EXAM DRILL

Introduction to Trigonometry

CHAPTER 8

SOLUTIONS

1. **(b) :** We have, $\dfrac{\sin A}{\tan A}+\dfrac{\cos A\cot A}{\operatorname{cosec} A}$

$=\dfrac{\sin A}{\dfrac{\sin A}{\cos A}}+\cos A\times\dfrac{\cos A}{\sin A}\times\sin A$

$\left[\because\ \tan A=\dfrac{\sin A}{\cos A},\ \cot A=\dfrac{\cos A}{\sin A},\ \dfrac{1}{\operatorname{cosec} A}=\sin A\right]$

$=\cos A+\cos^2 A$

2. **(a) :** We have, $\sin\theta=\cos\theta$

$\Rightarrow\ \dfrac{\sin\theta}{\cos\theta}=1\ \Rightarrow \tan\theta=1$ $\quad\left[\because\ \tan\theta=\dfrac{\sin\theta}{\cos\theta}\right]$

$\Rightarrow\ \tan\theta=\tan 45^\circ$ $\quad[\because \tan 45^\circ=1]$

$\Rightarrow\ \theta=45^\circ$

3. **(c) :** Given, $\sec(2x+17)^\circ=\sqrt{2}$

$\Rightarrow\ \sec(2x+17)^\circ=\sec 45^\circ$ $\quad[\because\ \sec 45^\circ=\sqrt{2}\,]$

$\Rightarrow\ 2x+17=45\ \Rightarrow\ 2x=45-17$

$\Rightarrow\ 2x=28\Rightarrow x=14$

4. **(b) :** We have, $\cos 1^\circ\cdot\cos 2^\circ\cdot\cos 3^\circ\cdot\cos 4^\circ\cdot\ \ldots\ \cos 100^\circ$

$=\cos 1^\circ\cdot\cos 2^\circ\cdot\cos 3^\circ\ldots\cos 90^\circ\ldots\cos 100^\circ$

$=0$ $\quad[\because\ \cos 90^\circ=0]$

5. **(b) :** $\sin(45^\circ+\theta)-\cos(45^\circ-\theta)$

$=\cos[90^\circ-(45^\circ+\theta)]-\cos(45^\circ-\theta)$ $\quad[\because \cos(90^\circ-\theta)=\sin\theta]$

$=\cos(45^\circ-\theta)-\cos(45^\circ-\theta)=0$

6. We have, $\sec 5A=\operatorname{cosec}(A+30^\circ)$

$\Rightarrow\ \sec 5A=\sec[90^\circ-(A+30^\circ)]$

$\Rightarrow\ \sec 5A=\sec(60^\circ-A)$

$\Rightarrow\ 5A=60^\circ-A\Rightarrow 6A=60^\circ$

$\Rightarrow\ A=10^\circ$

7. Given, $\sin\theta=\dfrac{a}{b}$

$\therefore\ \cos\theta=\sqrt{1-\sin^2\theta}$ $\quad[\because \sin^2\theta+\cos^2\theta=1]$

$=\sqrt{1-\left(\dfrac{a}{b}\right)^2}=\sqrt{1-\dfrac{a^2}{b^2}}=\dfrac{\sqrt{b^2-a^2}}{b}$

8. $\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}$

$=\dfrac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}=\dfrac{2}{1-\sin^2\theta}=\dfrac{2}{\cos^2\theta}=2\sec^2\theta$

9. In a ΔABC, we have $A+B+C=180^\circ$

$\Rightarrow\ B+C=180^\circ-A\ \Rightarrow\ \dfrac{B+C}{2}=90^\circ-\dfrac{A}{2}$

$\Rightarrow\ \tan\left(\dfrac{B+C}{2}\right)=\tan\left(90^\circ-\dfrac{A}{2}\right)$

$\Rightarrow\ \tan\left(\dfrac{B+C}{2}\right)=\cot\dfrac{A}{2}$

10. L.H.S. $=\sqrt{\cos\alpha\operatorname{cosec}\beta-\cos\alpha\sin\beta}$

$=\sqrt{\cos\alpha\operatorname{cosec}(90^\circ-\alpha)-\cos\alpha\sin(90^\circ-\alpha)}$

[Given $\alpha+\beta=90^\circ$]

$=\sqrt{\cos\alpha\sec\alpha-\cos\alpha\cos\alpha}=\sqrt{1-\cos^2\alpha}=\sin\alpha=$ R.H.S.

$[\because \sin^2\theta+\cos^2\theta=1]$

11. We have, $\dfrac{1}{x}\left[\dfrac{\sin^2 5^\circ+\sin^2 85^\circ}{\cos^2 5^\circ+\cos^2 85^\circ}\right]-\dfrac{3}{4}=1$

$\Rightarrow\ \dfrac{1}{x}\left[\dfrac{\sin^2 5^\circ+\sin^2(90^\circ-5^\circ)}{\cos^2(90^\circ-85^\circ)+\cos^2 85^\circ}\right]-\dfrac{3}{4}=1$

$\Rightarrow\ \dfrac{1}{x}\left[\dfrac{\sin^2 5^\circ+\cos^2 5^\circ}{\sin^2 85^\circ+\cos^2 85^\circ}\right]-\dfrac{3}{4}=1$

$[\because \sin(90^\circ-\theta)=\cos\theta,\ \cos(90^\circ-\theta)=\sin\theta]$

$\Rightarrow\ \dfrac{1}{x}\times 1-\dfrac{3}{4}=1\ \Rightarrow\ \dfrac{1}{x}=1+\dfrac{3}{4}\ \Rightarrow\ \dfrac{1}{x}=\dfrac{7}{4}\ \Rightarrow\ x=\dfrac{4}{7}$

12. Given, $x=b\sec^3\theta$ and $y=a\tan^3\theta$

$\Rightarrow\ \sec^3\theta=\dfrac{x}{b}$ and $\tan^3\theta=\dfrac{y}{a}$ $\quad$ ...(i)

Now consider, $\left(\dfrac{x}{b}\right)^{2/3}-\left(\dfrac{y}{a}\right)^{2/3}$

$=(\sec^3\theta)^{2/3}-(\tan^3\theta)^{2/3}$ $\quad$ (Using (i))

$=\sec^2\theta-\tan^2\theta=1$

13. We know that $\sin^2\theta+\cos^2\theta=1$

$\Rightarrow\ (\sin^2\theta+\cos^2\theta)^3=1^3$ $\quad$ [Cubing both the sides]

$\Rightarrow\ (\sin^2\theta)^3+(\cos^2\theta)^3+3\sin^2\theta\cos^2\theta\,(\sin^2\theta+\cos^2\theta)=1$

$\Rightarrow\ \sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta=1$

14. L.H.S. $=(\sin^4\theta-\cos^4\theta+1)\operatorname{cosec}^2\theta$

$=[(\sin^2\theta-\cos^2\theta)(\sin^2\theta+\cos^2\theta)+1]\operatorname{cosec}^2\theta$

$=(\sin^2\theta-\cos^2\theta+1)\operatorname{cosec}^2\theta$ $\quad[\because \sin^2\theta+\cos^2\theta=1]$

$=2\sin^2\theta\operatorname{cosec}^2\theta$ $\quad[\because 1-\cos^2\theta=\sin^2\theta]$

$=2=$ R.H.S.

15. We have, $2(\operatorname{cosec}^2\theta-1)\tan^2\theta$

$=2(\cot^2\theta)\tan^2\theta$ $\quad[\because \operatorname{cosec}^2\theta=1+\cot^2\theta]$

$=2$ $\quad[\because\ \tan\theta\cdot\cot\theta=1]$

16. L.H.S. $= \dfrac{\tan^2 A(1+\cot^2 A)}{(1+\tan^2 A)} = \dfrac{\tan^2 A(\text{cosec}^2 A)}{\sec^2 A}$

[$\because$ $1 + \cot^2 A = \text{cosec}^2 A$; $1 + \tan^2 A = \sec^2 A$]

$= \dfrac{\sin^2 A}{\cos^2 A}\cdot\dfrac{\cos^2 A}{\sin^2 A} = 1 =$ R.H.S.

17. **(i)** **(d) :** In ΔAPQ, $\tan\theta = \dfrac{AQ}{PQ} = \dfrac{1.2}{1.6} = \dfrac{3}{4}$

(ii) **(d) :** In ΔPBQ, $\cot B = \dfrac{QB}{PQ} = \dfrac{3}{1.6} = \dfrac{15}{8}$ ...(1)

(iii) **(c) :** In ΔAPQ, $\tan A = \dfrac{PQ}{AQ} = \dfrac{1.6}{1.2} = \dfrac{4}{3}$ ...(2)

(iv) **(d) :** We have, $\tan^2 A + 1 = \sec^2 A$

$\Rightarrow \sec A = \sqrt{\left(\dfrac{4}{3}\right)^2 + 1}$ [Using (2)]

$= \sqrt{\dfrac{16}{9} + 1} = \sqrt{\dfrac{25}{9}} = \dfrac{5}{3}$

(v) **(a) :** Since, $\text{cosec}B = \sqrt{\cot^2 B + 1}$

$= \sqrt{\left(\dfrac{15}{8}\right)^2 + 1}$ [Using (1)]

$= \dfrac{17}{8}$

18. $\because$ ΔPQR is a right angled triangle.

$\therefore$ $PR^2 + RQ^2 = PQ^2$

$\Rightarrow$ $PR^2 = (13)^2 - (12)^2 = 25 \Rightarrow PR = 5$ cm

(i) **(c) :** $\cos\theta = \dfrac{QR}{PQ} = \dfrac{12}{13}$

(ii) **(c) :** $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{13}{12}$

(iii) **(c) :** $\tan\theta = \dfrac{PR}{RQ} = \dfrac{5}{12}$ ...(1)

$\therefore \dfrac{\tan\theta}{1+\tan^2\theta} = \dfrac{\frac{5}{12}}{1+\frac{25}{144}} = \dfrac{\frac{5}{12}}{\frac{169}{144}} = \dfrac{60}{169}$

(iv) **(a) :** $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{12}{5}$ [Using (1)]

$\text{cosec}\theta = \dfrac{PQ}{PR} = \dfrac{13}{5}$

$\therefore$ $\cot^2\theta - \text{cosec}^2\theta = \dfrac{144}{25} - \dfrac{169}{25} = -1$

(v) **(b) :** $\sin^2\theta + \cos^2\theta = 1$ (Using identity)

19. We have, $KL = 4$ cm, $ML = 4\sqrt{3}$ cm, $KM = 8$ cm

(i) **(a) :** $\tan M = \dfrac{KL}{LM} = \dfrac{4}{4\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow$ $\tan M = \tan 30° \Rightarrow \angle M = 30°$

(ii) **(c) :** $\tan K = \dfrac{ML}{KL} = \dfrac{4\sqrt{3}}{4} = \sqrt{3} = \tan 60°$

$\Rightarrow$ $\angle K = 60°$

(iii) **(b)** **(iv)** **(c)**

(v) **(a) :** $\dfrac{\tan^2 45° - 1}{\tan^2 45° + 1} = \dfrac{(1)^2 - 1}{1^2 + 1} = \dfrac{0}{2} = 0$

20. We have, $AB = BC = 6\sqrt{2}$ m and $AC = 12$ m.

(i) **(d) :** $\because$ D is mid point of AC.

$\therefore$ $AD = DC = 6$ m

Now, $AB^2 = BD^2 + AD^2$ ($\because$ ΔABD is a right triangle)

$\Rightarrow$ $BD^2 = (6\sqrt{2})^2 - 6^2 = 72 - 36 = 36$

$\Rightarrow$ $BD = 6$ m ...(1)

(ii) **(c) :** In ΔABD, $\sin A = \dfrac{BD}{AB} = \dfrac{6}{6\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ [Using (1)]

$\Rightarrow$ $\sin A = \sin 45° \Rightarrow \angle A = 45°$

(iii) **(c) :** In ΔBDC, $\tan C = \dfrac{BD}{DC} = \dfrac{6}{6}$ [Using (1)]

$\Rightarrow$ $\tan C = 1 = \tan 45° \Rightarrow \angle C = 45°$

(iv) **(d) :** $\sin A = \dfrac{1}{\sqrt{2}}, \cos C = \cos 45° = \dfrac{1}{\sqrt{2}}$

$\therefore$ $\sin A + \cos C = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

(v) **(c) :** $\tan C = 1$, $\tan A = \tan 45° = 1$

$\Rightarrow$ $\tan^2 C + \tan^2 A = 1 + 1 = 2$

21. Given, $\sin\theta - \cos\theta = 0$

$\Rightarrow$ $\sin\theta = \cos\theta \Rightarrow \dfrac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow$ $\tan\theta = 1$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta} \text{ and } \tan 45° = 1\right]$

$\Rightarrow$ $\tan\theta = \tan 45° \Rightarrow \theta = 45°$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45° + \cos^4 45°$

$= \left(\dfrac{1}{\sqrt{2}}\right)^4 + \left(\dfrac{1}{\sqrt{2}}\right)^4$ $\left[\because \sin 45° = \cos 45° = \dfrac{1}{\sqrt{2}}\right]$

$= \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

22. Given that, $A = 60°$ and $B = 30°$

$\therefore$ $\cos A = \cos 60° = \dfrac{1}{2}$; $\cos B = \cos 30° = \dfrac{\sqrt{3}}{2}$

and $\cos(A + B) = \cos(60° + 30°) = \cos 90° = 0$

Now, $\cos A + \cos B = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2} \neq 0$

$\therefore$ $\cos(A + B) \neq \cos A + \cos B$.

23. L.H.S. = $\text{cosec}^2 60° \sec^2 30° \cos^2 0° \sin 45° \cot^2 60° \tan^2 60°$

$$=\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{2}{\sqrt{3}}\right)^2(1)^2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}\right)^2(\sqrt{3})^2 = \frac{4}{3}\times\frac{4}{3}\times\frac{1}{\sqrt{2}}\times\frac{1}{3}\times 3$$

$$= \frac{16}{9}\times\frac{1}{\sqrt{2}} = \frac{8\sqrt{2}}{9} = \text{R.H.S.}$$

OR

We have, $\sin(A + B) = 1$

$\Rightarrow$ $\sin(A + B) = \sin 90° \Rightarrow A + B = 90°$ ...(i)

Also, $\sin(A - B) = \frac{1}{2}$

$\Rightarrow$ $\sin(A - B) = \sin 30° \Rightarrow A - B = 30°$ ...(ii)

Adding (i) and (ii), we get

$A + B + A - B = 120° \Rightarrow 2A = 120° \Rightarrow A = 60°$

From (i), we have $60° + B = 90° \Rightarrow B = 30°$

24. We have, $\cos 15\theta = \sin 3\theta$

$\Rightarrow$ $\cos 15\theta = \cos(90° - 3\theta) \Rightarrow 15\theta = 90° - 3\theta$

$\Rightarrow$ $18\theta = 90° \Rightarrow \theta = \frac{90°}{18} = 5°$

$\therefore$ $\cot 9\theta + \tan 9\theta = \cot 45° + \tan 45° = 1 + 1 = 2$

25. Given, $2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow$ $2\sin^2\theta - (1 - \sin^2\theta) = 2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow$ $2\sin^2\theta + \sin^2\theta - 1 = 2$

$\Rightarrow$ $3\sin^2\theta = 3 \Rightarrow \sin^2\theta = 1$

$\Rightarrow$ $\sin\theta = 1 = \sin 90°$ $[\because \sin 90° = 1]$

$\therefore$ $\theta = 90°$

26. In ΔOPQ, we have $OQ^2 = OP^2 + PQ^2$

$\Rightarrow$ $(1 + PQ)^2 = OP^2 + PQ^2$ $[\because OQ - PQ = 1]$

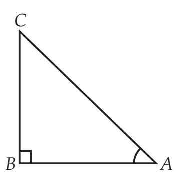
$\Rightarrow$ $1 + PQ^2 + 2PQ = OP^2 + PQ^2$

$\Rightarrow$ $1 + 2PQ = 7^2 \Rightarrow 2PQ = 48$

$\Rightarrow$ $PQ = 24$ cm and $OQ = 1 + PQ = 25$ cm

So, $\sin Q = \frac{OP}{OQ} = \frac{7}{25}$ and $\cos Q = \frac{PQ}{OQ} = \frac{24}{25}$

27. In ΔABC, $\tan A = \frac{BC}{AB} = 1$



$\Rightarrow$ $BC = AB$

Let $AB = BC = k$ units

$\therefore$ By Pythagoras theorem, we have

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{(k)^2 + (k)^2} = k\sqrt{2} \text{ units}$$

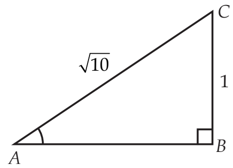
$\therefore$ $\sin A = \frac{BC}{AC} = \frac{k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$ and $\cos A = \frac{AB}{AC} = \frac{k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$

So, $2 \sin A \cos A = 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = 1$

Hence verified.

28. We have, $\text{cosec } A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\sqrt{10}}{1}$

So, we draw a ΔABC, right-angled at B such that $BC = 1$ unit and $AC = \sqrt{10}$ units.



By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow$ $(\sqrt{10})^2 = AB^2 + 1^2$

$\Rightarrow$ $AB^2 = 10 - 1 = 9$

$\Rightarrow$ $AB = \sqrt{9} = 3$ units

$\therefore$ $\sin A = \frac{BC}{AC} = \frac{1}{\sqrt{10}}$, $\cos A = \frac{AB}{AC} = \frac{3}{\sqrt{10}}$,

$\tan A = \frac{BC}{AB} = \frac{1}{3}$, $\sec A = \frac{1}{\cos A} = \frac{\sqrt{10}}{3}$

and $\cot A = \frac{1}{\tan A} = \frac{3}{1} = 3$

29. Since, $\tan R = \frac{1}{\sqrt{3}} = \tan 30° \Rightarrow R = 30°$

Again, $\cos P = \frac{1}{2} = \cos 60° \Rightarrow P = 60°$

(i) Now, $\cos P \cos R + \sin P \sin R$

$= \cos 60° \cos 30° + \sin 60° \sin 30°$

$$= \frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

(ii) $\cos(P - R) = \cos(60° - 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

From (i) and (ii), we get

$\cos(P - R) = \cos P \cos R + \sin P \sin R$

OR

We have, $\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°$

$$= \frac{\sin^2 22° + \sin^2(90° - 22°)}{\cos^2(90° - 68°) + \cos^2 68°} + \sin^2 63° + \cos 63° \sin(90° - 63°)$$

$$= \frac{\sin^2 22° + \cos^2 22°}{\sin^2 68° + \cos^2 68°} + \sin^2 63° + \cos 63° \cos 63°$$

$[\because \sin(90° - \theta) = \cos\theta \text{ and } \cos(90° - \theta) = \sin\theta]$

$= \frac{1}{1} + (\sin^2 63° + \cos^2 63°)$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= 1 + 1 = 2$

30. Given, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Squaring both the sides, we get

$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow$ $\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\Rightarrow$ $1 = 2\cos^2\theta - 2\cos\theta\sin\theta$

$\Rightarrow$ $1 - 2\cos^2\theta = -2\cos\theta\sin\theta$

$\Rightarrow$ $1 + 1 - 2\cos^2\theta = 1 - 2\cos\theta\sin\theta$

$\Rightarrow$ $2 - 2\cos^2\theta = \cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta$

$\Rightarrow$ $2(1 - \cos^2\theta) = (\cos\theta - \sin\theta)^2$

$\Rightarrow$ $2\sin^2\theta = (\cos\theta - \sin\theta)^2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow$ $\cos\theta - \sin\theta = \sqrt{2\sin^2\theta} = \sqrt{2}\sin\theta$

31. $\text{L.H.S.} = \dfrac{\cos^2(45°+\theta)+\cos^2(45°-\theta)}{\tan(60°+\theta)\cdot\tan(30°-\theta)}$

$= \dfrac{\cos^2(45°+\theta)+[\sin\{90°-(45°-\theta)\}]^2}{\tan(60°+\theta)\cdot\cot\{90°-(30°-\theta)\}}$

$[\because \sin(90° - \theta) = \cos\theta \text{ and } \cot(90° - \theta) = \tan\theta]$

$= \dfrac{\cos^2(45°+\theta)+\sin^2(45°+\theta)}{\tan(60°+\theta)\cdot\cot(60°+\theta)} = \dfrac{1}{\tan(60°+\theta)\cdot\dfrac{1}{\tan(60°+\theta)}}$

$[\because \sin^2\theta + \cos^2\theta = 1, \cot\theta = 1/\tan\theta]$

$= 1 = \text{R.H.S.}$

OR

$\text{L.H.S.} = \sqrt{\sec^2\theta + \text{cosec}^2\theta} = \sqrt{\dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}}$

$\left[\because \sec\theta = \dfrac{1}{\cos\theta} \text{ and } \text{cosec}\theta = \dfrac{1}{\sin\theta}\right]$

$= \sqrt{\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cdot\cos^2\theta}} = \sqrt{\dfrac{1}{\sin^2\theta\cdot\cos^2\theta}}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \dfrac{1}{\sin\theta\cdot\cos\theta} = \dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cdot\cos\theta}$ $[\because 1 = \sin^2\theta + \cos^2\theta]$

$= \dfrac{\sin^2\theta}{\sin\theta\cdot\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cdot\cos\theta} = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

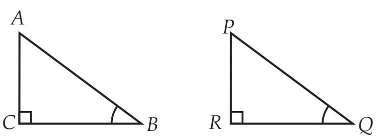
$= \tan\theta + \cot\theta$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \dfrac{\cos\theta}{\sin\theta}\right]$

$= \text{R.H.S.}$

32. Let us consider two triangles ABC and PQR, right angled at C and R respectively such that $\sin B = \sin Q$.

We have, $\sin B = \dfrac{AC}{AB}$ and $\sin Q = \dfrac{PR}{PQ}$

Then $\dfrac{AC}{AB} = \dfrac{PR}{PQ} \Rightarrow \dfrac{AC}{PR} = \dfrac{AB}{PQ} = k$ (say) ...(i)



Now, by using Pythagoras theorem, we have

$BC = \sqrt{AB^2 - AC^2}$ and $QR = \sqrt{PQ^2 - PR^2}$

So, $\dfrac{BC}{QR} = \dfrac{\sqrt{AB^2-AC^2}}{\sqrt{PQ^2-PR^2}}$

$= \dfrac{\sqrt{k^2PQ^2-k^2PR^2}}{\sqrt{PQ^2-PR^2}} = \dfrac{k\sqrt{PQ^2-PR^2}}{\sqrt{PQ^2-PR^2}} = k$ ...(ii)

From (i) and (ii), we have, $\dfrac{AC}{PR} = \dfrac{AB}{PQ} = \dfrac{BC}{QR}$

So, $\Delta ACB \sim \Delta PRQ$ and therefore, $\angle B = \angle Q$.

33. Given, $\sin(A + C - B) = \dfrac{\sqrt{3}}{2}$ and $\cot(B + C - A) = \sqrt{3}$

$\Rightarrow$ $\sin(A + C - B) = \sin 60°$ and $\cot(B + C - A) = \cot 30°$

$\Rightarrow$ $A + C - B = 60°$ ...(i)

and $B + C - A = 30°$ ...(ii)

Adding (i) and (ii), we have $2C = 90° \Rightarrow C = 45°$

Putting this value of C in (i), we get $A - B = 15°$ ...(iii)

Also, by angle sum property of a triangle

$A + B + C = 180°$

$\Rightarrow$ $A + B = 135°$ ...(iv)

Adding (iii) and (iv), we have

$2A = 150° \Rightarrow A = 75°$

From (iii), we have $B = 75° - 15° = 60°$

OR

We are given that, $\text{cosec}\,\theta = \dfrac{17}{12} \Rightarrow \sin\theta = \dfrac{12}{17}$

$\therefore$ $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\dfrac{12}{17}\right)^2} = \dfrac{\sqrt{145}}{17}$

So, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{12}{17}\times\dfrac{17}{\sqrt{145}} = \dfrac{12}{\sqrt{145}}$ ...(i)

Now, $\sqrt{\dfrac{(\sec\theta-1)(\sec\theta+1)}{(\text{cosec}\,\theta-1)(\text{cosec}\,\theta+1)}} = \sqrt{\dfrac{(\sec^2\theta-1)}{(\text{cosec}^2\theta-1)}}$

$= \sqrt{\dfrac{\tan^2\theta}{\cot^2\theta}} = \sqrt{\left(\dfrac{\tan\theta}{\cot\theta}\right)^2} = \dfrac{\tan\theta}{\cot\theta} = \tan\theta \times \tan\theta = \tan^2\theta$

$= \left(\dfrac{12}{\sqrt{145}}\right)^2$ [Using (i)]

$= \dfrac{144}{145}$

34. We have,

$\dfrac{\text{cosec}^2(90°-\theta)-\tan^2\theta}{2(\cos^2 37°+\cos^2 53°)} - \dfrac{2\tan^2 30°\sec^2 37°\sin^2 53°}{\text{cosec}^2 63°-\tan^2 27°}$

$= \dfrac{\sec^2\theta-\tan^2\theta}{2(\cos^2(90°-53°)+\cos^2 53°)}$

$- \dfrac{2\left(\dfrac{1}{\sqrt{3}}\right)^2\left[\dfrac{1}{\cos^2(90°-53°)}\times\sin^2 53°\right]}{\text{cosec}^2(90°-27°)-\tan^2 27°}$

$\left[\because \text{cosec}(90°-\theta) = \sec\theta \text{ and } \tan 30° = \dfrac{1}{\sqrt{3}}\right]$

$$= \frac{1}{2(\sin^2 53° + \cos^2 53°)} - \frac{2 \times \frac{1}{3}\left[\frac{1}{\sin^2 53°} \times \sin^2 53°\right]}{\sec^2 27° - \tan^2 27°}$$

$$= \frac{1}{2} - \frac{2}{3} = \frac{3-4}{6} = \frac{-1}{6}$$

OR

We have, $a \sin\theta = b \cos\theta$ ...(i)

Also, $a \sin^3\theta + b\cos^3\theta = \sin\theta \cos\theta$

$\Rightarrow (a \sin\theta) \sin^2\theta + b\cos^3\theta = \sin\theta \cos\theta$

$\Rightarrow b \cos\theta \,.\, \sin^2\theta + b\cos^3\theta = \sin\theta \cos\theta$ [Using (i)]

$\Rightarrow b \cos\theta\, [\sin^2\theta + \cos^2\theta] = \sin\theta \cos\theta$

$\Rightarrow b \cos\theta \times 1 = \sin\theta \cos\theta$

$\Rightarrow b = \sin\theta$ ...(ii)

From (i) and (ii), we have

$a \cdot b = b \cos\theta \Rightarrow a = \cos\theta$ ...(iii)

Squaring and adding (ii) and (iii), we get

$a^2 + b^2 = \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow a^2 + b^2 = 1$

Hence proved.

35. Given, $m = \cos A - \sin A$, $n = \cos A + \sin A$

Now, $\frac{m}{n} - \frac{n}{m} = \frac{m^2 - n^2}{mn}$

$$= \frac{(\cos A - \sin A)^2 - (\cos A + \sin A)^2}{(\cos A - \sin A)(\cos A + \sin A)}$$

$$= \frac{\cos^2 A + \sin^2 A - 2\cos A \sin A - \cos^2 A - \sin^2 A - 2\sin A \cos A}{\cos^2 A - \sin^2 A}$$

$$= \frac{-4\sin A \cos A}{\cos^2 A - \sin^2 A} \quad \text{...(i)}$$

Dividing numerator and denominator by $\sin A \cos A$, we get

$$\frac{-4}{\frac{\cos^2 A}{\sin A \cos A} - \frac{\sin^2 A}{\sin A \cos A}} = \frac{-4}{\cot A - \tan A} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{m}{n} - \frac{n}{m} = \frac{-4\sin A \cos A}{\cos^2 A - \sin^2 A} = \frac{-4}{\cot A - \tan A}$$