfrac{AB}{6} = \dfrac{24}{8} \Rightarrow AB = 18 \text{ cm}$

16. (i) (a) : In ΔPAB and ΔPQR,

$\angle P = \angle P$ (Common)

$\angle A = \angle Q$ (Corresponding angles)

By AA similarity criterion, $\Delta PAB \sim \Delta PQR$

$\therefore \quad \dfrac{AB}{QR} = \dfrac{PA}{PQ} \Rightarrow \dfrac{AB}{12} = \dfrac{6}{24} \Rightarrow AB = 3 \text{ m}$

(ii) (d) : Similarly, ΔPCD and ΔPQR are similar.

$\therefore \quad \dfrac{PC}{PQ} = \dfrac{CD}{QR} \Rightarrow \dfrac{14}{24} = \dfrac{CD}{12} \Rightarrow CD = 7 \text{ m}$

(iii) (a) : Area of whole empty land

$= \dfrac{1}{2} \times \text{base} \times \text{height} = \dfrac{1}{2} \times 12 \times 15 = 90 \text{ m}^2$

(iv) (b) : Since, $\Delta PAB \sim \Delta PQR$.

$\therefore \quad \dfrac{ar(\Delta PAB)}{ar(\Delta PQR)} = \left(\dfrac{PA}{PQ}\right)^2 = \left(\dfrac{6}{24}\right)^2 = \dfrac{1}{16}$

$\Rightarrow \quad ar(\Delta PAB) = \dfrac{1}{16} \times 90 = \dfrac{45}{8} \text{ m}^2$

$\left[\because ar(\Delta PQR) = 90 \text{ m}^2\right]$

(v) (d) : Since, $\Delta PCD \sim \Delta PQR$.

$\therefore \quad \dfrac{ar(\Delta PCD)}{ar(\Delta PQR)} = \left(\dfrac{PC}{PQ}\right)^2 = \left(\dfrac{14}{24}\right)^2 = \left(\dfrac{7}{12}\right)^2$

$\Rightarrow \quad ar(\Delta PCD) = \dfrac{90 \times 49}{144} = \dfrac{245}{8} \text{ m}^2$

17. (i) (b) : If ΔAED and ΔBEC, are similar by SAS similarity rule, then their corresponding proportional sides are $\dfrac{BE}{AE} = \dfrac{CE}{DE}$

(ii) (c) : By Pythagoras theorem, we have

$BC = \sqrt{CE^2 + EB^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9}$

$= \sqrt{25} = 5 \text{ cm}$

(iii) (a) : Since ΔADE and ΔBCE are similar.

$\therefore \quad \dfrac{\text{Perimeter of } \Delta ADE}{\text{Perimeter of } \Delta BCE} = \dfrac{AD}{BC}$

$\Rightarrow \quad \dfrac{2}{3} = \dfrac{AD}{5} \Rightarrow AD = \dfrac{5 \times 2}{3} = \dfrac{10}{3} \text{ cm}$

(iv) (b) : $\dfrac{\text{Perimeter of } \Delta ADE}{\text{Perimeter of } \Delta BCE} = \dfrac{ED}{CE}$

$\Rightarrow \quad \dfrac{2}{3} = \dfrac{ED}{4} \Rightarrow ED = \dfrac{4 \times 2}{3} = \dfrac{8}{3} \text{ cm}$

(v) (d) : $\dfrac{\text{Perimeter of } \Delta ADE}{\text{Perimeter of } \Delta BCE} = \dfrac{AE}{BE} \Rightarrow \dfrac{2}{3} BE = AE$

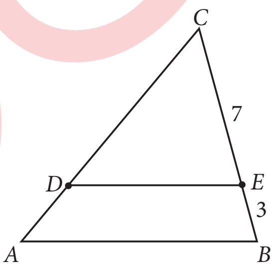
$\Rightarrow \quad AE = \dfrac{2}{3}\sqrt{BC^2 - CE^2}$

Also, in ΔAED, $AE = \sqrt{AD^2 - DE^2}$

18. (i) (b) : Let ΔABC is the triangle formed by both hotels and mountain top. ΔCDE is the triangle formed by both huts and mountain top.

Clearly, $DE \parallel AB$ and so

$\Delta ABC \sim \Delta DEC$ [By AA-similarity criterion]



Now, required ratio = Ratio of their corresponding sides

$= \dfrac{BC}{EC} = \dfrac{10}{7}$ *i.e.*, 10 : 7.

(ii) (c) : Since, $DE \parallel AB$, therefore

$\dfrac{CD}{AD} = \dfrac{CE}{EB} \Rightarrow \dfrac{10}{AD} = \dfrac{7}{3} \Rightarrow AD = \dfrac{10 \times 3}{7} = 4.29$ miles

(iii) (b) : Since, $\Delta ABC \sim \Delta DEC$

$\therefore \quad \dfrac{BC}{EC} = \dfrac{AB}{DE}$ [$\because$ Corresponding sides of similar triangles are proportional]

$\Rightarrow \quad \dfrac{10}{7} = \dfrac{AB}{8} \Rightarrow AB = \dfrac{80}{7} = 11.43$ miles

(iv) (a) : Given, $DC = 5 + BC$.

Clearly, $BC = 10 - 5 = 5$ miles

Now, $CE = \dfrac{7}{10} \times BC = \dfrac{7}{10} \times 5 = 3.5$ miles

(v) (d) : Clearly, the ratio of areas of two triangles (*i.e.*, ΔABC to ΔDEC)

$= \left(\dfrac{BC}{EC}\right)^2 = \left(\dfrac{10}{7}\right)^2 = \dfrac{100}{49}$

$\therefore \quad \text{Required ratio} = \dfrac{ar(\Delta CDE)}{ar(EBAD)} = \dfrac{49}{100 - 49} = \dfrac{49}{51}$

19. In ΔACF, $BP \parallel CF$

$\therefore \frac{AB}{BC} = \frac{AP}{PF}$ [By B.P.T.]

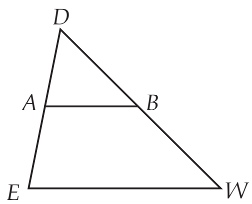
$\Rightarrow \frac{2}{8-2} = \frac{AP}{PF} \Rightarrow \frac{AP}{PF} = \frac{1}{3}$ ...(i)

In ΔAEF, $DP \parallel EF$

$\therefore \frac{AD}{DE} = \frac{AP}{PF}$ [By B.P.T.]

$\Rightarrow \frac{AD}{DE} = \frac{1}{3}$ [Using (i)]

20. In ΔDEW, $AB \parallel EW$,



$\therefore \frac{DA}{AE} = \frac{DB}{BW}$ [By B.P.T.]

$\Rightarrow \frac{DA}{DE-AD} = \frac{DB}{DW-DB}$

$\Rightarrow \frac{4}{12-4} = \frac{DB}{24-DB}$

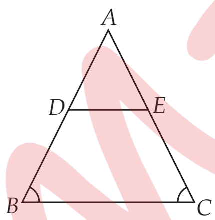
[DA = 4 cm, DE = 12 cm, DW = 24 cm]

$\Rightarrow \frac{4}{8} = \frac{DB}{24-DB} \Rightarrow \frac{1}{2} = \frac{DB}{24-DB}$

$\Rightarrow 24 - DB = 2DB \Rightarrow 24 = 3DB$

$\Rightarrow DB = 24/3 = 8$ cm

21. In ΔABC, we have



$\angle B = \angle C \Rightarrow AC = AB$

$\Rightarrow AE + EC = AD + DB$

$\Rightarrow AE + CE = AD + CE$

[$\because BD = CE$]

$\Rightarrow AE = AD$

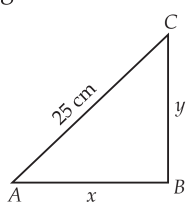
Thus, we have

$AD = AE$ and $BD = CE$

$\therefore \frac{AD}{BD} = \frac{AE}{CE}$

$\Rightarrow DE \parallel BC$ [By the converse of B.P.T.]

22. Let ABC be a right triangle right angled at B. Let $AB = x$ and $BC = y$.



In ΔABC, by Pythagoras theorem,

$AB^2 + BC^2 = AC^2$

$x^2 + y^2 = 25^2$ ...(i)

Now, $x + y + 25 = 60$

[$\because$ Perimeter of triangle is 60 cm]

$\Rightarrow x + y = 35$

$\Rightarrow (x + y)^2 = 35^2$ [On squaring both sides]

$\Rightarrow x^2 + y^2 + 2xy = 35^2$

$\Rightarrow 25^2 + 2xy = 35^2$ [Using (i)]

$\Rightarrow 2xy = 35^2 - 25^2$

$\Rightarrow 2xy = (35 + 25)(35 - 25) \Rightarrow 2xy = 60 \times 10$

$\Rightarrow xy = 300 \Rightarrow \frac{1}{2}xy = 150$

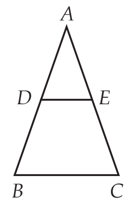
$\Rightarrow$ Area of $\Delta ABC = 150$ cm^2

23. In ΔADE and ΔABC, $DE \parallel BC$

$\Rightarrow \angle ADE = \angle ABC$ and $\angle AED = \angle ACB$

[Corresponding angles]

$\therefore \Delta ADE \sim \Delta ABC$ [By AA similarity criterion]



$\Rightarrow \frac{AB}{AD} = \frac{BC}{DE}$ ...(i)

Given, $\frac{AD}{DB} = \frac{2}{3} \Rightarrow \frac{DB}{AD} = \frac{3}{2}$

$\Rightarrow \frac{DB}{AD} + 1 = \frac{3}{2} + 1 \Rightarrow \frac{DB + AD}{AD} = \frac{3+2}{2}$

$\Rightarrow \frac{AB}{AD} = \frac{5}{2}$ ...(ii)

From (i) and (ii), we get $\frac{BC}{DE} = \frac{5}{2}$

24. Since, $AB \parallel DC$

$\therefore \angle OAB = \angle OCQ$ [Alternate angles]

and $\angle APO = \angle OQC$ [Alternate angles]

Now, in ΔOAP and ΔOCQ,

$\angle OAP = \angle OCQ$ [Proved above]

$\angle APO = \angle OQC$ [Proved above]

$\angle AOP = \angle QOC$ [Vertically opposite angles]

$\therefore \Delta OAP \sim \Delta OCQ$ [By AAA similarity criterion]

$\Rightarrow \frac{OA}{OC} = \frac{OP}{OQ} = \frac{AP}{CQ}$

$\Rightarrow OA \cdot CQ = OC \cdot AP$

25. In ΔABC, $\angle A + \angle B + \angle C = 180°$

[By angle sum property]

$\Rightarrow \angle A = 180° - 30° - 20° = 130°$

Also, $\frac{DE}{AC} = \frac{7}{63} = \frac{1}{9}$ and $\frac{EF}{AB} = \frac{5}{45} = \frac{1}{9}$

Now, in ΔABC and ΔEFD,

$\angle A = \angle E = 130°$

$\frac{DE}{AC} = \frac{EF}{AB}$

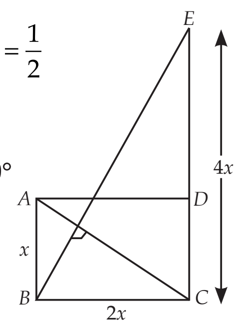
$\therefore \Delta ABC \sim \Delta EFD$ [By SAS similarity criterion]

$\Rightarrow \angle A = \angle E, \angle B = \angle F, \angle C = \angle D$

$\therefore \angle D = 20°$ and $\angle F = 30°$.

26. Let $AB = x$

$\Rightarrow BC = 2x$ and $CE = 4x$



Now, in ΔABC and ΔBCE, $\frac{AB}{BC} = \frac{x}{2x} = \frac{1}{2}$

$\frac{BC}{CE} = \frac{2x}{4x} = \frac{1}{2}$

$\therefore \frac{AB}{BC} = \frac{BC}{CE} = \frac{1}{2}$ and $\angle B = \angle C = 90°$

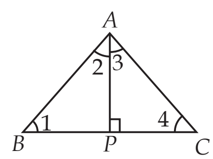
$\therefore \Delta ABC \sim \Delta BCE$

[By SAS similarity criterion]

$\therefore \frac{AB}{BC} = \frac{BC}{CE} = \frac{AC}{BE}$

$\Rightarrow \frac{AC}{BE} = \frac{1}{2}$ or $AC : BE = 1 : 2$

27. In ΔABC, $AP \perp BC$



and $AC^2 = BC^2 - AB^2$

$\Rightarrow BC^2 = AB^2 + AC^2$

$\therefore$ By the converse of Pythagoras theorem, ΔABC is a right triangle right angled at A.

$\Rightarrow \quad \angle 2 + \angle 3 = 90°$ ...(i)

Also, $\angle 1 + \angle 2 = 90°$ ...(ii) [$\because \angle APB = 90°$]

$\Rightarrow \quad \angle 1 = \angle 3$ [Using (i) and (ii)]

Similarly, $\angle 2 = \angle 4$

Now, in ΔBAP and ΔACP

$\angle 1 = \angle 3$ and $\angle 2 = \angle 4$

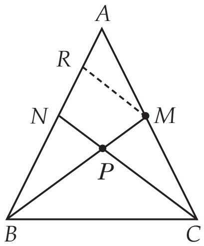
$\therefore \quad \Delta BAP \sim \Delta ACP$ [By AA similarity criterion]

$\Rightarrow \quad \frac{BP}{AP} = \frac{AP}{CP} \Rightarrow AP^2 = BP \times CP \Rightarrow PA^2 = PB \times CP$

28. Given, $AM : MC = 3 : 4$, $BP : PM = 3 : 2$ and $BN = 12$ cm

Draw MR parallel to CN which meets AB at the point R.

In ΔBMR, $PN \parallel MR$



$\therefore \quad \frac{BN}{NR} = \frac{BP}{PM}$ [By B.P.T.]

$\Rightarrow \quad \frac{12}{NR} = \frac{3}{2} \Rightarrow NR = \frac{12 \times 2}{3} = 8$ cm

In ΔANC, $RM \parallel NC$

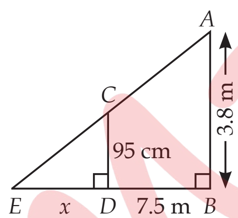
$\therefore \quad \frac{AR}{RN} = \frac{AM}{MC}$ [By B.P.T.]

$\Rightarrow \quad \frac{AR}{8} = \frac{3}{4} \Rightarrow AR = \frac{3 \times 8}{4} = 6$ cm

$\therefore \quad AN = AR + RN = 6 + 8 = 14$ cm

OR

Let AB be the lamp post and CD be the boy after walking 5 seconds. Let $DE = x$ m be the length of his shadow such that $BD = 1.5 \times 5 = 7.5$ m.



In ΔABE and ΔCDE,

$\angle B = \angle D$ [Each equals 90°]

$\angle E = \angle E$ [Common]

$\therefore \quad \Delta ABE \sim \Delta CDE$ [By AA similarity criterion]

$\Rightarrow \quad \frac{BE}{DE} = \frac{AB}{CD} \Rightarrow \frac{7.5 + x}{x} = \frac{3.8}{0.95}$

[$\because AB = 3.8$ m, $CD = 95$ cm $= 0.95$ m and $BE = BD + DE = (7.5 + x)$m]

$\Rightarrow \quad 7.125 + 0.95x = 3.8x \Rightarrow 7.125 = 3.8x - 0.95x$

$\Rightarrow \quad 7.125 = 2.85x \Rightarrow x = 7.125 \div 2.85 \Rightarrow x = 2.5$

Hence, the length of his shadow after 5 seconds is 2.5 m.

29. Since, $\Delta NSQ \cong \Delta MTR$

$\therefore \quad \angle SQN = \angle TRM$

$\Rightarrow \quad \angle Q = \angle R$

In ΔPQR,

$\angle P + \angle Q + \angle R = 180°$ [By angle sum property]

$\Rightarrow \quad \angle Q + \angle Q = 180° - \angle P$

$\Rightarrow \angle Q = \frac{1}{2}(180° - \angle P) \Rightarrow \angle Q = \angle R = 90° - \frac{1}{2}\angle P$ ...(i)

Again, in ΔPST, $\angle 1 = \angle 2$ [Given]

and $\angle P + \angle 1 + \angle 2 = 180°$ [By angle sum property]

$\Rightarrow \quad \angle 1 + \angle 1 = 180° - \angle P$

$\Rightarrow \quad \angle 1 = \frac{1}{2}(180° - \angle P)$

$\Rightarrow \quad \angle 1 = \angle 2 = 90° - \frac{1}{2}\angle P$ ...(ii)

Now, in ΔPTS and ΔPRQ

$\angle 1 = \angle Q$ [From (i) and (ii)]

$\angle P = \angle P$ [Common]

$\therefore \quad \Delta PTS \sim \Delta PRQ$ [By AA similarity criterion]

30. Let $CD = 4x$ and $DA = 3x$

Then $CA = CD + DA = 4x + 3x = 7x$

In ΔABC and ΔAED, we have

$\angle ABC = \angle AED$ [Corresponding angles]

$\angle ACB = \angle ADE$ [Corresponding angles]

$\therefore \quad \Delta ABC \sim \Delta AED$ [By AA similarity criterion]

$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta AED)} = \frac{(CA)^2}{(DA)^2}$

$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta ABC) - ar(\text{quad. } BCDE)} = \frac{(7x)^2}{(3x)^2} = \frac{49}{9}$

$\Rightarrow \quad 9\, ar(\Delta ABC) = 49\, ar(\Delta ABC) - 49\, ar(\text{quad. } BCDE)$

$\Rightarrow \quad 49\, ar(\text{quad. } BCDE) = 40\, ar(\Delta ABC)$

$\Rightarrow \quad \frac{ar(\text{quad.} BCDE)}{ar(\Delta ABC)} = \frac{40}{49}$

Hence, ar (quad. $BCDE$) : $ar(\Delta ABC) = 40 : 49$.

OR

We have,

$ar(\Delta BXY) = 2ar(\text{quad. } ACYX)$

$\Rightarrow \quad ar(\Delta BXY = 2[ar(\Delta BAC) - ar(\Delta BXY)]$

$\Rightarrow \quad ar(\Delta BXY) = 2ar(\Delta BAC) - 2\, ar(\Delta BXY)$

$\Rightarrow \quad 3ar(\Delta BXY) = 2ar(\Delta BAC)$

$\Rightarrow \quad \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{2}{3}$ ...(i)

In ΔBXY and ΔBAC,

$\angle B = \angle B$ [Common]

$\therefore \quad XY \parallel AC \Rightarrow \angle BXY = \angle BAC$ [Corresponding angles]

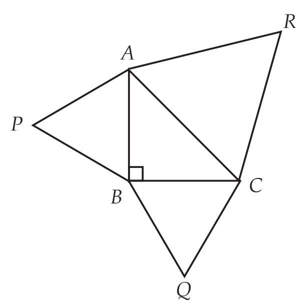
$\therefore \quad \Delta BXY \sim \Delta BAC$ [By AA similarity criterion]

$\Rightarrow \quad \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{BX^2}{BA^2} \Rightarrow \frac{2}{3} = \frac{BX^2}{BA^2}$

$\Rightarrow \quad \frac{BX}{BA} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow 1 - \frac{BX}{BA} = 1 - \frac{\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \quad \frac{BA - BX}{BA} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}} \Rightarrow \frac{AX}{AB} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}} = \frac{3 - \sqrt{6}}{3}$

31. Given, a right triangle ABC right angled at B. Equilateral triangles PAB, QBC and RAC are described on sides AB, BC and CA respectively.



Since, triangles PAB, QBC and RAC are equilateral. Therefore, they are equiangular and hence similar.

$\therefore \dfrac{\text{Area of } \Delta PAB}{\text{Area of } \Delta RAC} + \dfrac{\text{Area of } \Delta QBC}{\text{Area of } \Delta RAC} = \dfrac{AB^2}{AC^2} + \dfrac{BC^2}{AC^2}$

$= \dfrac{AB^2 + BC^2}{AC^2} = \dfrac{AC^2}{AC^2}$

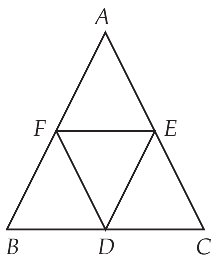
[$\because$ ΔABC is a right triangle with $\angle B = 90°$

$\therefore AC^2 = AB^2 + BC^2$]

$\Rightarrow \dfrac{\text{Area of } \Delta PAB + \text{Area of } \Delta QBC}{\text{Area of } \Delta RAC} = 1$

$\Rightarrow$ Area of ΔPAB + Area of ΔQBC = Area of ΔRAC

32. Given, ΔABC in which D, E and F are the mid-points of sides BC, CA and AB respectively.



Since, F and E are mid-points of AB and AC respectively.

$\therefore$ $FE \parallel BC$

$\Rightarrow$ $\angle AFE = \angle B$ [Corresponding angles]

Thus, in ΔAFE and ΔABC, we have

$\angle AFE = \angle B$ [Proved above]

and $\angle A = \angle A$ [Common]

$\therefore$ $\Delta AFE \sim \Delta ABC$ [By AA similarity criterion]

Similarly, we have

$\Delta FBD \sim \Delta ABC$ and $\Delta EDC \sim \Delta ABC$.

Now, we shall prove that $\Delta DEF \sim \Delta ABC$.

Clearly, $ED \parallel AF$ and $DF \parallel EA$.

$\therefore$ $AFDE$ is a parallelogram.

$\Rightarrow$ $\angle EDF = \angle A$

[$\because$ Opposite angles of a parallelogram are equal]

Similarily, $BDEF$ is a parallelogram.

$\therefore$ $\angle DEF = \angle B$

Thus, in ΔDEF and ΔABC, we have

$\angle EDF = \angle A$ and $\angle DEF = \angle B$

$\therefore$ $\Delta DEF \sim \Delta ABC$ [By AA similarity criterion]

Thus, each one of the triangles AFE, FBD, EDC and DEF is similar to ΔABC.

33. In ΔDFG and ΔDAB,

$AB \parallel FE \Rightarrow \angle 1 = \angle 2$ [Corresponding angles]

$\angle FDG = \angle ADB$ [Common]

$\therefore$ $\Delta DFG \sim \Delta DAB$ [By AA similarity criterion]

$\Rightarrow \dfrac{DF}{DA} = \dfrac{FG}{AB}$ ...(i)

In trapezium $ABCD$, we have

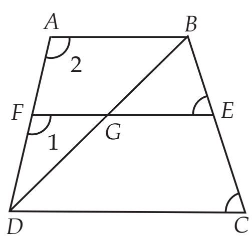
$EF \parallel AB \parallel DC$

$\therefore \dfrac{AF}{DF} = \dfrac{BE}{EC} \Rightarrow \dfrac{AF}{DF} = \dfrac{3}{4}$ $\left[\because \dfrac{BE}{EC} = \dfrac{3}{4}(\text{given})\right]$

$\Rightarrow \dfrac{AF}{DF} + 1 = \dfrac{3}{4} + 1$

$\Rightarrow \dfrac{AF + DF}{DF} = \dfrac{3+4}{4}$

$\Rightarrow \dfrac{AD}{DF} = \dfrac{7}{4} \Rightarrow \dfrac{DF}{AD} = \dfrac{4}{7}$ ...(ii)



From (i) and (ii), we get

$\dfrac{FG}{AB} = \dfrac{4}{7} \Rightarrow FG = \dfrac{4}{7}AB$ ...(iii)

In ΔBEG and ΔBCD,

$EF \parallel CD \Rightarrow \angle BEG = \angle BCD$ [Corresponding angles]

$\angle B = \angle B$ [Common]

$\therefore$ $\Delta BEG \sim \Delta BCD$ [By AA similarity criterion]

$\Rightarrow \dfrac{BE}{BC} = \dfrac{EG}{CD} \Rightarrow \dfrac{3}{7} = \dfrac{EG}{CD}$

$\left[\because \dfrac{BE}{EC} = \dfrac{3}{4} \Rightarrow \dfrac{EC}{BE} = \dfrac{4}{3} \Rightarrow \dfrac{EC}{BE} + 1 = \dfrac{4}{3} + 1 \Rightarrow \dfrac{BC}{BE} = \dfrac{7}{3}\right]$

$\Rightarrow EG = \dfrac{3}{7}CD \Rightarrow EG = \dfrac{3}{7} \times 2AB$ [$\because CD = 2AB$ (Given)]

$\Rightarrow EG = \dfrac{6}{7}AB$ ...(iv)

Adding (iii) and (iv), we get

$FG + EG = \dfrac{4}{7}AB + \dfrac{6}{7}AB \Rightarrow FE = \dfrac{10}{7}AB$

$\Rightarrow$ $7\,FE = 10\,AB$

OR

We have $\dfrac{XP}{PY} = \dfrac{XQ}{QZ}$

$\therefore$ $PQ \parallel YZ$ [By converse of B.P.T.]

In ΔXPQ and ΔXYZ

$\angle XPQ = \angle XYZ$ [Corresponding angles]

$\angle X = \angle X$ [Common]

$\therefore$ $\Delta XPQ \sim \Delta XYZ$ [By AA similarity criterion]

$\Rightarrow \dfrac{ar(\Delta XPQ)}{ar(\Delta XYZ)} = \dfrac{(XP)^2}{(XY)^2} = \dfrac{(XQ)^2}{(XZ)^2} = \dfrac{(PQ)^2}{(YZ)^2}$ ...(i)

[$\because$ The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides]

Now, $\dfrac{XP}{PY} = \dfrac{XQ}{QZ} = \dfrac{3}{1}$ [Given]

$\Rightarrow \dfrac{PY}{XP} = \dfrac{QZ}{XQ} = \dfrac{1}{3} \Rightarrow \dfrac{PY}{XP} + 1 = \dfrac{QZ}{XQ} + 1 = \dfrac{1}{3} + 1$

$\Rightarrow \dfrac{PY + XP}{XP} = \dfrac{QZ + XQ}{XQ} = \dfrac{1+3}{3}$

$\Rightarrow \dfrac{XY}{XP} = \dfrac{XZ}{XQ} = \dfrac{4}{3} \Rightarrow \dfrac{XP}{XY} = \dfrac{XQ}{XZ} = \dfrac{3}{4}$ ...(ii)

From (i) and (ii), we have

$\dfrac{ar(\Delta XPQ)}{ar(\Delta XYZ)} = \left(\dfrac{XP}{XY}\right)^2 = \left(\dfrac{3}{4}\right)^2 = \dfrac{9}{16}$

$\Rightarrow ar(\Delta XPQ) = \dfrac{9}{16} \times ar(\Delta XYZ)$

$\Rightarrow ar(\Delta XPQ) = \dfrac{9}{16} \times 32$ [$\therefore ar(\Delta XYZ) = 32\text{ cm}^2$ (Given)]

$\Rightarrow ar(\Delta XPQ) = 18\text{ cm}^2$

Now, $ar(\text{quad. } PYZQ) = ar(\Delta XYZ) - ar(\Delta XPQ)$

$= (32 - 18)\text{ cm}^2 = 14\text{ cm}^2$

34. In ΔABC, we have $DE \parallel BC$

$\Rightarrow$ $\angle ADE = \angle ABC$ and $\angle AED = \angle ACB$ [Corresponding angles]

$\therefore$ $\Delta ADE \sim \Delta ABC$ [By AA similarity criterion]

$\Rightarrow \dfrac{AD}{AB} = \dfrac{DE}{BC}$ ...(i)

Now, $\frac{AD}{DB}=\frac{5}{4} \Rightarrow \frac{DB}{AD}=\frac{4}{5} \Rightarrow \frac{DB}{AD}+1=\frac{4}{5}+1$

$\Rightarrow \frac{DB+AD}{AD}=\frac{4+5}{5} \Rightarrow \frac{AB}{AD}=\frac{9}{5}$

$\Rightarrow \frac{AD}{AB}=\frac{5}{9}$

$\Rightarrow \frac{DE}{BC}=\frac{5}{9}$ [Using (i)]

In ΔDEF and ΔCBF, we have

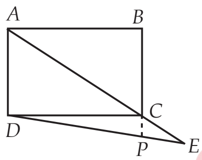
$\angle DFE = \angle CFB$ [Vertically opposite angles]

$\angle DEF = \angle FBC$ [Alternate angles]

$\therefore \Delta DEF \sim \Delta CBF$ [By AA similarity criterion]

$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta CBF)}=\frac{DE^2}{BC^2}=\left(\frac{5}{9}\right)^2=\frac{25}{81}$

OR



Given, $AB = 8$ cm and $BC = 6$ cm

$\therefore AC=\sqrt{8^2+6^2}=10$ cm

Also, $AC : CE = 2 : 1$ [Given]

Produce BC to meet DE at the point P.

Now $AD \parallel BC \Rightarrow AD \parallel CP$

In ΔECP and ΔEAD,

$\angle E = \angle E$ [Common]

$\angle ECP = \angle EAD$ [Corresponding angles]

$\therefore \Delta ECP \sim \Delta EAD$ [By AA similarity criterion]

$\Rightarrow \frac{CP}{AD}=\frac{CE}{AE} \Rightarrow \frac{CP}{6}=\frac{1}{3}$ $\left(\because \frac{CE}{AC}=\frac{1}{2} \Rightarrow \frac{CE}{AE}=\frac{1}{3}\right)$

$\Rightarrow CP = 2$ cm

In right ΔCPD, by Pythagoras theroem

$DP=\sqrt{CD^2+CP^2}=\sqrt{64+4}=\sqrt{68}=2\sqrt{17}$cm

Now, in ΔEAD, $CP \parallel AD$

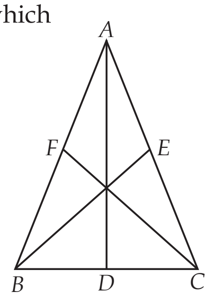
and $\frac{CE}{AC}=\frac{1}{2} \Rightarrow \frac{PE}{PD}=\frac{1}{2}$

$\therefore PE=\frac{1}{2}\times PD=\frac{1}{2}\times 2\sqrt{17}=\sqrt{17}$cm

So, $DE = DP + PE = 2\sqrt{17}+\sqrt{17}=3\sqrt{17}$cm

35. Given, ΔABC in which AD, BE and CF are three medians.

Since, in any triangle, the sum of the squares of any two sides is equal to twice the sum of square of half of the third side and the square of the median bisecting it.



Therefore, taking AD as the median which bisects side BC, we have

$AB^2 + AC^2 = 2[AD^2 + BD^2]$

$\Rightarrow AB^2+AC^2=2\left[AD^2+\left(\frac{1}{2}BC\right)^2\right]$

$\Rightarrow AB^2+AC^2=2\left[AD^2+\frac{1}{4}BC^2\right]$

$\Rightarrow AB^2+AC^2=2AD^2+\frac{1}{2}BC^2$

$\Rightarrow 2(AB^2 + AC^2) = 4AD^2 + BC^2$ ...(i)

Similarly, by taking BE and CF respectively as the medians, we get

$2(AB^2 + BC^2) = 4BE^2 + AC^2$ ...(ii)

and $2(AC^2 + BC^2) = 4CF^2 + AB^2$ ...(iii)

Adding (i), (ii) and (iii), we get

$4(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2) + BC^2 + AC^2 + AB^2$

$\Rightarrow 3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$