CHAPTER 11

Constructions

NCERT FOCUS — SOLUTIONS

EXERCISE - 11.1

1. Steps of Construction

Step 1 : Draw a line segment $AB = 7.6$ cm.

Step 2 : Draw a ray AX making an acute angle with AB.

Step 3 : Locate 13 (8 + 5) points at equal distance on AX and mark them as $X_1, X_2, X_3, \ldots\ldots, X_{13}$.

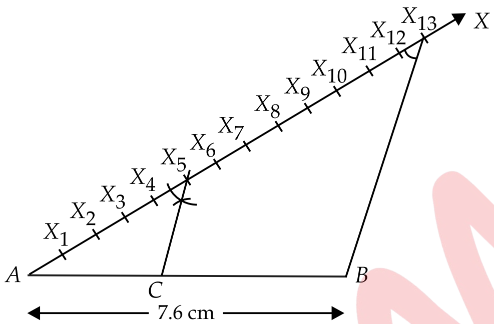
Step 4 : Join X_{13} to B.

Step 5 : From point X_5, draw $X_5C \parallel X_{13}B$, which meets AB at C.

Thus, C divides AB in the ratio 5 : 8.

On measuring the two parts, we get

$AC = 2.9$ cm and $CB = 4.7$ cm.



Justification:

In ΔABX_{13} and ΔACX_5, we have

$CX_5 \parallel BX_{13}$

$\therefore \quad \dfrac{AC}{CB} = \dfrac{AX_5}{X_5X_{13}} = \dfrac{5}{8}$ [By Thales theorem]

$\Rightarrow \quad AC : CB = 5 : 8.$

2. Steps of Construction

Step 1 : Draw a ΔABC such that $BC = 6$ cm, $AC = 5$ cm and $AB = 4$ cm.

Step 2 : Draw a ray BX making an acute angle $\angle CBX$.

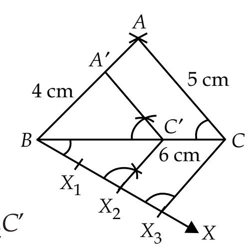
Step 3 : Mark three points X_1, X_2, X_3 on BX such that $BX_1 = X_1X_2 = X_2X_3$.

Step 4 : Join X_3C.

Step 5 : Draw a line through X_2 such that it is parallel to X_3C and meets BC at C'.

Step 6 : Draw a line through C' parallel to CA to intersect BA at A'.

Thus, $\Delta A'BC'$ is the required similar triangle.



Justification:

By construction, we have $X_3C \parallel X_2C'$

$\Rightarrow \quad \dfrac{BX_2}{X_2X_3} = \dfrac{BC'}{C'C}$ [Using Thales theorem]

But $\dfrac{BX_2}{X_2X_3} = \dfrac{2}{1}$

$\Rightarrow \quad \dfrac{BC'}{C'C} = \dfrac{2}{1} \Rightarrow \dfrac{C'C}{BC'} = \dfrac{1}{2}$

Adding 1 to both sides, we get

$\dfrac{C'C}{BC'} + 1 = \dfrac{1}{2} + 1 \Rightarrow \dfrac{C'C + BC'}{BC'} = \dfrac{1+2}{2} \Rightarrow \dfrac{BC}{BC'} = \dfrac{3}{2}$

Now, in $\Delta BC'A'$ and ΔBCA, we have $CA \parallel C'A'$

$\therefore$ Using AA similarity, we have

$\Delta BC'A' \sim \Delta BCA$

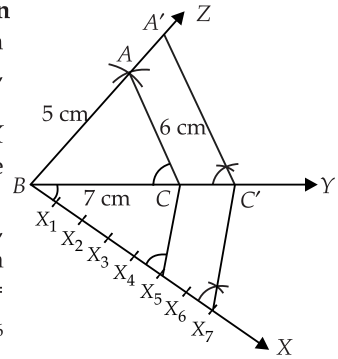
$\Rightarrow \quad \dfrac{A'B}{AB} = \dfrac{A'C'}{AC} = \dfrac{BC'}{BC} = \dfrac{2}{3}$

3. Steps of Construction

Step 1 : Construct a ΔABC such that $AB = 5$ cm, $BC = 7$ cm and $AC = 6$ cm.

Step 2 : Draw a ray BX such that $\angle CBX$ is an acute angle.

Step 3 : Mark 7 points $X_1, X_2, X_3, X_4, X_5, X_6$ and X_7 on BX such that $BX_1 = X_1X_2 = X_2X_3 = X_3X_4 = X_4X_5 = X_5X_6 = X_6X_7$.



Step 4 : Join X_5 to C.

Step 5 : Draw a line through X_7 intersecting BC produced at C' such that $X_5C \parallel X_7C'$.

Step 6 : Draw a line through C' parallel to CA to intersect BA produced at A'.

Thus, $\Delta A'BC'$ is the required triangle.

Justification:

By construction, we have $C'A' \parallel CA$

$\therefore$ Using AA similarity, $\Delta ABC \sim \Delta A'BC'$

$\dfrac{A'B}{AB} = \dfrac{A'C'}{AC} = \dfrac{BC'}{BC}$

Also, $X_7C' \parallel X_5C$ [By construction]

$\therefore \quad \Delta BX_7C' \sim \Delta BX_5C \Rightarrow \dfrac{BC}{BC'} = \dfrac{BX_5}{BX_7}$

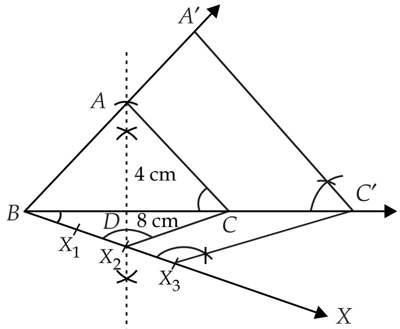
But $\dfrac{BX_5}{BX_7} = \dfrac{5}{7} \Rightarrow \dfrac{BC}{BC'} = \dfrac{5}{7}$ or $\dfrac{BC'}{BC} = \dfrac{7}{5}$

$\therefore \quad \dfrac{A'B}{AB} = \dfrac{A'C'}{AC} = \dfrac{BC'}{BC} = \dfrac{7}{5}$

4. Steps of Construction

Step 1 : Draw $BC = 8$ cm.

Step 2 : Draw the perpendicular bisector of BC which intersects BC at D.



Step 3 : Mark a point A on the perpendicular above BC such that $DA = 4$ cm.

Step 4 : Join AB and AC.

Thus, ΔABC is the required isosceles triangle.

Step 5 : Now, draw a ray BX such that $\angle CBX$ is an acute angle.

Step 6 : On BX, mark three points X_1, X_2 and X_3 such that $BX_1 = X_1X_2 = X_2X_3$.

Step 7 : Join X_2C.

Step 8 : Draw a line through X_3 parallel to X_2C intersecting BC extended at C'.

Step 9 : Draw a line through C' parallel to CA intersecting BA extended at A'.

Thus, $\Delta A'BC'$ is the required triangle.

Justification:

We have $C'A' \parallel CA$ [By construction]

$\therefore$ Using AA similarity, $\Delta A'BC' \sim \Delta ABC$

$\Rightarrow \dfrac{A'B}{AB} = \dfrac{A'C'}{AC} = \dfrac{BC'}{BC}$ ...(i)

In $\Delta BX_3C'$, $X_3C' \parallel X_2C$ [By construction]

$\Rightarrow \dfrac{BC'}{BC} = \dfrac{BX_3}{BX_2}$ [By BPT]

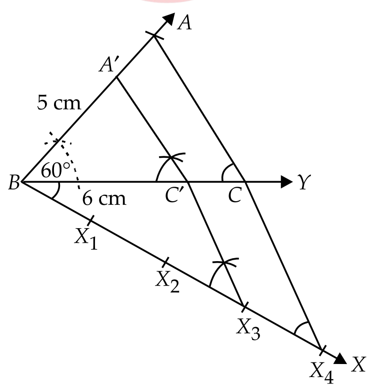
But $\dfrac{BX_3}{BX_2} = \dfrac{3}{2} \Rightarrow \dfrac{BC'}{BC} = \dfrac{3}{2}$

Thus, by (i) $\dfrac{A'B}{AB} = \dfrac{A'C'}{AC} = \dfrac{BC'}{BC} = \dfrac{3}{2}$

5. Steps of Construction

Step 1 : Construct a ΔABC such that $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60°$.

Step 2 : Draw a ray BX such that $\angle CBX$ is an acute angle.



Step 3 : Along BX, mark four points X_1, X_2, X_3, X_4 such that $BX_1 = X_1X_2 = X_2X_3 = X_3X_4$.

Step 4 : Join X_4C and draw a line through X_3 parallel to X_4C to intersect BC at C'.

Step 5 : Draw line through C' parallel to CA to intersect BA at A'.

Thus, $\Delta A'BC'$ is the required triangle.

Justification:

In ΔBX_4C, we have

$X_4C \parallel X_3C'$ [By construction]

$\therefore \dfrac{BX_3}{BX_4} = \dfrac{BC'}{BC}$ [By BPT]

But $\dfrac{BX_3}{BX_4} = \dfrac{3}{4}$ [By construction]

$\Rightarrow \dfrac{BC'}{BC} = \dfrac{3}{4}$ ...(i)

Also, $CA \parallel C'A'$ [By construction]

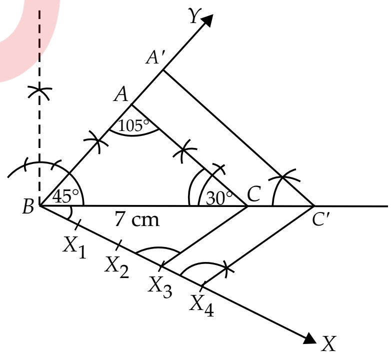
$\therefore \Delta BC'A' \sim \Delta BCA$ [Using AA similarity]

$\Rightarrow \dfrac{A'B}{AB} = \dfrac{A'C'}{AC} = \dfrac{BC'}{BC} = \dfrac{3}{4}$. [From (i)]

6. Steps of Construction

Step 1 : Construct a ΔABC such that $BC = 7$ cm, $\angle B = 45°$, $\angle A = 105°$ and $\angle C = 30°$.

Step 2 : Draw a ray BX making an acute angle $\angle CBX$ with BC.



Step 3 : On BX, mark four points X_1, X_2, X_3 and X_4 such that $BX_1 = X_1X_2 = X_2X_3 = X_3X_4$.

Step 4 : Join X_3C.

Step 5 : Draw a line through X_4 parallel to X_3C intersecting BC extended at C'.

Step 6 : Draw a line through C' parallel to CA intersecting the extended line segment BA at A'.

Thus, $\Delta A'BC'$ is the required triangle.

Justification:

By construction, we have

$C'A' \parallel CA$

$\therefore \Delta ABC \sim \Delta A'BC'$ [AA similarity]

$\Rightarrow \dfrac{A'B}{AB} = \dfrac{A'C'}{AC} = \dfrac{BC'}{BC}$ ...(i)

Also, in $\Delta BX_4C'$,

$X_4C' \parallel X_3C$ [By construction]

$\therefore \Delta BX_4C' \sim \Delta BX_3C$ [AA similarity]

$\therefore \dfrac{BC'}{BC} = \dfrac{BX_4}{BX_3}$

But $\frac{BX_4}{BX_3} = \frac{4}{3} \Rightarrow \frac{BC'}{BC} = \frac{4}{3}$ ...(ii)

From (i) and (ii), we have

$\frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{4}{3}$.

7. Steps of Construction

Step 1 : Construct the right triangle PQR such that $\angle Q = 90°$, $PQ = 4$ cm and $QR = 3$ cm.

Step 2 : Draw a ray QX such that an acute angle $\angle PQX$ is formed.

Step 3 : Mark 5 points X_1, X_2, X_3, X_4 and X_5 on QX such that $QX_1 = X_1X_2 = X_2X_3 = X_3X_4 = X_4X_5$.

Step 4 : Join X_3P.

Step 5 : Draw a line through X_5 parallel to X_3P intersecting the extended line segment QP at P'.

Step 6 : Draw another line through P' parallel to PR intersecting the extended line segment QR at R'.

Thus, $\Delta P'QR'$ is the required triangle.

Justification :

By construction, we have

$P'R' \parallel PR$

$\therefore \quad \Delta RQP \sim \Delta R'QP'$ [AA similarity]

$\Rightarrow \quad \frac{R'Q}{RQ} = \frac{R'P'}{RP} = \frac{QP'}{QP}$ ...(i)

Also, in $\Delta QX_5P'$, $X_5P' \parallel X_3P$ [By construction]

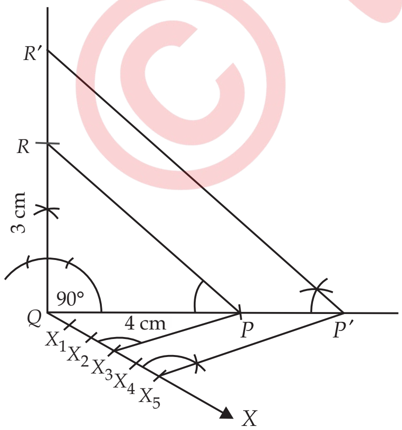
$\therefore \quad \Delta QX_5P' \sim \Delta QX_3P$ [AA similarity]

$\therefore \quad \frac{QP'}{QP} = \frac{QX_5}{QX_3}$

But $\frac{QX_5}{QX_3} = \frac{5}{3} \Rightarrow \frac{QP'}{QP} = \frac{5}{3}$ ...(ii)

From (i) and (ii), we get

$\frac{R'Q}{RQ} = \frac{R'P'}{RP} = \frac{QP'}{QP} = \frac{5}{3}$



EXERCISE - 11.2

1. Steps of Construction

Step 1 : Draw a circle of radius 6 cm. Mark its centre as O.

Step 2 : Take a point P such that $OP = 10$ cm. Join OP.

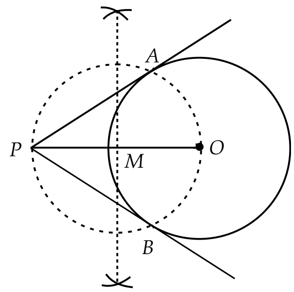
Step 3 : Bisect OP and let M be its mid-point.

Step 4 : Taking M as centre and MP or MO as radius, draw a circle. Let the new circle intersects the given circle at A and B.

Step 5 : Join PA and PB.

Thus, PA and PB are the required tangents.

By measurement, we have $PA = PB = 8$ cm.



Justification:

Join OA and OB

Since PO is a diameter.

$\therefore \quad \angle OAP = 90° = \angle OBP$ [Angles in a semi-circle]

Also, OA and OB are radii of the same circle.

$\Rightarrow \quad PA$ and PB are tangents to the circle.

2. Steps of Construction

Step 1 : Draw two concentric circles with centre O and radii 4 cm and 6 cm.

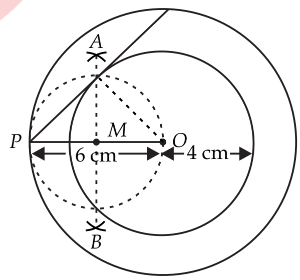
Step 2 : Take any point P on outer circle.

Step 3 : Join PO and bisect it and let the mid-point of PO is represented by M.

Step 4 : Taking M as centre and OM or MP as radius, draw a circle such that this circle intersects the inner circle (of radius 4 cm) at A and B.

Step 5 : Join AP.

Thus, PA is the required tangent.



By measurement, we have $PA = 4.5$ cm.

Justification:

Join OA. As PO is diameter,

$\therefore \quad \angle PAO = 90°$ [Angle in a semi-circle]

$\Rightarrow \quad PA \perp OA$

$\because \quad OA$ is a radius of the inner circle.

$\therefore \quad PA$ has to be a tangent to the inner circle.

Verification of length of PA: In right ΔPAO, $PO = 6$ cm, $OA = 4$ cm.

$\therefore \quad PA = \sqrt{6^2 - 4^2} = \sqrt{36 - 16} = \sqrt{20} = 4.47$ cm

Hence both lengths are approximately equal.

3. Steps of Construction

Step 1 : Draw a circle of radius 3 cm with centre O and draw a diameter.

Step 2 : Extend its diameter on both sides such that $OP = OQ = 7$ cm.

Step 3 : Bisect PO such that M be its mid-point.

Step 4 : Taking M as centre and MO as radius, draw a circle. Let it intersect the given circle at A and B.

Step 5 : Join PA and PB.

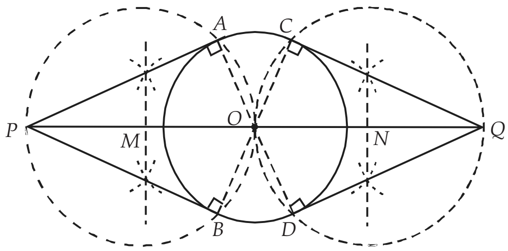
Thus, PA and PB are the two required tangents from P.

Step 6 : Now bisect OQ such that N be its mid-point.

Step 7 : Taking N as centre and NO as radius, draw a circle. Let it intersect the given circle at C and D.

Step 8 : Join QC and QD.

Thus, QC and QD are the required tangents from Q.



Justification :

Join OA to get $\angle OAP = 90°$ [Angle in a semi-circle]

$\Rightarrow$ $PA \perp OA \Rightarrow PA$ is a tangent.

Similarly, $PB \perp OB \Rightarrow PB$ is a tangent.

Now, join OC to get $\angle QCO = 90°$ [Angle in a semi-circle]

$\Rightarrow$ $QC \perp OC \Rightarrow QC$ is a tangent.

Similarly, $QD \perp OD \Rightarrow QD$ is a tangent.

4. Steps of Construction

Step 1 : With centre O and radius 5 cm, draw a circle.

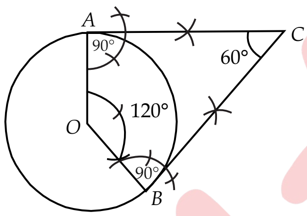
Step 2 : Taking a point A on the circle draw $\angle AOB = 120°$.

Step 3 : Draw a perpendicular on OA at A.

Step 4 : Draw another perpendicular on OB at B.

Step 5 : Let the two perpendiculars meet at C.

Thus, CA and CB are the two required tangents to the given circle which are inclined to each other at 60°.



Justification:

In a quadrilateral $OACB$, using angle sum property, we have

$\angle AOB + \angle OAC + \angle OBC + \angle ACB = 360°$

$\Rightarrow$ $120° + 90° + 90° + \angle ACB = 360°$

$\Rightarrow$ $300° + \angle ACB = 360° \Rightarrow \angle ACB = 360° - 300° = 60°$.

5. Steps of Construction

Step 1 : Draw a line segment $AB = 8$ cm.

Step 2 : Draw a circle with centre A and radius 4 cm, draw another circle with centre B and radius 3 cm.

Step 3 : Bisect the line segment AB. Let its mid-point be M.

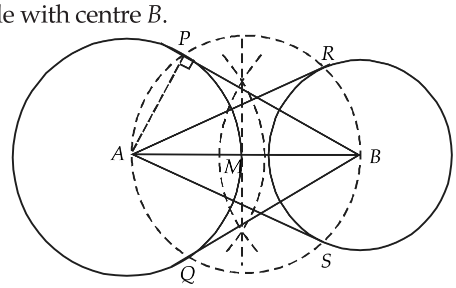
Step 4 : With centre as M and MA (or MB) as radius, draw a circle such that it intersects the two circles at points P, Q, R and S.

Step 5 : Join BP and BQ.

Thus, BP and BQ are the required two tangents from B to the circle with centre A.

Step 6 : Join RA and SA.

Thus, RA and SA are the required two tangents from A to the circle with centre B.



Justification : Let us join A and P.

$\because$ $\angle APB = 90°$ [Angle in a semi-circle]

$\therefore$ $BP \perp AP$

But AP is radius of the circle with centre A.

$\Rightarrow$ BP has to be a tangent to the circle with centre A.

Similarly, BQ has to be tangent to the circle with centre A.

Also, AR and AS are tangents to the circle with centre B.

6. Steps of Construction

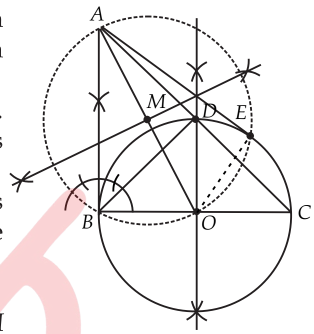
Step 1 : Draw ΔABC such that $AB = 6$ cm, $BC = 8$ cm and $\angle B = 90°$.

Step 2 : Draw $BD \perp AC$. Now, bisect BC and let its mid-point be O.

With O as centre OB as radius, draw the circle passing through B, C and D.

Step 3 : Join AO.

Step 4 : Bisect AO. Let M be the mid-point of AO.



Step 5 : Taking M as centre and MA as radius, draw a circle intersecting the given circle at B and E.

Step 6 : Join AB and AE.

Thus, AB and AE are the required two tangents to the given circle from A.

Justification:

Join OE, then $\angle AEO = 90°$ [Angle in a semi-circle]

$\therefore$ $AE \perp OE$.

But OE is a radius of the given circle.

$\Rightarrow$ AE has to be a tangent to the circle.

Similarly, AB is also a tangent to the given circle.

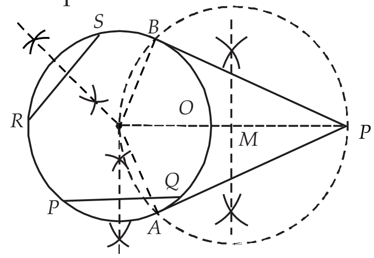
7. Steps of Construction

Step 1 : Draw a circle using a bangle.

Step 2 : Draw two non parallel chords PQ and RS on this circle.

Step 3 : Draw the perpendicular bisectors of PQ and RS such that they intersect at O. Therefore, O is the centre of the given circle.

Step 4 : Take a point P' outside this circle.



Step 5 : Join OP' and bisect it. Let M be the mid-point of OP'.

Step 6 : Taking M as centre and OM as radius, draw a circle. Let it intersect the given circle at A and B.

Step 7 : Join $P'A$ and $P'B$. Thus, $P'A$ and $P'B$ are the required two tangents.

Justification : Join OA and OB.

Since $\angle OAP' = 90°$ [Angle in a semi-circle]

$\therefore$ $P'A \perp OA$. Also OA is a radius.

$\therefore$ $P'A$ has to be a tangent to the given circle.

Similarly, $P'B$ is also a tangent to the given circle.