EXAM DRILL

Coordinate Geometry

CHAPTER 7

SOLUTIONS

1. **(b) :** Distance between $A(0, 6)$ and $B(0, -2)$,

$$AB=\sqrt{(0-0)^2+(-2-6)^2}=\sqrt{0+(-8)^2}=\sqrt{8^2}=8 \text{ units}$$

2. **(a) :** Let $A(4, p)$ and $B(1, 0)$ be the given points.
Given, $AB = 5$ units

$$\Rightarrow \left(\sqrt{(1-4)^2+(0-p)^2}\right)^2=(5)^2$$

$$\Rightarrow 9+p^2=25 \Rightarrow p^2=16 \Rightarrow p=\pm 4$$

3. **(a) :** Let $A(4, 7)$ be the given point. Suppose the perpendicular from A meet the y-axis at B.
So, the coordinates of B is $(0, 7)$.

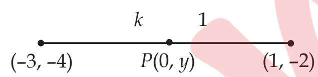
$$\therefore AB=\sqrt{(4-0)^2+(7-7)^2}=\sqrt{16+0}=4 \text{ units}$$

4. **(d) :** Let a point on x-axis be $(x_1, 0)$, then its distance from the point $(2, 3)$ = $\sqrt{(x_1-2)^2+9}=c$

$\Rightarrow (x_1-2)^2=c^2-9 \Rightarrow x_1-2=\sqrt{c^2-9}$

But $c < 3 \Rightarrow c^2-9<0$ $\therefore$ No real point exist.

5. **(c) :** Let the point $P(0, y)$ on the y-axis divides the line segment joining the points $(-3, -4)$ and $(1, -2)$ in the ratio $k : 1$.



Using section formula, we have $(0,y)=\left(\dfrac{k-3}{k+1},\dfrac{-2k-4}{k+1}\right)$

$$\Rightarrow \frac{k-3}{k+1}=0 \Rightarrow k-3=0 \Rightarrow k=3$$

Hence, the required ratio is 3 : 1.

6. **(c) :** Let $A(3, -5)$, $B(-7, 4)$ and $C(10, -k)$ be the given points.
Given, centroid of $\Delta ABC = (k, -1)$

$$\Rightarrow \left(\frac{3-7+10}{3},\frac{-5+4-k}{3}\right)=(k,-1)$$

$$\Rightarrow \left(\frac{6}{3},\frac{-1-k}{3}\right)=(k,-1) \Rightarrow \frac{6}{3}=k \text{ and } \frac{-1-k}{3}=-1$$

$\Rightarrow k = 2$ and $k = 3 - 1 = 2$

7. Let $A(0, 0)$, $B(2, 0)$, $C(0, 3)$ and $D(x, y)$ be the vertices of rectangle $ABCD$.
Since, diagonals of rectangle bisect each other.
$\therefore$ Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{0+0}{2},\frac{0+3}{2}\right)=\left(\frac{2+x}{2},\frac{0+y}{2}\right)$$

$$\Rightarrow \left(0,\frac{3}{2}\right)=\left(\frac{2+x}{2},\frac{y}{2}\right)$$

$$\Rightarrow \frac{2+x}{2}=0 \text{ and } \frac{y}{2}=\frac{3}{2}$$

$\Rightarrow x = -2$ and $y = 3$

8. Coordinate of the centroid G of ΔABC

$$=\left(\frac{-1+0-5}{3},\frac{3+4+2}{3}\right)=\left(\frac{-6}{3},\frac{9}{3}\right)=(-2,3)$$

Since, G lies on the median, $x - 2y + k = 0$

$\therefore -2-2(3)+k=0 \Rightarrow -2-6+k=0 \Rightarrow k=8$

9. Given, $A(x, 2)$, $B(-3, -4)$ and $C(7, -5)$ are collinear.
$\therefore$ Area of $\Delta ABC = 0$

$$\Rightarrow \frac{1}{2}[x(-4+5)+(-3)(-5-2)+7(2+4)]=0$$

$$\Rightarrow \frac{1}{2}[x+(-3)(-7)+7(6)]=0 \Rightarrow \frac{1}{2}[x+21+42]=0$$

$$\Rightarrow x+63=0 \Rightarrow x=-63$$

10. Required distance

$$=\sqrt{[c+a-(b+c)]^2+[a+b-(c+a)]^2}$$

$$=\sqrt{[c+a-b-c]^2+[a+b-c-a]^2}$$

$$=\sqrt{(a-b)^2+(b-c)^2}=\sqrt{a^2+2b^2+c^2-2ab-2bc} \text{ units}$$

11. Here, point A lies on Y-axis, so its abscissa is zero and given its ordinate is 5, therefore, its coordinates are $A(0, 5)$.

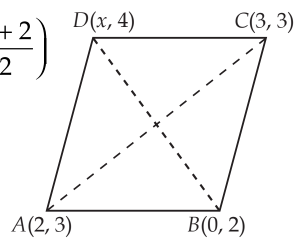
Now, $AB=\sqrt{(-5-0)^2+(3-5)^2}=\sqrt{25+4}=\sqrt{29}$ units

12. Diameter of circle, $d=\sqrt{(2-24)^2+(23-1)^2}$

$$=\sqrt{(-22)^2+(22)^2}=\sqrt{(22)^2(1+1)}=22\sqrt{2} \text{ units}$$

$\therefore$ Radius of a circle, $r=\dfrac{d}{2}=\dfrac{22\sqrt{2}}{2}=11\sqrt{2}$ units

13. As the diagonals of parallelogram bisect each other.
$\therefore$ Mid-point of AC = Mid-point of BD



$$\Rightarrow \left(\frac{2+3}{2},\frac{3+3}{2}\right)=\left(\frac{x+0}{2},\frac{4+2}{2}\right)$$

$$\Rightarrow \left(\frac{5}{2},3\right)=\left(\frac{x}{2},3\right)$$

$$\Rightarrow \frac{x}{2}=\frac{5}{2} \Rightarrow x=5$$

14. Let coordinates of P and Q be $(x, 0)$ and $(0, y)$ respectively.
Let $M(-2, -6)$ be the mid-point of PQ.
$\therefore$ By mid-point formula, we have

$$-2 = \frac{x+0}{2} \text{ and } -6 = \frac{0+y}{2} \Rightarrow -4 = x \text{ and } -12 = y$$

$\therefore$ Points are $P(-4, 0)$ and $Q(0, -12)$.

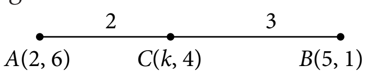
15. Let $A(1, 2)$, $B(4, 3)$ and $C(6, 6)$ and $D(x, y)$ be the vertices of parallelogram.
Since diagonals AC and BD bisect each other.
$\therefore$ Mid-point of BD = Mid-point of AC

$$\Rightarrow \left(\frac{4+x}{2}, \frac{3+y}{2}\right) = \left(\frac{1+6}{2}, \frac{2+6}{2}\right)$$

$$\Rightarrow \left(\frac{4+x}{2}, \frac{3+y}{2}\right) = \left(\frac{7}{2}, \frac{8}{2}\right)$$

$\Rightarrow$ $4 + x = 7$ and $3 + y = 8$ $\Rightarrow x = 3$ and $y = 5$
$\therefore$ Coordinates of D are $(3, 5)$.

16. We have, $A(2, 6)$, $B(5, 1)$ and $C(k, 4)$ divides the given line segment in the ratio $2 : 3$.



Using section formula, we have

$$(k, 4) = \left(\frac{2\times5+3\times2}{2+3}, \frac{2\times1+3\times6}{2+3}\right) = \left(\frac{16}{5}, \frac{20}{5}\right) \Rightarrow k = \frac{16}{5}$$

17. **(i)** **(a) :** We have, $OA = 2\sqrt{2}$ km

$\Rightarrow \sqrt{2^2 + y^2} = 2\sqrt{2}$

$\Rightarrow 4 + y^2 = 8 \Rightarrow y^2 = 4$

$\Rightarrow y = 2$ $\qquad$ ($\because$ $y = -2$ is not possible)

(ii) **(c) :** We have, $OB = 8\sqrt{2}$

$\Rightarrow \sqrt{x^2 + 8^2} = 8\sqrt{2}$

$\Rightarrow x^2 + 64 = 128 \Rightarrow x^2 = 64$

$\Rightarrow x = 8$ $\qquad$ ($\because$ $x = -8$ is not possible)

(iii) **(c) :** Coordinates of A and B are $(2, 2)$ and $(8, 8)$ respectively, therefore coordinates of point M are $\left(\frac{2+8}{2}, \frac{2+8}{2}\right)$, *i.e.*, $(5, 5)$

(iv) **(d) :** Let A divides OM in the ratio $k : 1$.

Then, $2 = \frac{5k+0}{k+1} \Rightarrow 2k + 2 = 5k \Rightarrow 3k = 2 \Rightarrow k = \frac{2}{3}$

$\therefore$ Required ratio = $2 : 3$

(v) **(b) :** Since M is the mid-point of A and B therefore $AM = MB$. Hence, he should try his luck moving towards B.

18. Consider the house is at origin $(0, 0)$, then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively $(2, 3)$, $(-4, -6)$, $(6, -8)$ and $(-6, 8)$

(i) **(d) :** Since, grocery store is at $(2, 3)$ and food cart is at $(6, -8)$

$\therefore$ Required distance $= \sqrt{(6-2)^2 + (-8-3)^2}$

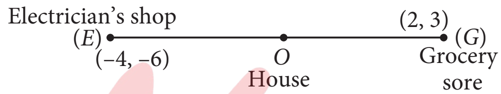
$= \sqrt{4^2 + 11^2} = \sqrt{16+121} = \sqrt{137}$ cm

(ii) **(b) :** Required distance

$= \sqrt{(-6)^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$ cm

(iii) **(c) :** Let O divides EG in the ratio $k : 1$, then

$0 = \frac{2k-4}{k+1}$



$\Rightarrow 2k = 4$

$\Rightarrow k = 2$

Thus, O divides EG in the ratio $2 : 1$
Hence, required ratio = $OG : OE$ *i.e.*, $1 : 2$

(iv) **(c) :** Since, $(0, 0)$ is the mid-point of $(-6, 8)$ and $(6, -8)$, therefore both bus stand and food cart are at equal distances from the house.
Hence, required ratio is $1 : 1$.

(v) **(d) :** Mid-point of grocery store and electrician's shop is $\left(\frac{2-4}{2}, \frac{3-6}{2}\right)$, *i.e.*, $\left(-1, \frac{-3}{2}\right)$

Thus, the diagonals does not bisect each other
[$\because$ Mid-point are not same]
Hence, they form a quadrilateral.

19. **(i)** **(c) :** The coordinates of point A are $(9, 27)$, therefore its distance from x-axis = 27 units.

(ii) **(b) :** Coordinates of B and C are $(4, 19)$ and $(14, 19)$

$\therefore$ Required distance $= \sqrt{(14-4)^2 + (19-19)^2}$

$= \sqrt{10^2} = 10$ units

(iii) **(c) :** Coordinates of F and G are $(2, 6)$ and $(16, 6)$ respectively.

$\therefore$ Required distance $= \sqrt{(16-2)^2 + (6-6)^2}$

$= \sqrt{14^2} = 14$ units

(iv) **(a) :** Since the coordinates of F and G are $(2, 6)$ and $(16, 6)$ respectively therefore mid-point of FG is

$$\left(\frac{2+16}{2}, \frac{6+6}{2}\right) = (9, 6)$$

Thus, the mid-point of FG will lie on the line represented by $x = 9$.

(v) **(d) :** Coordinates of L and N are $(6, 4)$ and $(7, 1)$ respectively.

Length of $LN = \sqrt{(7-6)^2 + (1-4)^2}$

$= \sqrt{1+9} = \sqrt{10}$ units

$\Rightarrow$ Length of $MP = \sqrt{10}$ units

Now, perimeter of $LMPN = LN + LM + MP + NP$

$= \sqrt{10} + 6 + \sqrt{10} + 4 = (2\sqrt{10} + 10)$ units

[$\because$ $LM = 12 - 6 = 6$ units and $NP = 11 - 7 = 4$ units]

20. **(i)** **(a) :** Coordinates of Q are (9, 5).

$\therefore$ Distance of point Q from y-axis = 9 units

(ii) **(b) :** Coordinates of point U are (8, 2).

(iii) **(d) :** We have, $P(2, 5)$ and $Q(9, 5)$

$\therefore$ $PQ = \sqrt{(2-9)^2 + (5-5)^2} = \sqrt{49+0} = 7$ units

(iv) **(c) :** Point $A(x, y)$ is equidistant from $R(3, 8)$ and $T(3, 2)$.

$\therefore$ $AR = AT \Rightarrow AR^2 = AT^2$

$\Rightarrow$ $(x-3)^2 + (y-8)^2 = (x-3)^2 + (y-2)^2$

$\Rightarrow$ $y^2 + 64 - 16y = y^2 + 4 - 4y$

$\Rightarrow$ $16y - 4y = 64 - 4 \Rightarrow 12y = 60 \Rightarrow y = 5$

(v) **(d) :** Length of TU = 5 units and of TL = 2 units

$\therefore$ Perimeter of image of a rectangular face = 2(5 + 2) = 14 units

21. We have, $P(x, y)$, $A(5, 1)$ and $B(-1, 5)$

Given, $AP = BP \Rightarrow AP^2 = BP^2$

$\Rightarrow$ $(x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$

$\Rightarrow$ $x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 + 2x + y^2 + 25 - 10y$

$\Rightarrow$ $-10x - 2y = 2x - 10y \Rightarrow -12x = -8y \Rightarrow 3x = 2y$

22. Here, $AB = \sqrt{(4-1)^2 + (5-2)^2}$ units

$= \sqrt{18}$ units $= 3\sqrt{2}$ units

$BC = \sqrt{(-1-4)^2 + (0-5)^2}$ units

$= \sqrt{50}$ units $= 5\sqrt{2}$ units

and $AC = \sqrt{(-1-1)^2 + (0-2)^2}$ units

$= \sqrt{4+4}$ units $= 2\sqrt{2}$ units

Now, $AB + AC = (3\sqrt{2} + 2\sqrt{2})$ units $= 5\sqrt{2}$ units $= BC$

Hence, A, B and C lie on a straight line. In other words, A, B, C are collinear.

23. Let $A(6, -2)$ and $B(-2, y)$ be the given points.

Length of the line segment AB = 10 units (Given)

$\Rightarrow$ $\sqrt{(-2-6)^2 + (y+2)^2} = 10$

$\Rightarrow$ $\sqrt{64 + y^2 + 4y + 4} = 10$

$\Rightarrow$ $y^2 + 4y + 68 = 100 \Rightarrow y^2 + 4y - 32 = 0$

$\Rightarrow$ $y^2 + 8y - 4y - 32 = 0 \Rightarrow y(y+8) - 4(y+8) = 0$

$\Rightarrow$ $(y+8)(y-4) = 0 \Rightarrow y = 4$ or $y = -8$

So, ordinate of B will be 4 or –8.

OR

The given points are $A(0, 2)$, $B(3, p)$ and $C(p, 5)$.

Since, A is equidistant from B and C.

$\therefore$ $AB = AC \Rightarrow AB^2 = AC^2$

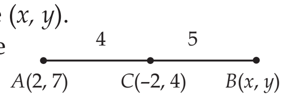
$\Rightarrow$ $(3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$\Rightarrow$ $9 + p^2 + 4 - 4p = p^2 + 9 \Rightarrow 4 - 4p = 0$

$\Rightarrow$ $4p = 4 \Rightarrow p = 1$

24. Let the coordinates of B be (x, y).

Using section formula, we have coordinates of C are



$\left(\dfrac{4x + 5(2)}{4+5}, \dfrac{4y + 5(7)}{4+5}\right) = \left(\dfrac{4x+10}{9}, \dfrac{4y+35}{9}\right)$

Also, the coordinates of C are (–2, 4). (Given)

$\therefore$ $\dfrac{4x+10}{9} = -2$ and $\dfrac{4y+35}{9} = 4$

$\Rightarrow$ $4x + 10 = -18$ and $4y + 35 = 36$

$\Rightarrow$ $4x = -18 - 10$ and $4y = 36 - 35$

$\Rightarrow$ $4x = -28$ and $4y = 1$

$\Rightarrow$ $x = -7$ and $y = \dfrac{1}{4}$

25. Let $A(a, a^2)$, $B(b, b^2)$ and $C(0, 0)$

For the points A, B and C to be collinear, area of ΔABC must be zero.

Now, area of $\Delta ABC = \dfrac{1}{2}[a(b^2 - 0) + b(0 - a^2) + 0(a^2 - b^2)]$

$= \dfrac{1}{2}(ab^2 - ba^2) = \dfrac{ab}{2}(b - a) \neq 0$ [Since, $a \neq b \neq 0$]

$\therefore$ A, B and C are not collinear.

26. Let $A(t, t-2)$, $B(t+2, t+2)$, $C(t+3, t)$ be the given points.

$\therefore$ Area of triangle ABC

$= \dfrac{1}{2}[t(t+2-t) + (t+2)(t-t+2) + (t+3)(t-2-t-2)]$

$= \dfrac{1}{2}[2t + 2t + 4 - 4t - 12] = \dfrac{1}{2}(-8)$

= 4 sq. units [$\therefore$ Area of triangle can't be negative]

$\Rightarrow$ Area of triangle is independent of t.

27. Given, points $A(-1, -4)$, $B(b, c)$ and $C(5, -1)$ are collinear.

$\therefore$ Area of $\Delta ABC = 0$

$\Rightarrow$ $\dfrac{1}{2}[-1(c - (-1)) + b(-1 - (-4)) + 5(-4 - c)] = 0$

$\Rightarrow$ $-c - 1 - b + 4b - 20 - 5c = 0$

$\Rightarrow$ $3b - 6c = 21 \Rightarrow b - 2c = 7$ ...(i)

Also, given that $2b + c = 4$

$\Rightarrow$ $2(7 + 2c) + c = 4$ (From (i))

$\therefore$ $14 + 4c + c = 4$

$\Rightarrow$ $5c = -10 \Rightarrow c = -2$

Put $c = -2$ in (i), we get $b = 7 + 2c = 7 - 4 = 3$

Hence, we have $b = 3$, $c = -2$

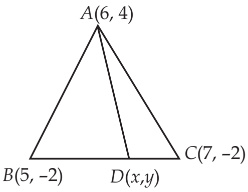
28. Using distance formula, we have

$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$ units

$AC = \sqrt{(6-7)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$ units

$BC = \sqrt{(5-7)^2 + (-2+2)^2}$

$= \sqrt{4+0} = 2$ units



Now, $AB = AC \neq BC$.
So, ΔABC is an isosceles triangle.
Let D (x, y) be the mid-point of BC.

Using mid-point formula, we have

$$(x, y) = \left(\frac{5+7}{2}, \frac{-2-2}{2}\right) = \left(\frac{12}{2}, \frac{-4}{2}\right) = (6, -2)$$

$\therefore$ Coordinates of D are (6, –2)

$\therefore$ Length of median, $AD = \sqrt{(6-6)^2 + (-2-4)^2}$

$= \sqrt{0+36} = 6$ units

OR

Since C is equidistant from A and B.

$\therefore$ $AC = CB \Rightarrow AC^2 = CB^2$

$\Rightarrow$ $(3+2)^2 + (-1-3)^2 = (x+2)^2 + (8-3)^2$

$\Rightarrow$ $25 + 16 = x^2 + 4 + 4x + 25$

$\Rightarrow$ $x^2 + 4x - 12 = 0 \Rightarrow x^2 + 6x - 2x - 12 = 0$

$\Rightarrow$ $x(x+6) - 2(x+6) = 0 \Rightarrow (x+6)(x-2) = 0$

$\Rightarrow$ $x = -6$ or $x = 2$

Now, using distance formula

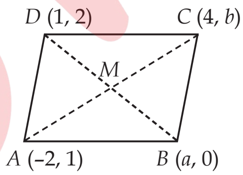
$BC = \sqrt{(-6+2)^2 + (8-3)^2} = \sqrt{16+25} = \sqrt{41}$ units

or $\sqrt{(2+2)^2 + (8-3)^2} = \sqrt{16+25} = \sqrt{41}$ units

$AB = \sqrt{(-6-3)^2 + (8+1)^2} = \sqrt{81+81} = 9\sqrt{2}$ units

or $\sqrt{(2-3)^2 + (8+1)^2} = \sqrt{1+81} = \sqrt{82}$ units

29. Let the given points are $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$.



We know, diagonals of a parallelogram bisect each other.
$\therefore$ Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{-2+4}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{0+2}{2}\right)$$

$$\Rightarrow \left(\frac{2}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{2}{2}\right) \Rightarrow \frac{2}{2} = \frac{a+1}{2}$$

$\Rightarrow$ $1 + a = 2$ $\Rightarrow$ $a = 1$ and $\frac{1+b}{2} = \frac{2}{2}$

$\Rightarrow$ $b + 1 = 2$ $\Rightarrow$ $b = 1$

Hence, $a = 1$, $b = 1$

Now, $AB = CD$ and $BC = AD$

($\because$ Opposite sides of a parallelogram are equal)

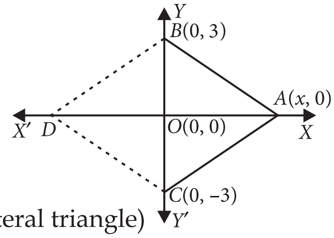
$\therefore$ $AB = CD = \sqrt{(-2-1)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10}$ units

and $BC = AD = \sqrt{(1-4)^2 + (0-1)^2} = \sqrt{9+1} = \sqrt{10}$ units

30. $\because$ O is the mid-point of the base BC.

$\therefore$ Coordinates of point B are (0, 3)

So, $BC = \sqrt{(0-0)^2 + (-3-3)^2} = \sqrt{6^2} = 6$ units.



Let the coordinates of point A be $(x, 0)$.

$\therefore$ $AB = \sqrt{(0-x)^2 + (3-0)^2}$

$= \sqrt{x^2 + 9}$

Also, $AB = BC$

($\because$ ΔABC is an equilateral triangle)

$\Rightarrow$ $\sqrt{x^2+9} = 6$ $\Rightarrow$ $x^2 + 9 = 36$

$\Rightarrow$ $x^2 = 27$ $\Rightarrow$ $x = \pm 3\sqrt{3}$

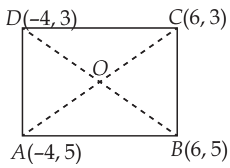
$\therefore$ Coordinates of point $A = (x, 0) = (3\sqrt{3}, 0)$

Since, $BACD$ is a rhombus.

$\therefore$ $AB = AC = CD = DB$

$\therefore$ Coordinates of point $D = (-3\sqrt{3}, 0)$

31. We have $ABCD$ is a rectangle, where AC and BD are its diagonal.



Now, $AC = \sqrt{[6-(-4)]^2 + (3-5)^2}$

$= \sqrt{(10)^2 + (-2)^2}$

$= \sqrt{100+4} = \sqrt{104}$ units

$BD = \sqrt{(-4-6)^2 + (3-5)^2} = \sqrt{(-10)^2 + (-2)^2}$

$= \sqrt{100+4} = \sqrt{104}$ units

$\Rightarrow$ $AC = BD$

Hence, diagonals of rectangle $ABCD$ are equal.

Let O is the mid-point of both AC and BD.

Using mid-point formula, we have

coordinates of O from $AC = \left(\frac{6+(-4)}{2}, \frac{3+5}{2}\right)$

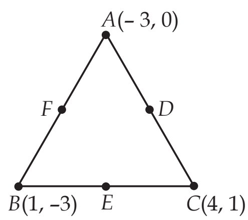
$= \left(\frac{2}{2}, \frac{8}{2}\right) = (1, 4)$

Coordinates of O from $BD = \left(\frac{-4+6}{2}, \frac{3+5}{2}\right)$

$= \left(\frac{2}{2}, \frac{8}{2}\right) = (1, 4)$

$\Rightarrow$ AC and BD bisect each other at O.

OR



Let D, E and F be the mid-points of the sides AC, BC and AB respectively.

Then the coordinates of D are

$$\left(\frac{-3+4}{2}, \frac{0+1}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

Coordinates of E are $\left(\frac{1+4}{2}, \frac{-3+1}{2}\right) = \left(\frac{5}{2}, \frac{-2}{2}\right) = \left(\frac{5}{2}, -1\right)$

Coordinates of F are $\left(\frac{-3+1}{2}, \frac{0-3}{2}\right) = \left(\frac{-2}{2}, \frac{-3}{2}\right) = \left(-1, \frac{-3}{2}\right)$

Using distance formula, lengths of medians are

$\therefore\ AE = \sqrt{\left[\frac{5}{2} - (-3)\right]^2 + (-1-0)^2}$

$= \sqrt{\left(\frac{11}{2}\right)^2 + 1} = \sqrt{\frac{121}{4} + 1} = \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2}$ units

$BD = \sqrt{\left(\frac{1}{2} - 1\right)^2 + \left[\frac{1}{2} - (-3)\right]^2} = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{7}{2}\right)^2}$

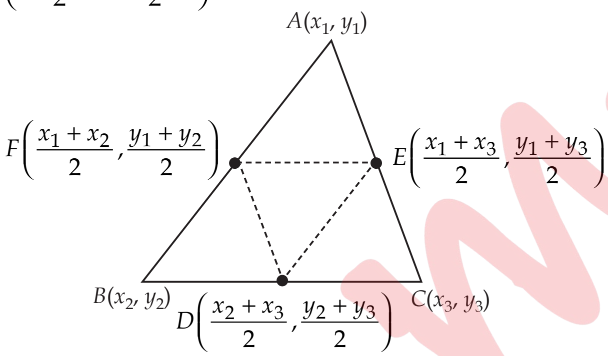
$= \sqrt{\frac{1}{4} + \frac{49}{4}} = \sqrt{\frac{50}{4}} = \frac{\sqrt{50}}{2}$ units

$CF = \sqrt{(-1-4)^2 + \left(\frac{-3}{2} - 1\right)^2} = \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{100+25}{4}}$

$= \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2}$ units

32. Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ be the vertices of ΔABC. Then, the coordinates of D, E and F are

$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$, $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$ and $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ respectively.



Δ_1 = Area of ΔABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Δ_2 = Area of ΔDEF

$= \frac{1}{2}\left[\left(\frac{x_2+x_3}{2}\right)\left(\frac{y_1+y_3}{2} - \frac{y_1+y_2}{2}\right) + \left(\frac{x_1+x_3}{2}\right)\left(\frac{y_1+y_2}{2} - \frac{y_2+y_3}{2}\right) + \left(\frac{x_1+x_2}{2}\right)\left(\frac{y_2+y_3}{2} - \frac{y_1+y_3}{2}\right)\right]$

$\Rightarrow\ \Delta_2 = \frac{1}{8}[(x_2 + x_3)(y_3 - y_2) + (x_1 + x_3)(y_1 - y_3) + (x_1 + x_2)(y_2 - y_1)]$

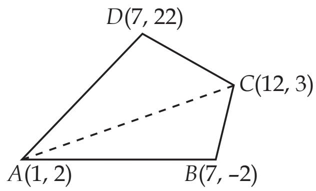
$\Rightarrow\ \Delta_2 = \frac{1}{8}[x_1(y_1 - y_3 + y_2 - y_1) + x_2(y_3 - y_2 + y_2 - y_1) + x_3(y_3 - y_2 + y_1 - y_3)]$

$\Rightarrow\ \Delta_2 = \frac{1}{8}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\Rightarrow\ \Delta_2 = \frac{1}{4}$ (Area of ΔABC) $= \frac{1}{4}\Delta_1$

Hence, Area of $\Delta DEF = \frac{1}{4}$ (Area of ΔABC)

33. We have, $A(1, 2)$, $B(7, -2)$, $C(12, 3)$ and $D(7, 22)$



Area of quadrilateral $ABCD$
= Area of ΔABC
+ Area of ΔACD ...(i)

Now, Area of ΔABC =

$\frac{1}{2}$ [1 (– 2 – 3) + 7(3 – 2) + 12 (2 + 2)]

$= \frac{1}{2}[1(-5) + 7(1) + 12(4)] = \frac{1}{2}[-5 + 7 + 48]$

$= \frac{1}{2}[50] = 25$ sq. units

Also, area of $\Delta ACD = \frac{1}{2}[1(3 - 22) + 12(22 - 2) + 7(2 - 3)]$

$= \frac{1}{2}[1(-19) + 12(20) + 7(-1)] = \frac{1}{2}[-19 + 240 - 7]$

$= \frac{1}{2}[214] = 107$ sq. units

From (i), we have

Area of quadrilateral $ABCD$ = (25 + 107) sq. units = 132 sq. units

34. Let the vertices of an equilateral triangle are $A(-6, 5)$, $B(6, 5)$ and $C(x, y)$.

We know that, in an equilateral triangle all three sides are equal.

$\therefore\ AB = BC = CA \Rightarrow AB^2 = BC^2 = CA^2$

Consider, $AB^2 = BC^2$

$\Rightarrow\ (6 + 6)^2 + (5 - 5)^2 = (x - 6)^2 + (y - 5)^2$

$\Rightarrow\ 144 + 0 = x^2 + 36 - 12x + y^2 + 25 - 10y$

$\Rightarrow\ x^2 + y^2 - 12x - 10y + 61 = 144$

$\Rightarrow\ x^2 + y^2 - 12x - 10y = 83$ ...(i)

Now, consider $AB^2 = CA^2$

$\Rightarrow\ (6 + 6)^2 + (5 - 5)^2 = (x + 6)^2 + (y - 5)^2$

$\Rightarrow\ 144 = 36 + x^2 + 12x + 25 + y^2 - 10y$

$\Rightarrow\ x^2 + y^2 + 12x - 10y + 61 = 144$

$\Rightarrow\ x^2 + y^2 + 12x - 10y = 83$ ...(ii)

From (i) and (ii) we get

$x^2 + y^2 - 12x - 10y = x^2 + y^2 + 12x - 10y$

$\Rightarrow\ 12x + 12x = 0 \Rightarrow 24x = 0 \Rightarrow x = 0$

Puting $x = 0$ in (i), we get

$0 + y^2 - 12(0) - 10y = 83$

$\Rightarrow\ y^2 - 10y - 83 = 0$

$\therefore\ y = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-83)}}{2 \times 1}$

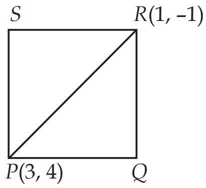
$\Rightarrow\ y = \frac{10 \pm \sqrt{100 + 332}}{2}$

$\Rightarrow\ y = \frac{10 \pm \sqrt{432}}{2}$

$\therefore\ y = \frac{10 + \sqrt{432}}{2}$ or $y = \frac{10 - \sqrt{432}}{2}$

Hence, the third vertex is $\left(0, \frac{10+\sqrt{432}}{2}\right)$ or $\left(0, \frac{10-\sqrt{432}}{2}\right)$.

OR



Let *PQRS* be a square and let *P*(3, 4) and *R*(1, –1) be the given opposite angular points.
Let *Q*(*x*, *y*) be the unknown vertex.
Since, all sides of square are equal.

$\therefore \quad PQ = QR$

$\Rightarrow \quad PQ^2 = QR^2$

$\Rightarrow \quad (x-3)^2 + (y-4)^2 = (x-1)^2 + (y+1)^2$

$\Rightarrow \quad x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 - 2x + 1 + y^2 + 2y + 1$

$\Rightarrow \quad -6x - 8y + 25 = -2x + 2y + 2$

$\Rightarrow \quad -6x + 2x - 8y - 2y = 2 - 25$

$\Rightarrow \quad -4x - 10y = -23 \Rightarrow \quad 4x + 10y = 23$

$\Rightarrow \quad x = \frac{23-10y}{4}$ ...(i)

In right angled triangle *PQR*, we have

$PQ^2 + QR^2 = PR^2$

$\Rightarrow \quad (x-3)^2 + (y-4)^2 + (x-1)^2 + (y+1)^2 = (3-1)^2 + (4+1)^2$

$\Rightarrow \quad x^2 - 6x + 9 + y^2 - 8y + 16 + x^2 - 2x + 1 + y^2 + 2y + 1 = 4 + 25$

$\Rightarrow \quad 2x^2 + 2y^2 - 8x - 6y + 27 = 29$

$\Rightarrow \quad 2x^2 + 2y^2 - 8x - 6y = 2$

$\Rightarrow \quad x^2 + y^2 - 4x - 3y - 1 = 0$ ...(ii)

Substitute the value of *x* from (i) into (ii), we get

$$\left(\frac{23-10y}{4}\right)^2 + y^2 - 4\left(\frac{23-10y}{4}\right) - 3y - 1 = 0$$

$$\Rightarrow \left(\frac{529+100y^2-460y}{16}\right) + y^2 - 23 + 10y - 3y - 1 = 0$$

$$\Rightarrow \left(\frac{529+100y^2-460y}{16}\right) + y^2 + 7y - 24 = 0$$

$$\Rightarrow \frac{529+100y^2-460y+16y^2+112y-384}{16} = 0$$

$\Rightarrow \quad 116y^2 - 348y + 145 = 0$

$\Rightarrow \quad 4y^2 - 12y + 5 = 0$ [On dividing by 29]

$\Rightarrow \quad 4y^2 - 10y - 2y + 5 = 0$

$\Rightarrow \quad 2y(2y-5) - 1(2y-5) = 0$

$\Rightarrow \quad (2y-5)(2y-1) = 0$

$\Rightarrow \quad y = \frac{5}{2}$ or $y = \frac{1}{2}$

On putting $y = \frac{1}{2}$ in (i), we get

$$x = \frac{23-10\left(\frac{1}{2}\right)}{4} = \frac{23-5}{4} = \frac{18}{4} = \frac{9}{2}$$

On putting $y = \frac{5}{2}$ in (i) we get

$$x = \frac{23-10\left(\frac{5}{2}\right)}{4} = \frac{23-25}{4} = \frac{-2}{4} = \frac{-1}{2}$$

Hence, the other two vertices are $\left(\frac{9}{2}, \frac{1}{2}\right)$ and $\left(\frac{-1}{2}, \frac{5}{2}\right)$.

35. Given, ΔAOB is a right angle triangle right angled at *O* and *AB* is hypotenuse and *C* is the mid-point of *AB*.
Let the coordinates of *B* and *A* are (0, *b*) and (*a*, 0) respectively.

$\Rightarrow \quad OA = a$ and $OB = b$

So, the coordinates of $C = \left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Now, $CO = \sqrt{\left(0-\frac{a}{2}\right)^2 + \left(0-\frac{b}{2}\right)^2}$

$= \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \sqrt{\frac{a^2+b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2}$ units

$CA = \sqrt{\left(a-\frac{a}{2}\right)^2 + \left(0-\frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$= \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \sqrt{\frac{a^2+b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2}$ units

$CB = \sqrt{\left(0-\frac{a}{2}\right)^2 + \left(b-\frac{b}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$

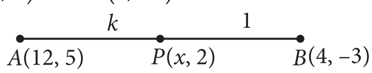
$= \sqrt{\frac{a^2+b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2}$ units

$\Rightarrow \quad CO = CA = CB$

Therefore, *C* is equidistant from the three vertices of ΔAOB.

OR

Let the point *P*(*x*, 2) divides the line segment joining the points *A*(12, 5) and *B*(4, –3) in the ratio *k* : 1.



Using section formula, we have

coordinates of *P* are $\left(\frac{4k+12}{k+1}, \frac{-3k+5}{k+1}\right)$

Now, the coordinates of *P* are (*x*, 2). (Given)

$\therefore \quad \frac{4k+12}{k+1} = x$ and $\frac{-3k+5}{k+1} = 2$

$\Rightarrow \quad -3k + 5 = 2k + 2 \Rightarrow 5k = 3 \Rightarrow k = 3/5$

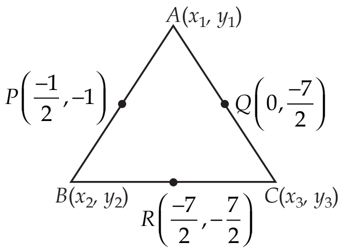
Substituting, $k = \frac{3}{5}$ in $\frac{4k+12}{k+1} = x$, we get

$$x = \frac{4\times\frac{3}{5}+12}{\frac{3}{5}+1} = \frac{12+60}{3+5} = \frac{72}{8} = 9$$

Thus, the value of x is 9.

Also, the point P divides the line segment joining the points $A(12, 5)$ and $B(4, -3)$ in the ratio $\frac{3}{5} : 1$ *i.e.*, $3 : 5$.

36. Let the coordinates of the vertices A, B and C of ΔABC be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively.



As, $P\left(\frac{-1}{2}, -1\right)$ is the mid-point of AB.

$\therefore \quad \frac{x_1 + x_2}{2} = \frac{-1}{2}$ and $\frac{y_1 + y_2}{2} = -1$

$\Rightarrow \quad x_1 + x_2 = -1$ and $y_1 + y_2 = -2$ ...(i)

Point R is the mid-point of BC.

$\therefore \quad \frac{x_2 + x_3}{2} = \frac{-7}{2}$ and $\frac{y_2 + y_3}{2} = \frac{-7}{2}$

$\Rightarrow \quad x_2 + x_3 = -7$ and $y_2 + y_3 = -7$ ...(ii)

Point Q is the mid-point of AC.

$\therefore \quad \frac{x_1 + x_3}{2} = 0$ and $\frac{y_1 + y_3}{2} = \frac{-7}{2}$

$\Rightarrow \quad x_1 + x_3 = 0$ and $y_1 + y_3 = -7$ ...(iii)

Adding (i), (ii) and (iii), we get

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = -1 - 7 + 0$ and $y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = -2 - 7 - 7$

$\Rightarrow \quad 2(x_1 + x_2 + x_3) = -8$ and $2(y_1 + y_2 + y_3) = -16$

$\Rightarrow \quad x_1 + x_2 + x_3 = -4$ and $y_1 + y_2 + y_3 = -8$ ...(iv)

From (i) and (iv), we get

$-1 + x_3 = -4$ and $-2 + y_3 = -8$

$\Rightarrow \quad x_3 = -3$ and $y_3 = -6$

So, coordinates of C are $(-3, -6)$.

From (ii) and (iv) we get

$x_1 + (-7) = -4$ and $y_1 + (-7) = -8$

$\Rightarrow \quad x_1 = 3$ and $y_1 = -1$

$\therefore$ Coordinates of A are $(3, -1)$

From (iii) and (iv), we get

$x_2 + 0 = -4$ and $y_2 + (-7) = -8$

$\Rightarrow \quad x_2 = -4$ and $y_2 = -1$

$\therefore$ Coordinates of B are $(-4, -1)$.

$\therefore$ Area of $\Delta ABC = \frac{1}{2}[3(-1+6) + (-4)(-6+1) + (-3)(-1+1)]$

$= \frac{1}{2}[3(5) + (-4)(-5) + 0]$

$= \frac{1}{2}[15 + 20] = \frac{35}{2}$ sq. units.