Areas Related to Circles

CHAPTER 12

NCERT FOCUS

SOLUTIONS

EXERCISE - 12.1

1. Let $r_1 = 19$ cm and $r_2 = 9$ cm

$\therefore$ Circumference of circle-I = $2\pi r_1 = 2\pi(19)$ cm

and circumference of circle-II = $2\pi r_2 = 2\pi\,(9)$ cm

Sum of the circumference of circle-I and circle-II

= $2\pi(19) + 2\pi(9) = 2\pi(19 + 9)$ cm = $2\pi(28)$ cm

Let R be the radius of the circle-III.

$\therefore$ Circumference of circle-III = $2\pi R$

According to the condition, $2\pi R = 2\pi(28)$

$\Rightarrow\ R = \dfrac{2\pi(28)}{2\pi} = 28\,\text{cm}$

Thus, the radius of the new circle = 28 cm.

2. We have, Radius of circle-I, $r_1 = 8$ cm

Radius of circle-II, $r_2 = 6$ cm

$\therefore$ Area of circle-I = $\pi r_1^2 = \pi(8)^2$ cm^2

Area of circle-II = $\pi r_2^2 = \pi(6)^2$ cm^2

Let the radius of the circle-III be R.

$\therefore$ Area of circle-III = πR^2

Now, according to the given condition, we have

$\pi r_1^2 + \pi r_2^2 = \pi R^2$

$\Rightarrow\ \pi(8)^2 + \pi(6)^2 = \pi R^2$

$\Rightarrow\ \pi(8^2 + 6^2) = \pi R^2$

$\Rightarrow\ 8^2 + 6^2 = R^2 \ \Rightarrow\ 64 + 36 = R^2$

$\Rightarrow\ 100 = R^2 \ \Rightarrow\ 10^2 = R^2 \Rightarrow R = 10$

Thus, the radius of the new circle = 10 cm.

3. Diameter of the innermost region = 21 cm

$\therefore$ Radius of the innermost (Gold Scoring) region

$= \dfrac{21}{2} = 10.5$ cm

$\therefore$ Area of Gold region = $\pi(10.5)^2$ cm^2

$= \dfrac{22}{7}\times\left(\dfrac{105}{10}\right)^2 \text{cm}^2 = \dfrac{22}{7}\times\dfrac{105}{10}\times\dfrac{105}{10}\ \text{cm}^2$

$= \dfrac{22\times15\times105}{100}\text{cm}^2 = 346.50\,\text{cm}^2$

Area of the Red region = $\pi(10.5 + 10.5)^2 - \pi(10.5)^2$

$= \pi(21)^2 - \pi(10.5)^2 = \pi[(21)^2 - (10.5)^2]$

$= \dfrac{22}{7}\ [(21 + 10.5)\ (21 - 10.5)]\text{cm}^2 = \dfrac{22}{7}\times31.5\times10.5\ \text{cm}^2$

$= 22\times\dfrac{315}{10}\times\dfrac{15}{10}\text{cm}^2 = 1039.50\ \text{cm}^2$

Since each band is 10.5 cm wide.

$\therefore$ Radius of Gold and Red region = (10.5 + 10.5) = 21 cm.

Area of Blue region = $\pi[(21 + 10.5)^2 - (21)^2]$cm^2

$= \dfrac{22}{7}\ [(31.5)^2 - (21)^2]$ cm^2

$= \dfrac{22}{7}\ [(31.5 + 21)\ (31.5 - 21)]\text{cm}^2 = \dfrac{22}{7}\times52.5\times10.5\ \text{cm}^2$

$= 22\times\dfrac{75}{10}\times\dfrac{105}{10}\text{cm}^2 = 1732.50\ \text{cm}^2$

Similarly, area of Black region

$= \pi[(31.5 + 10.5)^2 - (31.5)^2]\ \text{cm}^2 = \dfrac{22}{7}\ [(42)^2 - (31.5)^2]\ \text{cm}^2$

$= \dfrac{22}{7}\ [(42 - 31.5)\ (42 + 31.5)]\ \text{cm}^2 = \dfrac{22}{7}\times10.5\times73.5\ \text{cm}^2$

$= 22\times\dfrac{15}{10}\times\dfrac{735}{10}\text{cm}^2 = 2425.50\ \text{cm}^2$

Area of White region

$= \pi[(42 + 10.5)^2 - (42)^2]\ \text{cm}^2 = \pi[(52.5)^2 - (42)^2]\ \text{cm}^2$

$= \pi[(52.5 + 42)(52.5 - 42)]\ \text{cm}^2$

$= \dfrac{22}{7}\times94.5\times10.5 = 22\times\dfrac{945}{10}\times\dfrac{15}{10} = 3118.5\ \text{cm}^2$

4. Diameter of a wheel = 80 cm

$\therefore$ Radius of the wheel = $\dfrac{80}{2} = 40$ cm

So, circumference of the wheel = $2\pi r = 2\times\dfrac{22}{7}\times40$ cm

$\Rightarrow$ Distance covered by a wheel in one revolution

$= \dfrac{2\times22\times40}{7}\text{cm}$

Distance travelled by the car in 1 hour = 66 km

= 66 × 1000 × 100 cm

$\therefore$ Distance travelled in 10 minutes

$= \dfrac{66\times1000\times100}{60}\times10\ \text{cm} = 11\times100000\ \text{cm}$

Now, number of revolutions

$= \dfrac{[\text{Distance travelled in 10 minutes}]}{[\text{Distance travelled in one revolution}]}$

$= \dfrac{[1100000]}{\left[\dfrac{2\times22\times40}{7}\right]} = \dfrac{1100000\times7}{2\times22\times40} = 4375$

Thus, the required number of revolutions = 4375.

5. **(a) :** We have, Area of the circle = Circumference of the circle

$\Rightarrow\ \pi r^2 = 2\pi r \Rightarrow \pi r^2 - 2\pi r = 0$

$\Rightarrow\ r^2 - 2r = 0$

$\Rightarrow\ r(r - 2) = 0 \Rightarrow r = 0$ or $r = 2$

But r cannot be zero

$\therefore\ r = 2$ units.

Thus, the radius of circle is 2 units.

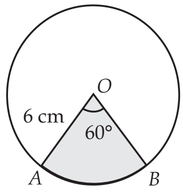
EXERCISE - 12.2

1. Here $r = 6$ cm and $\theta = 60°$

$\therefore$ Area of a sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{60°}{360°} \times \frac{22}{7} \times 6 \times 6 \text{ cm}^2$

$= \frac{22}{7} \times 6 \text{ cm}^2 = \frac{132}{7} \text{ cm}^2.$



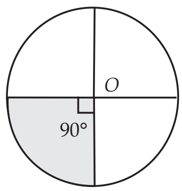
2. Let radius of the circle $= r$

Given, circumference of circle = 22 cm

$\therefore \quad 2\pi r = 22 \quad \Rightarrow \quad 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow \quad r = 22 \times \frac{7}{22} \times \frac{1}{2} = \frac{7}{2}$ cm

Here, $\theta = 90°$



$\therefore$ Area of quadrant of the circle

$= \frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{ cm}^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{cm}^2 = \frac{77}{8} \text{cm}^2.$

3. Length of minute hand = radius of the circle

$\Rightarrow r = 14$ cm

$\because$ Angle swept by the minute hand in 60 minutes = 360°

$\therefore$ Angle swept by the minute hand in 5 minutes

$= \frac{360°}{60°} \times 5 = 30°$

Now, area of the sector with $r = 14$ cm and $\theta = 30°$

$= \frac{\theta}{360°} \times \pi r^2 = \frac{30°}{360°} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$

$= \frac{11 \times 14}{3} \text{ cm}^2 = \frac{154}{3} \text{cm}^2$

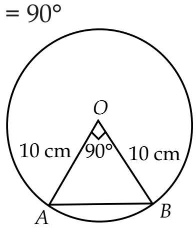
Thus, the required area swept by the minute hand in 5 minutes $= \frac{154}{3} \text{cm}^2.$

4. Length of the radius $(r) = 10$ cm, $\theta = 90°$

Area of the sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{90°}{360°} \times \frac{314}{100} \times 10 \times 10 \text{ cm}^2$

$= \frac{1}{4} \times 314 \text{ cm}^2 = \frac{157}{2} \text{cm}^2 = 78.5 \text{ cm}^2$



(i) Area of the minor segment

= [Area of the minor sector] – [Area of right ΔAOB]

$= [78.5 \text{ cm}^2] - \left[\frac{1}{2} \times 10 \times 10 \text{ cm}^2\right]$

$= 78.5 \text{ cm}^2 - 50 \text{ cm}^2 = 28.5 \text{ cm}^2$

(ii) Area of the major sector

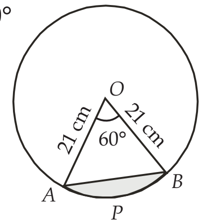
= [Area of the circle] – [Area of the minor sector]

$= \pi r^2 - 78.5 \text{ cm}^2 = \left[\frac{314}{100} \times 10 \times 10 - 78.5\right] \text{cm}^2$

$= (314 - 78.5) \text{ cm}^2 = 235.5 \text{ cm}^2.$

5. Here, radius, $r = 21$ cm and $\theta = 60°$

(i) Length of arc APB



$= \frac{\theta}{360°} \times 2\pi r = \left(\frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21\right) \text{cm}$

$= \left(\frac{1}{6} \times 2 \times 22 \times 3\right) \text{cm}$

$= \left(\frac{1}{6} \times 132\right) \text{cm} = 22 \text{ cm}$

(ii) Area of the sector with sector angle 60°

$= \frac{\theta}{360°} \times \pi r^2 = \frac{60°}{360°} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$

$= 11 \times 21 \text{ cm}^2 = 231 \text{ cm}^2$

(iii) Area of the segment APB

= [Area of the sector $AOBP$] – [Area of ΔAOB] ...(1)

In ΔAOB, $OA = OB = 21$ cm

$\therefore \quad \angle A = \angle B = 60°$ [$\because \angle O = 60°$]

$\Rightarrow$ AOB is an equilateral triangle.

$\therefore$ $AB = 21$ cm

$\therefore$ Area of $\Delta AOB = \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 21 \times 21 \text{ cm}^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2$ ...(2)

From (1) and (2), we have

Area of segment $= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2$

6. Here, radius $(r) = 15$ cm and Sector angle $(\theta) = 60°$

$\therefore$ Area of the sector $= \frac{\theta}{360°} \times \pi r^2$

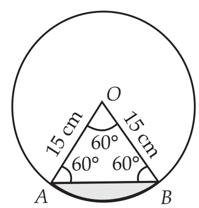
$= \frac{60°}{360°} \times \frac{314}{100} \times 15 \times 15 \text{ cm}^2 = \frac{157 \times 3}{4} = 117.75 \text{ cm}^2$

Since $\angle O = 60°$ and $OA = OB = 15$ cm

$\Rightarrow \quad \angle A = \angle B = 60°$

$\therefore$ AOB is an equilateral triangle.

$\therefore$ $AB = 15$ cm



Now, area of $\Delta AOB = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 15 \times 15 \text{ cm}^2 = \frac{225\sqrt{3}}{4} \text{cm}^2$

$= \frac{225 \times 1.73}{4} \text{cm}^2 = 97.3125 \text{ cm}^2$

Now, area of the minor segment

= (Area of minor sector) – (Area of ΔAOB)

$= (117.75 - 97.3125) \text{ cm}^2 = 20.4375 \text{ cm}^2$

$\therefore$ Area of the major segment

= [Area of the circle] – [Area of the minor segment]

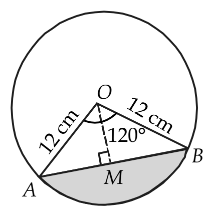
$= \pi r^2 - 20.4375 \text{ cm}^2 = \left[\frac{314}{100} \times 15^2\right] - 20.4375 \text{ cm}^2$

$= 706.5 - 20.4375 \text{ cm}^2 = 686.0625 \text{ cm}^2.$

7. Here, $\theta = 120°$ and $r = 12$ cm

$\therefore$ Area of the sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{120°}{360°} \times \frac{314}{100} \times 12 \times 12 \text{ cm}^2$



$= \frac{314 \times 4 \times 12}{100} \text{ cm}^2 = \frac{15072}{100} \text{ cm}^2 = 150.72 \text{ cm}^2$ ...(1)

Draw, $OM \perp AB$

$\Rightarrow$ OM is the perpendicular bisector of AB.

$\therefore$ $AM = BM = \frac{1}{2} AB$

In ΔAOB, $\angle O = 120°$

$\Rightarrow$ $\angle A + \angle B = 180° - 120° = 60°$

$\because$ $OB = OA = 12$ cm $\Rightarrow$ $\angle A = \angle B = 30°$

So, $\frac{OM}{OA} = \sin 30° = \frac{1}{2} \Rightarrow OM = OA \times \frac{1}{2} = 12 \times \frac{1}{2} = 6$ cm

and $\frac{AM}{OA} = \cos 30° = \frac{\sqrt{3}}{2}$

$\Rightarrow$ $AM = \frac{\sqrt{3}}{2} OA = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3}$ cm

$\therefore$ $AB = 2AM = 12\sqrt{3}$ cm

Now, area of $\Delta AOB = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 12\sqrt{3} \times 6 \text{ cm}^2 = 36\sqrt{3} \text{ cm}^2$

$= 36 \times 1.73 \text{ cm}^2 = 62.28 \text{ cm}^2$ ...(2)

From (1) and (2), we have

Area of the minor segment

= [Area of sector] – [Area of ΔAOB]

= [150.72 cm^2] – [62.28 cm^2] = 88.44 cm^2

8. Here, length of the rope = 5 m

$\therefore$ Radius of the circular region grazed by the horse = 5 m

(i) Area of the circular portion grazed

$= \frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times \frac{314}{100} \times 5 \times 5 \text{ m}^2 = \frac{1}{4} \times \frac{314}{4} \text{m}^2$

$= \frac{157}{8} \text{m}^2 = 19.625 \text{ m}^2$

(ii) When length of the rope is increased to 10 m

$\therefore$ $r = 10$ m

$\Rightarrow$ Area of the new circular portion grazed

$= \frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times \frac{314}{100} \times (10)^2 \text{ m}^2$

$= \frac{1}{4} \times 314 \text{ m}^2 = 78.5 \text{ m}^2$

$\therefore$ Increase in the grazing area

= (78.5 – 19.625) m^2 = 58.875 m^2.

9. Diameter of the circle = 35 mm

$\therefore$ Radius (r) = $\frac{35}{2}$mm

(i) Circumference of circle = $2\pi r$

$= 2 \times \frac{22}{7} \times \frac{35}{2} \text{mm} = 22 \times 5 = 110$ mm

Length of 1 piece of wire used to make diameter to divide the circle into 10 equal sectors = 35 mm

$\therefore$ Length of 5 pieces = 5 × 35 = 175 mm

$\therefore$ Total length of the silver wire

= (110 + 175) mm = 285 mm

(ii) Since the circle is divided into 10 equal sectors.

$\therefore$ Sector angle, $\theta = \frac{360°}{10} = 36°$

Now, area of each sector

$= \frac{\theta}{360°} \times \pi r^2 = \frac{36°}{360°} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \text{mm}^2$

$= \frac{11 \times 35}{4} \text{mm}^2 = \frac{385}{4} \text{mm}^2.$

10. Here, radius (r) = 45 cm

Since circle is divided into 8 equal parts.

$\therefore$ Sector angle corresponding to each part, $\theta = \frac{360°}{8} = 45°$

$\therefore$ Area of a sector (part)

$= \frac{\theta}{360°} \times \pi r^2 = \frac{45°}{360°} \times \frac{22}{7} \times 45 \times 45 \text{ cm}^2$

$= \frac{11 \times 45 \times 45}{4 \times 7} \text{cm}^2 = \frac{22275}{28} \text{cm}^2$

$\therefore$ The required area between the two consecutive ribs

$= \frac{22275}{28} \text{cm}^2$

11. Here, radius (r) = 25 cm

Sector angle (θ) = 115°

$\therefore$ Total area cleaned by each sweep of the blades

$= \left[\frac{\theta}{360°} \times \pi r^2\right] \times 2$ [$\because$ There are 2 blades]

$= \left[\frac{115°}{360°} \times \frac{22}{7} \times 25 \times 25\right] \times 2 \text{ cm}^2$

$= \frac{23 \times 11 \times 25 \times 25}{18 \times 7} \text{cm}^2 = \frac{158125}{126} \text{cm}^2$

12. Here, radius (r) = 16.5 km

Sector angle (θ) = 80°

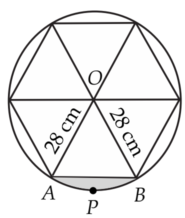
$\therefore$ Area of the sea surface over which the ships are warned $= \frac{\theta}{360°} \times \pi r^2 = \frac{80°}{360°} \times \frac{314}{100} \times \frac{165}{10} \times \frac{165}{10} \text{km}^2$

$= \frac{157 \times 11 \times 11}{100} \text{km}^2 = \frac{18997}{100} \text{km}^2 = 189.97 \text{ km}^2$

13. Here, r = 28 cm

Since, the circle is divided into six equal sectors.

$\therefore$ Sector angle, $\theta = \frac{360°}{6} = 60°$



$\therefore$ Area of each sector

$= \frac{60°}{360°} \times \frac{22}{7} \times 28 \times 28 \text{ cm}^2$

$= \frac{44 \times 28}{3} \text{cm}^2 = 410.67 \text{cm}^2$ ...(1)

Now, area of 1 design = Area of segment APB

= Area of sector $APBO$ – Area of ΔAOB ...(2)

In ΔAOB, $\angle AOB = 60°$, $OA = OB = 28$ cm

$\therefore$ $\angle OAB = 60°$ and $\angle OBA = 60°$

$\Rightarrow$ ΔAOB is an equilateral triangle.

$\therefore$ Area of $\Delta AOB = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 28 \times 28 = 14 \times 14\sqrt{3} \text{ cm}^2$

$= 14 \times 14 \times 1.7\ \text{cm}^2 = 333.2\ \text{cm}^2$ ...(3)

Now, from (1), (2) and (3), we have

Area of segment $APB = 410.67\ \text{cm}^2 - 333.2\ \text{cm}^2$

$= 77.47\ \text{cm}^2$

$\Rightarrow$ Area of 1 design $= 77.47\ \text{cm}^2$

$\therefore$ Area of the 6 equal designs $= 6 \times (77.47)\ \text{cm}^2$

$= 464.82\ \text{cm}^2$

So, cost of making the design at the rate of ₹ 0.35 per cm²

= ₹ (0.35 × 464.82) = ₹ 162.68

14. **(d) :** Here, radius $(r) = R$

Angle of sector $(\theta) = p$

$\therefore$ Area of the sector

$$= \frac{\theta}{360°} \times \pi r^2 = \frac{p}{360} \times \pi R^2 = \frac{2}{2} \times \left(\frac{p}{360} \times \pi R^2\right) = \frac{p}{720} \times 2\pi R^2$$

EXERCISE - 12.3

1. Since O is the centre of the circle.

$\therefore$ QOR is a diameter.

$\Rightarrow$ $\angle RPQ = 90°$ [$\because$ Angle in a semi-circle is 90°]

Now, in right ΔRPQ,

$RQ^2 = PQ^2 + PR^2$ [By Pythagoras theorem]

$\Rightarrow$ $RQ^2 = 24^2 + 7^2 = 576 + 49 = 625$

$\Rightarrow$ $RQ = \sqrt{625} = 25$ cm

$\therefore$ Radius of circle $= \frac{25}{2}$ cm

$\therefore$ Area of ΔRPQ

$$= \frac{1}{2} \times PQ \times PR = \frac{1}{2} \times 24 \times 7\text{cm}^2 = 12 \times 7\ \text{cm}^2 = 84\ \text{cm}^2$$

Now, area of semi-circle

$$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} = \frac{11 \times 625}{7 \times 4}\text{cm}^2$$

$$= \frac{6875}{28}\text{cm}^2 = 245.54\ \text{cm}^2$$

$\therefore$ Area of the shaded portion $= 245.54\ \text{cm}^2 - 84\ \text{cm}^2$

$= 161.54\ \text{cm}^2$

2. Radius of the outer circle, $R = 14$ cm and $\theta = 40°$

$\therefore$ Area of the sector $AOC = \frac{\theta}{360°} \times \pi R^2$

$$= \frac{40°}{360°} \times \frac{22}{7} \times 14 \times 14\text{cm}^2$$

$$= \frac{1}{9} \times 22 \times 2 \times 14\text{cm}^2 = \frac{616}{9}\text{cm}^2$$

Radius of the inner circle, $r = 7$ cm and $\theta = 40°$

$\therefore$ Area of the sector $BOD = \frac{\theta}{360°} \times \pi r^2$

$$= \left(\frac{40°}{360°} \times \frac{22}{7} \times 7 \times 7\right)\text{cm}^2 = \left(\frac{1}{9} \times 22 \times 7\right)\text{cm}^2 = \frac{154}{9}\text{cm}^2$$

Now, area of the shaded region = Area of sector AOC

– Area of sector BOD

$$= \left(\frac{616}{9} - \frac{154}{9}\right)\text{cm}^2 = \frac{1}{9}(616 - 154)\text{cm}^2$$

$$= \frac{1}{9} \times 462\ \text{cm}^2 = \frac{154}{3}\ \text{cm}^2$$

3. Side of the square = 14 cm

$\therefore$ Area of the square $ABCD = 14 \times 14\ \text{cm}^2 = 196\ \text{cm}^2$

Now, diameter of the semi-circle = Side of the square

= 14 cm

$\Rightarrow$ Radius of each of the semi-circles $= \frac{14}{2} = 7\text{cm}$

$\therefore$ Area of the semi-circle APD

$$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77\text{cm}^2$$

Area of the semi-circle $BPC = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77\text{cm}^2$

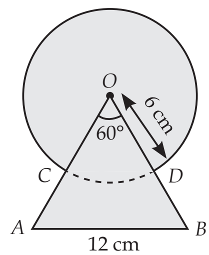
$\therefore$ Area of the shaded region

= Area of the square – [Area of semi-circle APD

+ Area of semi-circle BPC]

$= 196 - [77 + 77] = (196 - 154)\ \text{cm}^2 = 42\ \text{cm}^2$

4. Area of the circle with radius 6 cm $= \pi r^2$



$$= \frac{22}{7} \times 6 \times 6\ \text{cm}^2 = \frac{792}{7}\text{cm}^2$$

Area of equilateral triangle, having side, $a = 12$ cm, is given by

$$\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 12 \times 12\text{cm}^2 = 36\sqrt{3}\ \text{cm}^2$$

$\because$ Each angle of an equilateral triangle = 60°

$\therefore$ $\angle AOB = 60°$

$\therefore$ Area of sector $COD = \frac{\theta}{360°} \times \pi r^2$

$$= \frac{60°}{360°} \times \frac{22}{7} \times 6 \times 6\ \text{cm}^2 = \frac{22 \times 6}{7}\text{cm}^2 = \frac{132}{7}\text{cm}^2$$

Now, area of the shaded region = [Area of the circle]

+ [Area of the equilateral triangle] –

[Area of the sector COD]

$$= \left[\frac{792}{7} + 36\sqrt{3} - \frac{132}{7}\right]\text{cm}^2 = \left[\frac{660}{7} + 36\sqrt{3}\right]\text{cm}^2.$$

5. Side of the square = 4 cm

$\therefore$ Area of the square $ABCD = 4 \times 4\ \text{cm}^2 = 16\ \text{cm}^2$

$\because$ Each corner has a quadrant of circle of radius 1 cm.

$\therefore$ Area of all the 4 quadrants of circle $= 4 \times \frac{1}{4}\pi r^2 = \pi r^2$

$$= \frac{22}{7} \times 1 \times 1\text{cm}^2 = \frac{22}{7}\text{cm}^2$$

Diameter of the middle circle = 2 cm

$\Rightarrow$ Radius of the middle circle = 1 cm

$\therefore$ Area of the middle circle $= \pi r^2$

$$= \frac{22}{7} \times 1 \times 1\text{cm}^2 = \frac{22}{7}\text{cm}^2$$

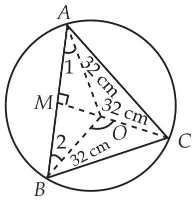
Now, area of the shaded region = [Area of the square $ABCD$] – [(Area of the 4 quadrants of circle)

+ (Area of the middle circle)]

$$= [16] - \left[\frac{22}{7} + \frac{22}{7}\right] = 16 - 2 \times \frac{22}{7}$$

$$= 16 - \frac{44}{7} = \frac{112 - 44}{7} = \frac{68}{7}\text{cm}^2$$

6. Area of the circle = πr^2



$= \frac{22}{7} \times 32 \times 32 \text{ cm}^2 = \frac{22528}{7} \text{cm}^2$

Let O is the centre of the circle.

$\therefore \quad AO = OB = OC = 32$ cm

$\Rightarrow \quad \angle AOB = \angle BOC = \angle AOC = 120°$

Now, in ΔAOB, $\angle 1 + \angle 2 = 180° - 120° = 60°$

and $OA = OB \quad \Rightarrow \quad \angle 1 = \angle 2$

$\therefore \quad \angle 1 = 30°$

Draw, $OM \perp AB$, then

$\frac{OM}{OA} = \sin 30° = \frac{1}{2} \Rightarrow OM = OA \times \frac{1}{2}$

$\Rightarrow \quad OM = 32 \times \frac{1}{2} = 16 \text{ cm}$

Also, $\frac{AM}{AO} = \cos 30° = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad AM = \frac{\sqrt{3}}{2} \times AO = \frac{\sqrt{3}}{2} \times 32$

$\Rightarrow \quad 2AM = AB = 2 \times \left(\frac{\sqrt{3}}{2} \times 32\right) = 32\sqrt{3} \text{ cm}$

Area of equilateral $\Delta ABC = \frac{\sqrt{3}}{4}(AB)^2$

$= \frac{\sqrt{3}}{4}(32\sqrt{3})^2 = 768\sqrt{3} \text{ cm}^2$

Now, area of the design = [Area of the circle] – [Area of the equilateral ΔABC]

$= \left(\frac{22528}{7} - 768\sqrt{3}\right) \text{cm}^2$

7. Side of the square $ABCD$ = 14 cm

$\therefore$ Area of the square $ABCD$ = 14 × 14 cm^2 = 196 cm^2.

$\because$ Circles touch each other externally

$\therefore$ Radius of the circle $= \frac{14}{2} = 7 \text{cm}$

Now, area of a sector of radius 7 cm and sector angle

$(\theta)\ 90° = \frac{90°}{360°} \times \frac{22}{7} \times 7 \times 7 \text{cm}^2 = \frac{11 \times 7}{2} \text{cm}^2$

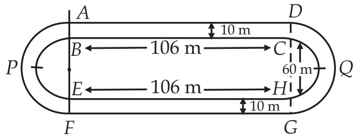
$\therefore$ Area of 4 sectors $= 4 \times \left[\frac{11 \times 7}{2}\right]$

= 2 × 11 × 7cm^2 = 154 cm^2

Area of the shaded region = [Area of the square $ABCD$] – [Area of the 4 sectors]

= 196 cm^2 – 154 cm^2 = 42 cm^2.

8. (i)



Distance around the track along its inner edge

$= BC + EH + \widehat{BPE} + \widehat{CQH}$

$= 106 + 106 + \frac{1}{2}(2\pi r) + \frac{1}{2}(2\pi r)$

$= 212 + \frac{1}{2}\left(2 \times \frac{22}{7} \times 30\right) + \frac{1}{2}\left(2 \times \frac{22}{7} \times 30\right)$

$\left[\because \ r = \frac{1}{2}BE = \frac{1}{2} \times 60 = 30 \text{ m}\right]$

$= \left(212 + \frac{1320}{7}\right) \text{m} = \frac{2804}{7} \text{m}$

(ii) Now, area of the track = Area of the shaded region = (Area of rectangle $ABCD$) + (Area of rectangle $EFGH$) + 2 [(Area of semi-circle of radius 40 m) – (Area of semi-circle of radius 30 cm)]

$\Rightarrow$ Area of the track = (106 × 10 m^2) + (106 × 10 m^2)

$+ 2\left[\frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2\right] \text{m}^2$

$= 1060 \text{ m}^2 + 1060 \text{ m}^2 + 2\left[\frac{1}{2} \times \frac{22}{7} \times (40^2 - 30^2)\right] \text{m}^2$

$= 2120 \text{m}^2 + 2 \times \frac{1}{2} \times \frac{22}{7}$ [(40 + 30) × (40 – 30)] m^2

$= 2120 \text{m}^2 + \frac{22}{7} \times 70 \times 10 \text{ m}^2$

= 2120 m^2 + 2200 m^2 = 4320 m^2

9. Given, O is the centre of the circle, OA = 7 cm

$\Rightarrow \quad AB = 2OA = 2 \times 7 = 14$ cm

Now, $OC = OA = 7$ cm

$\because AB$ and CD are perpendicular to each other

$\Rightarrow \quad OC \perp AB$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times AB \times OC$

$= \frac{1}{2} \times 14 \text{ cm} \times 7 \text{cm} = 49 \text{ cm}^2$

Again, $OD = OA = 7$ cm

$\therefore$ Radius of the smaller circle $= \frac{1}{2}(OD) = \frac{1}{2} \times 7 = \frac{7}{2}$ cm

$\therefore$ Area of the smaller circle $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{cm}^2$

$= \frac{11 \times 7}{2} = \frac{77}{2} \text{cm}^2$

$\because$ Radius of the bigger circle = 7 cm

$\therefore$ Area of the semi-circle $OACB$

$= \frac{1}{2}\left(\frac{22}{7} \times 7 \times 7\right) \text{cm}^2$ = 11 × 7 cm^2 = 77 cm^2

Now, Area of the shaded region = [Area of the smaller circle] + [Area of the bigger semi-circle $OACB$] – [Area of ΔABC]

$= \frac{77}{2} \text{cm}^2 + 77 \text{cm}^2 - 49 \text{cm}^2 = \left(\frac{77 + 154 - 98}{2}\right) \text{cm}^2$

$= \left(\frac{231 - 98}{2}\right) \text{cm}^2 = \frac{133}{2} \text{cm}^2 = 66.5 \text{ cm}^2.$

10. Given, area of ΔABC = 17320.5 cm^2

$\therefore \quad \frac{\sqrt{3}}{4} \times (\text{side})^2 = 17320.5 \Rightarrow \frac{1.73205}{4}(\text{side})^2 = 17320.5$

$\Rightarrow \frac{173205}{400000}(\text{side})^2 = \frac{173205}{10} \Rightarrow (\text{side})^2 = \frac{173205}{10} \times \frac{400000}{173205}$

$\Rightarrow \quad (\text{side})^2 = 40000 \Rightarrow (\text{side})^2 = (200)^2 \Rightarrow \text{side} = 200 \text{ cm}$

$\therefore$ Radius of each circle, $r = \frac{200}{2} = 100$ cm

Since each angle of an equilateral triangle is 60°.

$\therefore$ $\angle A = \angle B = \angle C = 60°$

$\therefore$ Area of a sector having angle of sector as 60° and radius 100 cm $= \frac{\theta}{360°} \times \pi r^2 = \frac{60°}{360°} \times \frac{314}{100} \times 100 \times 100 \text{ cm}^2$

$= \frac{1}{3} \times \frac{157}{100} \times 100 \times 100 \text{ cm}^2 = \frac{15700}{3} \text{cm}^2$

Area of 3 equal sectors $= 3 \times \frac{15700}{3} \text{cm}^2 = 15700 \text{ cm}^2$

Now, area of the shaded region

= [Area of the equilateral triangle ABC]

– [Area of 3 equal sectors]

$= 17320.5 \text{ cm}^2 - 15700 \text{ cm}^2 = 1620.5 \text{ cm}^2$

11. $\because$ The circles touch each other externally.

$\therefore$ The side of the square $ABCD$

= 3 × diameter of a circle = 3 × (2 × radius of a circle)

= 3 × (2 × 7cm) = 42 cm

$\therefore$ Area of the square $ABCD = 42 \times 42 \text{ cm}^2 = 1764 \text{ cm}^2$.

Now, area of one circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7 \text{cm}^2 = 154 \text{ cm}^2$

$\therefore$ Total area of 9 circles $= 154 \times 9 = 1386 \text{ cm}^2$

$\therefore$ Area of the remaining portion of the handkerchief = $(1764 - 1386) \text{ cm}^2 = 378 \text{ cm}^2$.

12. (i) Here, centre of the circle is O and radius = 3.5 cm.

$\therefore$ Area of the quadrant $OACB = \frac{1}{4} \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \text{cm}^2 = \frac{11 \times 7}{8} \text{cm}^2 = \frac{77}{8} \text{cm}^2$

(ii) Area of $\Delta BOD = \frac{1}{2} \times OB \times OD = \frac{1}{2} \times 3.5 \times 2$

$= \frac{1}{2} \times \frac{35}{10} \times 2 \text{cm}^2 = \frac{7}{2} \text{cm}^2$

$\therefore$ Area of the shaded region = (Area of the quadrant $OACB$) – (Area of ΔBOD)

$= \left(\frac{77}{8} - \frac{7}{2}\right) \text{cm}^2 = \frac{77 - 28}{8} \text{cm}^2 = \frac{49}{8} \text{cm}^2$.

13. $OABC$ is a square such that its side OA = 20 cm

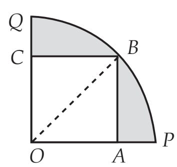
$\therefore$ $OB^2 = OA^2 + AB^2$

$\Rightarrow$ $OB^2 = 20^2 + 20^2$

$= 400 + 400 = 800$

$\Rightarrow$ $OB = \sqrt{800} = 20\sqrt{2}$ cm

$\Rightarrow$ Radius of the circle $= 20\sqrt{2}$ cm



Now, area of the quadrant $OPBQ = \frac{1}{4} \pi r^2$

$= \frac{1}{4} \times \frac{314}{100} \times 800 \text{ cm}^2 = 314 \times 2 = 628 \text{ cm}^2$

Area of the square $OABC = 20 \times 20 \text{ cm}^2 = 400 \text{ cm}^2$

$\therefore$ Area of the shaded region $= 628 \text{ cm}^2 - 400 \text{ cm}^2 = 228 \text{ cm}^2$.

14. $\because$ Radius of bigger circle, R = 21 cm and sector angle $\theta = 30°$

$\therefore$ Area of the sector $OAB = \frac{\theta}{360°} \times \pi R^2$

$= \frac{30°}{360°} \times \frac{22}{7} \times 21 \times 21 \text{cm}^2 = \frac{11 \times 21}{2} \text{cm}^2 = \frac{231}{2} \text{cm}^2$

Again, radius of the smaller circle, r = 7 cm

Also, the sector angle is 30°

$\therefore$ Area of the sector $OCD = \frac{\theta}{360°} \times \pi r^2$

$= \frac{30°}{360°} \times \frac{22}{7} \times 7 \times 7 \text{cm}^2 = \frac{77}{6} \text{cm}^2$

$\therefore$ Area of the shaded region

$= \left[\frac{231}{2} - \frac{77}{6}\right] \text{cm}^2 = \frac{693 - 77}{6} \text{cm}^2 = \frac{616}{6} \text{cm}^2 = \frac{308}{3} \text{cm}^2$.

15. Radius of the quadrant, r = 14 cm

Therefore, area of the quadrant $ABPC$

$= \frac{1}{4} \pi r^2 = \left[\frac{1}{4} \times \frac{22}{7} \times 14 \times 14\right] \text{cm}^2 = 22 \times 7 \text{ cm}^2 = 154 \text{ cm}^2$

Area of right $\Delta ABC = \frac{1}{2} \times 14 \times 14 \text{ cm}^2 = 98 \text{ cm}^2$

$\Rightarrow$ Area of segment $BPC = (154 - 98) \text{ cm}^2 = 56 \text{ cm}^2$

Now, in right ΔABC, $AC^2 + AB^2 = BC^2$

$\Rightarrow$ $14^2 + 14^2 = BC^2$ $\Rightarrow$ $196 + 196 = BC^2$

$\Rightarrow$ $BC^2 = 392$ $\Rightarrow$ $BC = 14\sqrt{2}$ cm.

$\therefore$ Radius of the semi-circle $BQC = \frac{14\sqrt{2}}{2} \text{cm} = 7\sqrt{2}$ cm

$\therefore$ Area of the semi-circle BQC

$= \frac{1}{2} \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times (7\sqrt{2})^2 = 11 \times 7 \times 2 \text{ cm}^2 = 154 \text{ cm}^2$

Now, area of the shaded region

= [Area of semi-circle BQC] – [Area of segment BPC]

$= 154 \text{ cm}^2 - 56 \text{ cm}^2 = 98 \text{ cm}^2$.

16. $\because$ Side of the square = 8 cm

$\therefore$ Area of the square $(ABCD) = 8 \times 8 \text{ cm}^2 = 64 \text{ cm}^2$

Now, radius of the quadrant $ADQB$ = 8 cm

$\therefore$ Area of the quadrant $ADQB = \frac{1}{4} \times \frac{22}{7} \times 8 \times 8 \text{cm}^2$

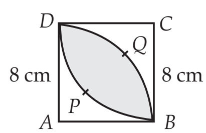
$= \frac{1}{4} \times \frac{22}{7} \times 64 \text{ cm}^2 = \frac{22 \times 16}{7} \text{cm}^2$

Similarly, area of the quadrant

$BPDC = \frac{22 \times 16}{7} \text{cm}^2$

Sum of the two quadrant

$= 2\left[\frac{22 \times 16}{7}\right] \text{cm}^2 = \frac{704}{7} \text{cm}^2$



Now, area of design = [Sum of the area of the two quadrant] – [Area of the square $ABCD$]

$= \frac{704}{7} \text{cm}^2 - 64 \text{cm}^2 = \frac{704 - 448}{7} \text{cm}^2 = \frac{256}{7} \text{cm}^2$.