Circles

CHAPTER 10

NCERT FOCUS

SOLUTIONS

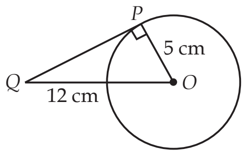
EXERCISE - 10.1

1. A circle can have an infinite number of tangents.

2. (i) exactly one (ii) secant
(iii) two (iv) point of contact

3. **(d) :** PQ is a tangent to the circle.

$\therefore$ $OP \perp PQ$

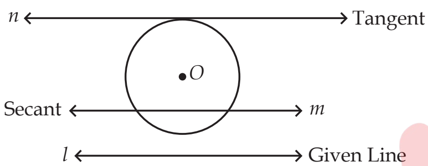


In right ΔPOQ, $OQ^2 = OP^2 + PQ^2$

$\Rightarrow$ $PQ = \sqrt{OQ^2 - OP^2}$

$= \sqrt{12^2 - 5^2} = \sqrt{144 - 25} = \sqrt{119}$ cm

4. We have the required figure, as shown

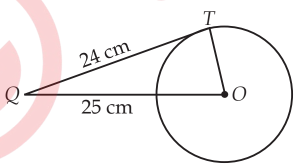


Here, l is the given line and a circle with centre O is drawn.

The line n is drawn which is parallel to l and tangent to the circle. Also, m is drawn parallel to line l and is a secant to the circle.

EXERCISE - 10.2

1. **(a) :** $\because$ QT is a tangent to the circle at T and OT is radius.



$\therefore$ $OT \perp QT$

Also, $OQ = 25$ cm

and $QT = 24$ cm

$\therefore$ Using Pythagoras theorem in ΔQTO, we get

$OQ^2 = QT^2 + OT^2$

$\Rightarrow$ $OT^2 = OQ^2 - QT^2 = 25^2 - 24^2 = 49 \Rightarrow OT = 7$ cm

Thus, the required radius is 7 cm.

2. **(b) :** $\because$ TQ and TP are tangents to a circle with centre O.

$\therefore$ $OP \perp PT$ and $OQ \perp QT \Rightarrow \angle OPT = \angle OQT = 90°$

Now, in the quadrilateral $TPOQ$,

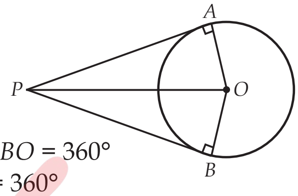
$\angle PTQ + 90° + 110° + 90° = 360°$

[By angle sum property of a quadrilateral]

$\Rightarrow$ $\angle PTQ + 290° = 360° \Rightarrow \angle PTQ = 360° - 290° = 70°$

3. **(a) :** Since, O is the centre of the circle and two tangents from P to the circle are PA and PB.

$\therefore$ $OA \perp AP$ and $OB \perp BP$



$\Rightarrow$ $\angle OAP = \angle OBP = 90°$

Now, in quadrilateral $PAOB$, we have

$\angle APB + \angle PAO + \angle AOB + \angle PBO = 360°$

$\Rightarrow$ $80° + 90° + \angle AOB + 90° = 360°$

$\Rightarrow$ $260° + \angle AOB = 360°$

$\Rightarrow$ $\angle AOB = 360° - 260°$

$\Rightarrow$ $\angle AOB = 100°$

In right ΔOAP and right ΔOBP, we have

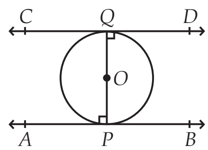
	$OP = OP$	[Common]
	$\angle OAP = \angle OBP$	[Each 90°]
	$OA = OB$	[Radii of the same circle]
$\therefore$	$\Delta OAP \cong \Delta OBP$	[By RHS congruence criterion]
$\Rightarrow$	$\angle POA = \angle POB$	[By CPCT]

$\therefore$ $\angle POA = \frac{1}{2}\angle AOB = \frac{1}{2} \times 100° = 50°.$

4. In the figure, PQ is diameter of the given circle and O is its centre.

Let tangents AB and CD be drawn at the end points of the diameter PQ.

Since, the tangent at a point to a circle is perpendicular to the radius through the point of contact.



$\therefore$ $PQ \perp AB$

$\Rightarrow$ $\angle APQ = 90°$

And $PQ \perp CD$

$\Rightarrow$ $\angle PQD = 90°$

$\Rightarrow$ $\angle APQ = \angle PQD$

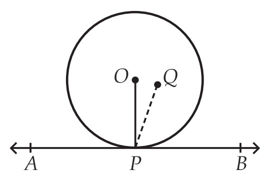
But they form a pair of alternate angles.

$\therefore$ $AB || CD$.

Hence, the two tangents are parallel.

5. In the figure, the centre of the circle is O and tangent AB touches the circle at P. If possible, let PQ be perpendicular to AB such that it is not passing through O. Join OP.

Since tangent at a point to a circle is perpendicular to the radius through that point.



$\therefore$ $OP \perp AB$

$\Rightarrow$ $\angle OPB = 90°$ ...(1)

But by construction,

$PQ \perp AB$

$\Rightarrow$ $\angle QPB = 90°$ ...(2)

From (1) and (2),

$\angle QPB = \angle OPB$,

which is possible only when O and Q coincide. Thus, the perpendicular at the point of contact to the tangent to a circle passes through the centre.

6. $\because$ The tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OTA = 90°$

Now, in the right ΔOTA, we have

$OA^2 = OT^2 + AT^2$ [By Pythagoras theorem]

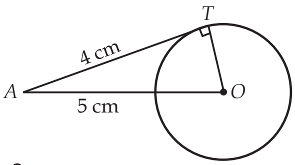
$\Rightarrow \quad 5^2 = OT^2 + 4^2$

$\Rightarrow \quad OT^2 = 5^2 - 4^2$

$\Rightarrow \quad OT^2 = 25 - 16$

$\Rightarrow \quad OT^2 = 9 = 3^2$

$\Rightarrow \quad OT = 3$ cm



Thus, the radius of the circle is 3 cm.

7. In the figure, O is the common centre, of the given concentric circles.

AB is a chord of the bigger circle such that it is a tangent to the smaller circle at P.

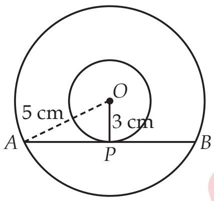
Since OP is the radius of the smaller circle.

$\therefore \quad OP \perp AB \Rightarrow \angle APO = 90°$

Also, radius perpendicular to a chord bisects the chord.

$\therefore \quad OP$ bisects AB

$\Rightarrow \quad AP = \dfrac{1}{2}AB$



Now, in right ΔAPO, $OA^2 = AP^2 + OP^2$

$\Rightarrow \quad 5^2 = AP^2 + 3^2 \Rightarrow AP^2 = 5^2 - 3^2 = 25 - 9 = 16$

$\Rightarrow \quad AP^2 = 4^2 \Rightarrow AP = 4$ cm

$\Rightarrow \quad \dfrac{1}{2}AB = 4 \Rightarrow AB = 2 \times 4 = 8$ cm

Hence, the required length of the chord AB is 8 cm.

8. Since, the sides of quadrilateral $ABCD$, *i.e.*, AB, BC, CD and DA touch the circle at P, Q, R and S respectively, and the lengths of two tangents to a circle from an external point are equal.

$\therefore \quad AP = AS$, $BP = BQ$, $CR = CQ$ and $DR = DS$

Adding all these equations, we get

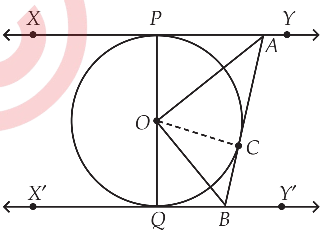
$(AP + BP) + (CR + DR) = (BQ + CQ) + (DS + AS)$

$\Rightarrow \quad AB + CD = BC + DA$

9. Join OC.

$\because$ The tangents drawn to a circle from an external point are equal.

$\therefore \quad AP = AC$ ...(1)



In ΔPAO and ΔCAO, we have,

$AO = AO$ [Common]

$OP = OC$ [Radii of the same circle]

$AP = AC$ [Using (1)]

$\therefore \quad \Delta PAO \cong \Delta CAO$ [By SSS congruency criterion]

$\Rightarrow \quad \angle PAO = \angle CAO$ [By CPCT]

$\Rightarrow \quad \angle PAC = 2\angle CAO$ ...(2)

Similarly, $\angle CBQ = 2\angle CBO$ ...(3)

Again, we know that sum of internal angles on the same side of a transversal is 180°.

$\therefore \quad \angle PAC + \angle CBQ = 180°$

$\Rightarrow \quad 2\angle CAO + 2\angle CBO = 180°$ [From (2) and (3)]

$\Rightarrow \quad \angle CAO + \angle CBO = \dfrac{180°}{2} = 90°$ ...(4)

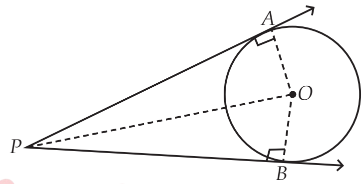
Also, in ΔAOB, $\angle BAO + \angle ABO + \angle AOB = 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad \angle CAO + \angle CBO + \angle AOB = 180°$

$\Rightarrow \quad 90° + \angle AOB = 180°$ [Using (4)]

$\Rightarrow \quad \angle AOB = 180° - 90° \Rightarrow \angle AOB = 90°$.

10. Let PA and PB be two tangents drawn from an external point P to a circle with centre O.



Now, in right ΔOAP and right ΔOBP, we have

$PA = PB$ [Tangents to a circle from an external point]

$OA = OB$ [Radii of the same circle]

$OP = OP$ [Common]

$\therefore \quad \Delta OAP \cong \Delta OBP$ [By SSS congruency criterion]

$\Rightarrow \quad \angle OPA = \angle OPB$ and $\angle AOP = \angle BOP$ [By CPCT]

$\Rightarrow \quad \angle APB = 2\angle OPA$ and $\angle AOB = 2\angle AOP$ ...(1)

But $\angle AOP = 90° - \angle OPA \Rightarrow 2\angle AOP = 180° - 2\angle OPA$

$\Rightarrow \quad \angle AOB = 180° - \angle APB$ [Using (1)]

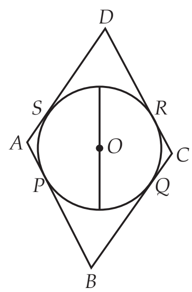
$\Rightarrow \quad \angle AOB + \angle APB = 180°$.

11. We have $ABCD$, a parallelogram which circumscribes a circle (*i.e.*, its sides touch the circle) with centre O.

Since, tangents to a circle from an external point are equal in length.

$\therefore \quad AP = AS \quad \Rightarrow \quad BP = BQ$

$CR = CQ \quad \Rightarrow \quad DR = DS$



Adding above all, we get

$(AP + BP) + (CR + DR)$

$= (AS + DS) + (BQ + CQ)$

$\Rightarrow \quad AB + CD = AD + BC$

But $AB = CD$ [$\because$ Opposite sides of parallelogram]

and $BC = AD$

$\therefore \quad AB + CD = AD + BC \Rightarrow 2AB = 2BC \Rightarrow AB = BC$

Similarly, $AB = DA$ and $DA = CD$

Thus, $AB = BC = CD = AD$

Hence, $ABCD$ is a rhombus.

12. Join OA, OE, OF, OB and OC.

Here, ΔABC circumscribe the circle with centre O. Also, radius = 4 cm

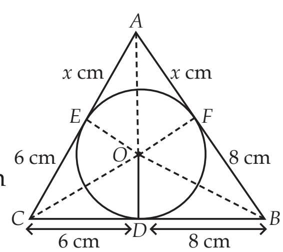
$\because$ The sides BC, CA and AB touch the circle at D, E and F respectively.

$\therefore \quad BF = BD = 8$ cm

$CD = CE = 6$ cm

$AF = AE = x$ cm (say)

So, the sides of the triangle are 14 cm, $(x + 6)$ cm and $(x + 8)$ cm



Perimeter of ΔABC

$= [14 + (x + 6) + (x + 8)]$ cm

$= [14 + 6 + 8 + 2x]$ cm $= (28 + 2x)$ cm

$\Rightarrow$ Semi perimeter of ΔABC (s)

$$= \frac{1}{2}[28 + 2x] \text{ cm} = (14 + x) \text{ cm},$$

where $a = BC$, $b = AC$, $c = AB$

$\therefore$ $s - a = (14 + x) - (14) = x$

$s - b = (14 + x) - (6 + x) = 8$

$s - c = (14 + x) - (8 + x) = 6$

$\therefore$ Area of $\Delta ABC = \sqrt{(14+x)(x)(8)(6)}$ cm^2

$$= \sqrt{(14+x)48x} \text{ cm}^2 \quad ...(1)$$

Now, $ar(\Delta OBC) = \frac{1}{2} \times BC \times OD$

$$= \frac{1}{2} \times 14 \times 4 = 28 \text{ cm}^2 \quad [\because OD = \text{Radius}]$$

$ar\,(\Delta OCA) = \frac{1}{2} CA \times OE = \frac{1}{2} \times (x+6) \times 4 = (2x + 12) \text{ cm}^2$

$ar\,(\Delta OAB) = \frac{1}{2} \times AB \times OF = \frac{1}{2} \times (x+8) \times 4 = (2x + 16) \text{ cm}^2$

$\therefore$ $ar\,(\Delta ABC) = ar\,(\Delta OBC) + ar\,(\Delta OCA) + ar\,(\Delta OAB)$

$= 28 \text{ cm}^2 + (2x + 12) \text{ cm}^2 + (2x + 16) \text{ cm}^2$

$= (28 + 12 + 16 + 4x) \text{ cm}^2 = (56 + 4x) \text{ cm}^2 \quad ...(2)$

From (1) and (2), we have

$56 + 4x = \sqrt{(14+x)48x}$

$\Rightarrow$ $4(14 + x) = 4\sqrt{(14+x) \times 3x}$

$\Rightarrow$ $14 + x = \sqrt{(14+x)3x}$

Squaring both sides, we get $(14 + x)^2 = (14 + x)3x$

$\Rightarrow$ $196 + x^2 + 28x = 42x + 3x^2$

$\Rightarrow$ $2x^2 + 14x - 196 = 0$

$\Rightarrow$ $x^2 + 7x - 98 = 0$

$\Rightarrow$ $(x - 7)(x + 14) = 0$

$\Rightarrow$ $x - 7 = 0$ or $x + 14 = 0 \Rightarrow x = 7$ or $x = -14$

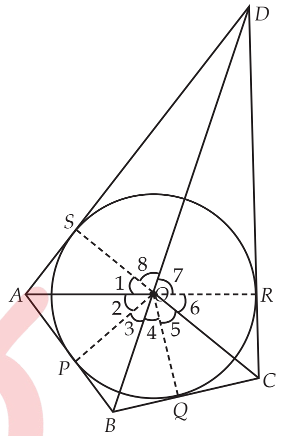
But $x = -14$ is rejected.

$\therefore$ $x = 7$ cm

Thus, $AB = 8 + 7 = 15$ cm

and $CA = 6 + 7 = 13$ cm.

13. We have a circle with centre O. A quadrilateral $ABCD$ is such that the sides AB, BC, CD and DA touch the circle at P, Q, R and S respectively.



Let us join OP, OQ, OR and OS.

We know that tangents drawn from an external point to a circle subtend equal angles at the centre.

$\therefore$ $\angle 1 = \angle 2$

$\angle 3 = \angle 4$

$\angle 5 = \angle 6$

and $\angle 7 = \angle 8$

Also, the sum of all the angles around a point is 360°.

$\therefore$ $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\therefore$ $2(\angle 1 + \angle 8 + \angle 5 + \angle 4) = 360°$

$\Rightarrow$ $(\angle 1 + \angle 8) + (\angle 5 + \angle 4) = 180° \quad ...(1)$

And $2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

$\Rightarrow$ $(\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180° \quad ...(2)$

Since, $\angle 2 + \angle 3 = \angle AOB$,

$\angle 6 + \angle 7 = \angle COD$

$\angle 1 + \angle 8 = \angle AOD$,

$\angle 4 + \angle 5 = \angle BOC$

$\therefore$ From (1) and (2), we have

$\angle AOD + \angle BOC = 180°$

and $\angle AOB + \angle COD = 180°$