Surface Areas and Volumes

CHAPTER 13

NCERT FOCUS

SOLUTIONS

EXERCISE - 13.1

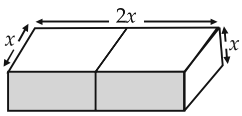
1. Let the edge of each cube = x

Given, volume of each cube

$= 64\text{ cm}^3$

$\therefore \quad x^3 = 64\text{ cm}^3$

$\Rightarrow \quad x = 4\text{ cm}$



Now, length of the resulting cuboid (l) = $2x$ cm = 8 cm

Breadth of the resulting cuboid (b) = x cm = 4 cm

Height of the resulting cuboid (h) = x cm = 4 cm

$\therefore$ Surface area of the cuboid = $2(lb + bh + hl)$

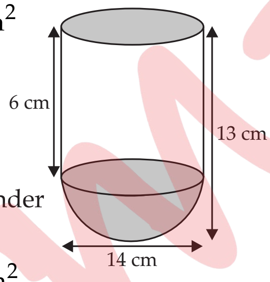
$= 2[(8 \times 4) + (4 \times 4) + (4 \times 8)]\text{ cm}^2$

$= 2[32 + 16 + 32]\text{ cm}^2 = 2[80]\text{ cm}^2 = 160\text{ cm}^2$

2. For hemispherical part, radius (r) $= \dfrac{14}{2} = 7\text{ cm}$

$\therefore$ Curved surface area of hemisphere = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 7 \times 7\text{cm}^2 = 308\text{ cm}^2$$



Total height of vessel = 13 cm

$\therefore$ Height of cylinder

$= (13 - 7)\text{ cm} = 6\text{ cm}$

and radius(r) = 7 cm

$\therefore$ Curved surface area of cylinder

$= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 6\text{cm}^2 = 264\text{cm}^2$$

$\therefore$ Inner surface area of vessel $= (308 + 264)\text{ cm}^2$

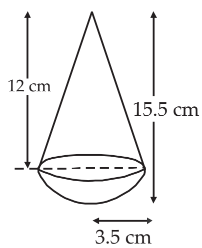
$= 572\text{ cm}^2$

3. Given radius of cone (r)

= radius of hemisphere (r)

= 3.5 cm, height of cone (h)

= (15.5 – 3.5) cm = 12 cm



Also, slant height (l) $= \sqrt{h^2 + r^2}$

$= \sqrt{12^2 + (3.5)^2} = \sqrt{156.25}$

= 12.5 cm

Total surface area of the toy = curved surface area of conical part + curved surface area of hemispherical part

$$= \pi rl + 2\pi r^2 = \pi r(l + 2r) = \frac{22}{7} \times \frac{35}{10}(12.5 + 2 \times 3.5)\text{ cm}^2$$

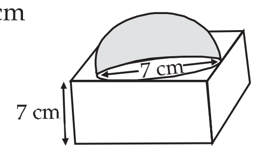
$= 11 \times (12.5 + 7)\text{ cm}^2 = 11 \times 19.5\text{ cm}^2 = 214.5\text{ cm}^2$

4. Let side of the block, (a) = 7 cm

$\therefore$ The greatest diameter of the hemisphere = 7 cm

Radius of hemisphere, (r)

= 7/2 cm



Surface area of the solid

= [Total surface area of the cubical block]

+ [Curved surface of the hemisphere]

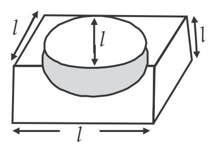
– [Base area of the hemisphere]

$= (6 \times a^2) + 2\pi r^2 - \pi r^2$

$$= 6a^2 + \pi r^2 = (6 \times 7^2) + \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) = \left(294 + \frac{77}{2}\right) = 332.5\text{ cm}^2$$

5. Given, side of the cube = diameter of the hemisphere $= l$

$\Rightarrow$ Radius of the hemisphere $= \dfrac{l}{2}$



$\therefore$ Surface area of hemisphere = $2\pi r^2$

$$= 2 \times \pi \times \frac{l}{2} \times \frac{l}{2} = \frac{\pi l^2}{2}\text{ sq. units}$$

Base area of the hemisphere $= \pi\left(\dfrac{l}{2}\right)^2 = \dfrac{\pi l^2}{4}$ sq. units

Surface area of the cube = $6 \times l^2 = 6l^2$ sq. units

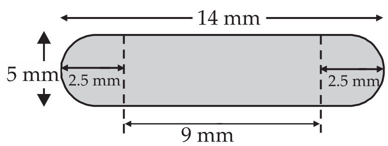
$\therefore$ Surface area of the remaining solid

$$= 6l^2 + \frac{\pi l^2}{2} - \frac{\pi l^2}{4} = \frac{24l^2 + 2\pi l^2 - \pi l^2}{4} = \frac{24l^2 + \pi l^2}{4}$$

$$= \frac{l^2}{4}(24 + \pi)\text{ sq.units.}$$

6. Radius of the hemispherical part (r) $= \dfrac{5}{2}$mm

= 2.5 mm



Curved surface area of one hemispherical part = $2\pi r^2$

$\therefore$ Surface area of both hemispherical parts

$$= 2(2\pi r^2) = 4\pi r^2 = 4 \times \frac{22}{7} \times \left(\frac{25}{10}\right)^2\text{ mm}^2$$

$$= 4 \times \frac{22}{7} \times \frac{25}{10} \times \frac{25}{10}\text{mm}^2$$

Entire length of capsule = 14 mm

$\therefore$ Length of cylindrical part = 14 – 2 × 2.5 = 9 mm

$\therefore$ Area of cylindrical part = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.5 \times 9\text{ mm}^2 = 2 \times \frac{22}{7} \times \frac{25}{10} \times 9\text{ mm}^2$$

$\therefore$ Total surface area of capsule

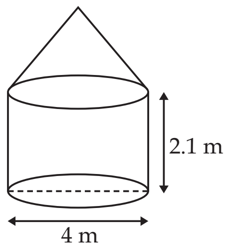
$$= \left[2 \times \frac{22}{7} \times \frac{25}{10} \times 9\right] + \left[4 \times \frac{22}{7} \times \frac{25}{10} \times \frac{25}{10}\right]\text{mm}^2$$

$$=\left(2\times\frac{22}{7}\times\frac{25}{10}\right)\left[9+\frac{50}{10}\right]\text{mm}^2=\frac{44\times25}{70}\times14\text{ mm}^2=220\text{ mm}^2$$

7. For cylindrical part :

Radius $(r)=\frac{4}{2}\text{m}=2\text{m}$ and

height (h) = 2.1 m



$\therefore$ Curved surface area

$$=2\pi rh=2\times\frac{22}{7}\times2\times\frac{21}{10}\text{m}^2$$

For conical part :

Slant height (l) = 2.8 m and base radius (r) = 2 m

$\therefore$ Curved surface area $=\pi rl=\frac{22}{7}\times2\times\frac{28}{10}\text{m}^2$

$\therefore$ Total surface area

= [Curved surface area of the cylindrical part] + [Curved surface area of conical part]

$$=\left[2\times\frac{22}{7}\times2\times\frac{21}{10}\right]+\left[\frac{22}{7}\times2\times\frac{28}{10}\right]\text{m}^2$$

$$=2\times\frac{22}{7}\left[\frac{42}{10}+\frac{28}{10}\right]\text{m}^2=2\times\frac{22}{7}\times\frac{70}{10}\text{ m}^2=44\text{ m}^2$$

$\because$ Cost of 1 m^2 of canvas = ₹ 500

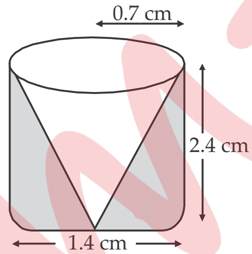
$\therefore$ Cost of 44 m^2 of canvas = ₹ (500 × 44) = ₹ 22000.

8. For cylindrical part :

Height (h) = 2.4 cm and

diameter = 1.4 cm

$\Rightarrow$ Radius (r) = 0.7 cm



$\therefore$ Total surface area of the cylindrical part

$=2\pi rh+2\pi r^2=2\pi r\,[h+r]$

$$=2\times\frac{22}{7}\times\frac{7}{10}[2.4+0.7]$$

$$=\frac{44}{10}\times3.1=\frac{44\times31}{100}=\frac{1364}{100}\text{cm}^2$$

For conical part :

Base radius (r) = 0.7 cm and height (h) = 2.4 cm

$\therefore$ Slant height $(l)=\sqrt{r^2+h^2}=\sqrt{(0.7)^2+(2.4)^2}$

$=\sqrt{0.49+5.76}=\sqrt{6.25}=2.5$ cm

$\therefore$ Curved surface area of the conical part

$$=\pi rl=\frac{22}{7}\times0.7\times2.5\text{ cm}^2=\frac{550}{100}\text{cm}^2$$

Base area of the conical part

$$=\pi r^2=\frac{22}{7}\times\left(\frac{7}{10}\right)^2\text{cm}^2=\frac{22\times7}{100}\text{cm}^2=\frac{154}{100}\text{cm}^2$$

Total surface area of the remaining solid

= [(Total surface area of cylindrical part) + (Curved surface area of conical part) – (Base area of the conical part)]

$$=\left[\frac{1364}{100}+\frac{550}{100}-\frac{154}{100}\right]\text{cm}^2=\frac{1760}{100}\text{cm}^2=17.6\text{ cm}^2.$$

Hence, total surface area to the nearest cm^2 is 18 cm^2.

9. Radius of the cylinder (r) = 3.5 cm

Height of the cylinder (h) = 10 cm

$\therefore$ Curved surface area = $2\pi rh$

$$=2\times\frac{22}{7}\times\frac{35}{10}\times10\text{cm}^2=220\text{cm}^2$$

Curved surface area of a hemisphere = $2\pi r^2$

$\therefore$ Curved surface area of both hemispheres

$$=2\times2\pi r^2=4\pi r^2=4\times\frac{22}{7}\times\frac{35}{10}\times\frac{35}{10}\text{cm}^2=154\text{ cm}^2$$

Total surface area of the remaining solid

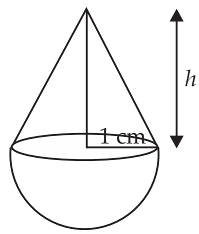
= (220 + 154) cm^2 = 374 cm^2

EXERCISE - 13.2

1. Here, r = 1 cm and h = 1 cm.

Volume of the conical part $=\frac{1}{3}\pi r^2h$

and volume of the hemispherical part $=\frac{2}{3}\pi r^3$



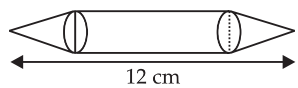
$\therefore$ Volume of the solid shape

$$=\frac{1}{3}\pi r^2h+\frac{2}{3}\pi r^3=\frac{1}{3}\pi r^2[h+2r]$$

$$=\frac{1}{3}\pi(1)^2[1+2(1)]\text{ cm}^3=\frac{1}{3}\pi\times1\times3\text{ cm}^3=\pi\text{ cm}^3$$

2. Here, diameter = 3 cm

$\Rightarrow$ Radius $(r)=\frac{3}{2}$ cm



Total height = 12 cm

Height of a cone (h) = 2 cm

$\therefore$ Height of both cones = 2 × 2 = 4 cm

$\Rightarrow$ Height of the cylinder (h_1) = (12 – 4) cm = 8 cm

Now, volume of the cylindrical part = πr^2h

Volume of both conical parts $=2\left[\frac{1}{3}\pi r^2h\right]$

$\therefore$ Volume of the whole model

$$=\pi r^2h_1+\frac{2}{3}\pi r^2h=\pi r^2\left[h_1+\frac{2}{3}h\right]$$

$$=\frac{22}{7}\times\left(\frac{3}{2}\right)^2\left[8+\frac{2}{3}(2)\right]=\frac{22}{7}\times\frac{9}{4}\times\left(\frac{24+4}{3}\right)$$

$$=\frac{22}{7}\times\frac{9}{4}\times\frac{28}{3}\text{cm}^3=66\text{ cm}^3.$$

3. Since, a gulab jamun is like a cylinder with hemispherical ends.

Total height of the gulab jamun = 5 cm.

Diameter = 2.8 cm $\Rightarrow$ Radius (r) = 1.4 cm

$\therefore$ Length (height) of the cylindrical part (h)

= 5 cm – (1.4 + 1.4) cm = 5 cm – 2.8 cm = 2.2 cm

Now, volume of the cylindrical part = πr^2h

and volume of both the hemispherical ends

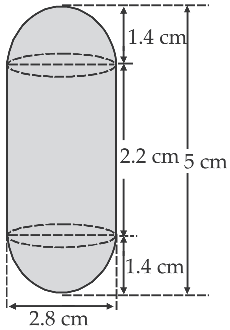
$= 2\left(\frac{2}{3}\pi r^3\right) = \frac{4}{3}\pi r^3$

$\therefore$ Volume of a gulab jamun

$= \pi r^2 h + \frac{4}{3}\pi r^3 = \pi r^2\left[h + \frac{4}{3}r\right]$

$= \frac{22}{7} \times (1.4)^2\left[2.2 + \frac{4}{3}(1.4)\right]$

$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10}\left[\frac{22}{10} + \frac{56}{30}\right]$



$= \frac{22 \times 2 \times 14}{10 \times 10}\left[\frac{66+56}{30}\right] = \frac{44 \times 14}{100} \times \frac{122}{30}\text{cm}^3$

Volume of 45 gulab jamuns

$= 45 \times \left[\frac{44 \times 14}{100} \times \frac{122}{30}\right] = \frac{15 \times 44 \times 14 \times 122}{1000}\text{cm}^3$

Since, the quantity of syrup in gulab jamuns

$= 30\%$ of [volume] $= 30\%$ of $\left[\frac{15 \times 44 \times 14 \times 122}{1000}\right]$

$= \frac{30}{100} \times \frac{15 \times 44 \times 14 \times 122}{1000} = 338.184\text{ cm}^3$

$= 338\text{ cm}^2$ (approx.)

4. Dimensions of the cuboid are 15 cm, 10 cm and 3.5 cm.

$\therefore$ Volume of the cuboid $= 15 \times 10 \times \frac{35}{10} = 525\text{ cm}^3$

Since each depression is conical in shape with base radius (r) = 0.5 cm and depth (h) = 1.4 cm.

$\therefore$ Volume of each depression

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{5}{10}\right)^2 \times \frac{14}{10} = \frac{11}{30}\text{cm}^3$

Since there are 4 depressions.

$\therefore$ Total volume of 4 depressions $= 4 \times \frac{11}{30} = \frac{44}{30}\text{cm}^3$

Now, volume of the wood in entire stand

= [Volume of the wooden cuboid]

– [Volume of 4 depressions]

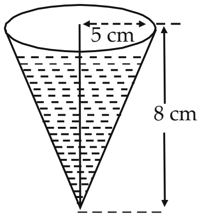
$= 525 - \frac{44}{30} = \frac{15750 - 44}{30} = \frac{15706}{30} = 523.53\text{ cm}^3$

5. Height of the conical vessel (h) = 8 cm

Base radius (r) = 5 cm

Volume of water in conical vessel

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 8 = \frac{4400}{21}\text{cm}^3$



Now, total volume of lead shots

$= \frac{1}{4}$ of [Volume of water in the cone]

$= \frac{1}{4} \times \frac{4400}{21} = \frac{1100}{21}\text{cm}^3$

Since, radius of spherical lead shot (r) = 0.5 cm

$\therefore$ Volume of 1 lead shot $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{5}{10}\text{cm}^3$

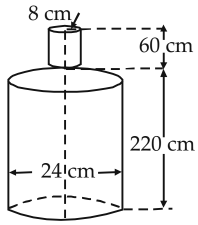
$\therefore$ Number of lead shots $= \frac{\text{Total volume of lead shots}}{\text{Volume of 1 lead shot}}$

$= = \frac{\left[\frac{1100}{21}\right]}{\left[\frac{4 \times 22 \times 5 \times 5 \times 5}{3 \times 7 \times 1000}\right]} = 100$

Thus, the required number of lead shots = 100

6. Height of the big cylinder (h)

= 220 cm



Base radius (r) $= \frac{24}{2}\text{cm} = 12\text{ cm}$

$\therefore$ Volume of the big cylinder

$= \pi r^2 h = \pi(12)^2 \times 220\text{ cm}^3$

Also, height of smaller cylinder (h_1)

= 60 cm

Base radius (r_1) = 8 cm

$\therefore$ Volume of the smaller cylinder $= \pi r_1^2 h_1 = \pi(8)^2 \times 60\text{ cm}^3$

$\therefore$ Volume of iron pole = [Volume of big cylinder]

+ [Volume of the smaller cylinder]

$= (\pi \times 220 \times 12^2 + \pi \times 60 \times 8^2)\text{ cm}^3$

$= 3.14[220 \times 12 \times 12 + 60 \times 8 \times 8]\text{ cm}^3$

$= \frac{314}{100}[220 \times 144 + 60 \times 64]\text{ cm}^3$

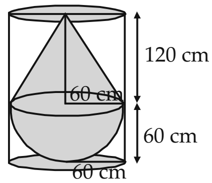
$= \frac{314}{100}[31680 + 3840]\text{ cm}^3 = \frac{314}{100} \times 35520\text{ cm}^3$

Mass of pole $= \frac{8 \times 314 \times 35520}{100}\text{g} = \frac{89226240}{100}\text{g}$

$= \frac{8922624}{10000}\text{kg} = 892.2624\text{ kg} = 892.26\text{ kg}.$

7. Height of the conical part (h)

= 120 cm.



Base radius of the conical part (r)

= 60 cm.

$\therefore$ Volume of the conical part

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120\text{ cm}^3$

Radius of the hemispherical part (r) = 60 cm

$\therefore$ Volume of the hemispherical part

$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (60)^3\text{cm}^3$

$\therefore$ Volume of the solid = [Volume of conical part]

+ [Volume of hemispherical part]

$= \left[\frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120\right] + \left[\frac{2}{3} \times \frac{22}{7} \times 60^3\right]$

$= \frac{2}{3} \times \frac{22}{7} \times 60^2[60 + 60]$

$= \frac{2}{3} \times \frac{22}{7} \times 60 \times 60 \times 120 = \frac{6336000}{7}\text{ cm}^3$

Radius of cylinder (r_1) = 60 cm

and, Height of cylinder (h_1) = 180 cm

Volume of the cylinder = $\pi r_1^2 h_1$

$$= \frac{22}{7} \times 60^2 \times 180 = \frac{14256000}{7} \text{cm}^3$$

$\Rightarrow$ Volume of water in the cylinder $= \frac{14256000}{7} \text{cm}^3$

$\therefore$ Volume of the water left in the cylinder

$$= \left[\frac{14256000}{7} - \frac{6336000}{7}\right] = \frac{7920000}{7}$$

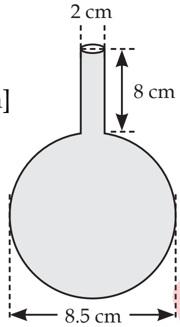
$= 1131428.571 \text{ cm}^3 = \frac{1131428.571}{1000000} \text{m}^3$

$= 1.131428571 \text{ m}^3 = 1.131 \text{ m}^3$ (approx).

8. Volume of the cylindrical part = $\pi r^2 h$

$= 3.14 \times 1^2 \times 8 = \frac{314}{100} \times 8 \text{ cm}^3$

$\left[\because \text{Radius}(r) = \frac{2}{2} = 1 \text{ cm}, \text{height}(h) = 8 \text{ cm}\right]$



Radius of spherical part $(r_1) = \frac{8.5}{2}$ cm

Volume of the spherical part $= \frac{4}{3}\pi r_1^3$

$= \frac{4}{3} \times \frac{314}{100} \times \frac{85}{20} \times \frac{85}{20} \times \frac{85}{20} \text{cm}^3$

Total volume of the glass-vessel

$= \left[\frac{314}{100} \times 8\right] + \left[\frac{4}{3} \times \frac{314}{100} \times \frac{85 \times 85 \times 85}{8000}\right]$

$= \frac{314}{100}\left[8 + \frac{4 \times 85 \times 85 \times 85}{24000}\right] = \frac{314}{100}\left[8 + \frac{614125}{6000}\right]$

$= \frac{314}{100}\left[\frac{48000 + 614125}{6000}\right] = \frac{314}{100}\left[\frac{662125}{6000}\right]$

$= 346.51 \text{ cm}^3$ (approx.)

$\Rightarrow$ Volume of water in the vessel = 346.51 cm^3

Since, the child finds the volume as 345 cm^3

$\therefore$ The child's answer is not correct

The correct answer is 346.51 cm^3.

EXERCISE - 13.3

1. Radius of the sphere $(r) = 4.2$ cm

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} \text{cm}^3$

Radius of the cylinder $(r_1) = 6$ cm

Let h be the height of the cylinder.

$\therefore$ Volume of the cylinder $= \pi r_1^2 h = \frac{22}{7} \times 6 \times 6 \times h \text{ cm}^3$

Since, volume of the metallic sphere = volume of the cylinder

$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} = \frac{22}{7} \times 6 \times 6 \times h$

$\Rightarrow h = \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} \times \frac{7}{22} \times \frac{1}{6} \times \frac{1}{6}$

$= \frac{4 \times 7 \times 7 \times 14}{10 \times 10 \times 10} = \frac{2744}{1000} = 2.744 \text{ cm.}$

Hence, height of the cylinder = 2.744 cm

2. Radii of the given spheres are:

$r_1 = 6$ cm, $r_2 = 8$ cm and $r_3 = 10$ cm

$\Rightarrow$ Volume of the given spheres are:

$V_1 = \frac{4}{3}\pi r_1^3, V_2 = \frac{4}{3}\pi r_2^3 \text{ and } V_3 = \frac{4}{3}\pi r_3^3$

$\therefore$ Total volume of the given spheres = $V_1 + V_2 + V_3$

$= \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi[r_1^3 + r_2^3 + r_3^3]$

$= \frac{4}{3} \times \frac{22}{7} \times [6^3 + 8^3 + 10^3] = \frac{4}{3} \times \frac{22}{7} \times [216 + 512 + 1000]$

$= \frac{4}{3} \times \frac{22}{7} \times 1728 \text{ cm}^3$

Let the radius of the new big sphere be R.

$\therefore$ Volume of the new sphere

$= \frac{4}{3} \times \pi \times R^3 = \frac{4}{3} \times \frac{22}{7} \times R^3$

Since, the two volume must be equal.

$\therefore \frac{4}{3} \times \frac{22}{7} \times R^3 = \frac{4}{3} \times \frac{22}{7} \times 1728$

$\Rightarrow R^3 = 1728 \Rightarrow R = 12$ cm

Thus, the required radius of the resulting sphere = 12 cm.

3. Diameter of the cylindrical well = 7 m

$\Rightarrow$ Radius of the cylindrical well $(r) = \frac{7}{2}$m

Depth of the cylindrical well $(h) = 20$ m

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 22 \times 7 \times 5 \text{ m}^3$

$\Rightarrow$ Volume of the earth taken out = $22 \times 7 \times 5 \text{ m}^3$

Now this earth is spread out to form a cuboidal platform having length = 22 m and breadth = 14 m.

Let h_1 be the height of the platform.

$\therefore$ Volume of the platform = $22 \times 14 \times h_1$

$\therefore$ $22 \times 14 \times h_1 = 22 \times 7 \times 5$

$\Rightarrow h_1 = \frac{22 \times 7 \times 5}{22 \times 14} = \frac{5}{2} = 2.5 \text{ m}$

Thus, the required height of the platform is 2.5 m.

4. Diameter of cylindrical well = 3 m

$\Rightarrow$ Radius of the cylindrical well $(r) = \frac{3}{2} = 1.5$ m

Depth of the well $(h) = 14$ m

$\therefore$ Volume of the cylindrical well

$= \pi r^2 h = \frac{22}{7} \times \left(\frac{15}{10}\right)^2 \times 14 = \frac{22 \times 15 \times 15 \times 14}{7 \times 10 \times 10} = 99 \text{ m}^3$

Let the height of the embankment be H metre.

Internal radius of the embankment $(r) = 1.5$ m.

External radius of the embankment

$R = (4 + 1.5)$ m = 5.5 m.

$\therefore$ Volume of the embankment

$= \pi R^2 H - \pi r^2 H = \pi H [R^2 - r^2] = \pi H (R + r)(R - r)$

$= \frac{22}{7} \times H(5.5 + 1.5)(5.5 - 1.5) = \frac{22}{7} \times H \times 7 \times 4 \text{ m}^3$

Since, volume of the embankment = volume of the cylindrical well

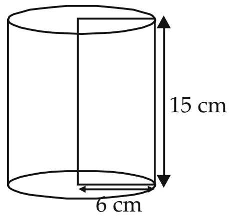
$\therefore \quad \frac{22}{7} \times H \times 7 \times 4 = 99 \Rightarrow H = 99 \times \frac{7}{22} \times \frac{1}{7} \times \frac{1}{4}$

$\Rightarrow \quad H = \frac{9}{8} = 1.125$

Thus, the required height of the embankment = 1.125 m.

5. For the right circular cylinder:

Diameter = 12 cm



$\Rightarrow$ Radius $(r) = \frac{12}{2} = 6$ cm

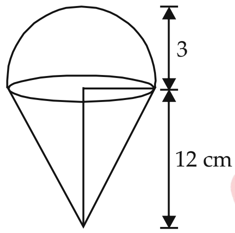
and height (h) = 15 cm

$\therefore$ Volume of total ice cream

$= \pi r^2 h = \frac{22}{7} \times 6 \times 6 \times 15 \text{ cm}^3$

For conical and hemispherical part of ice-cream:

Diameter = 6 cm



$\Rightarrow$ radius (R) = 3 cm

Height of conical part (H) = 12 cm

Volume of ice-cream = (Volume of the conical part) + (Volume of the hemispherical part)

$= \frac{1}{3}\pi R^2 H + \frac{2}{3}\pi R^3 = \frac{1}{3}\pi R^2 [H + 2R]$

$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 [12 + 2 \times 3] \text{ cm}^3 = \frac{22 \times 3}{7} \times 18 \text{ cm}^3$

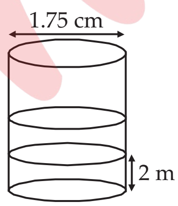
Let number of ice-cream cones required to fill the total ice cream = n.

$\therefore \quad n\left[\frac{22 \times 3}{7} \times 18\right] = \frac{22}{7} \times 6 \times 6 \times 15 \Rightarrow n = \frac{6 \times 6 \times 15}{3 \times 18} = 10$

Thus, the required number of cones is 10.

6. For a circular coin :

Diameter = 1.75 cm

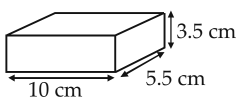


$\Rightarrow$ Radius $(r) = \frac{175}{200}$ cm

Thickness $(h) = 2 \text{ mm} = \frac{2}{10}$ cm

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \left(\frac{175}{200}\right)^2 \times \frac{2}{10} \text{ cm}^3$

For a cuboid:



Length (l) = 10 cm,

Breadth (b) = 5.5 cm

and height (h) = 3.5 cm

$\therefore$ Volume $= l \times b \times h = 10 \times \frac{55}{10} \times \frac{35}{10} \text{ cm}^3$

$$\text{Number of coins} = \frac{\text{Volume of cuboid}}{\text{Volume of one coin}}$$

$$= \frac{10 \times \frac{55}{10} \times \frac{35}{10}}{\frac{22}{7} \times \left(\frac{175}{200}\right)^2 \times \frac{2}{10}} = 400$$

Thus, the required number of coins = 400.

7. For the cylindrical bucket:

Radius (r) = 18 cm and height (h) = 32 cm

Volume of cylindrical bucket $= \pi r^2 h = \frac{22}{7} \times (18)^2 \times 32 \text{ cm}^3$

$\Rightarrow$ Volume of the sand $= \left(\frac{22}{7} \times 18 \times 18 \times 32\right) \text{cm}^3$

For the conical heap:

Height (H) = 24 cm and let radius of the base be R.

$\therefore$ Volume of conical heap

$= \frac{1}{3}\pi R^2 H = \left[\frac{1}{3} \times \frac{22}{7} \times R^2 \times 24\right] \text{cm}^3$

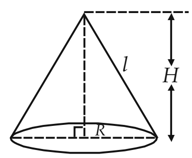
$\because$ Volume of the conical heap = Volume of the sand

$\therefore \quad \frac{1}{3} \times \frac{22}{7} \times R^2 \times 24 = \frac{22}{7} \times 18 \times 18 \times 32$

$\Rightarrow \quad R^2 = \frac{18 \times 18 \times 32 \times 3}{24} = 18^2 \times 2^2$

$\Rightarrow \quad R = \sqrt{18^2 \times 2^2} = 18 \times 2 \text{ cm} = 36 \text{ cm}$

Let 'l' be the slant height of the conical heap of the sand.



$\therefore \quad l = \sqrt{R^2 + H^2}$

$= \sqrt{36^2 + 24^2} = \sqrt{1872} = 12\sqrt{13}$ cm

Thus, the required radius = 36 cm

and slant height $= 12\sqrt{13}$ cm.

8. Width of the canal = 6 m, Depth of the canal = 1.5 m

Length of the water column in 1 hr = 10 km

$\therefore$ Length of the water column in 30 minutes $\left(i.e., \frac{1}{2}\text{hr}\right)$

$= \frac{10}{2}$ km = 5 km = 5000 m

$\therefore$ Volume of water flows in $\frac{1}{2}$hr

$= 6 \times 1.5 \times 5000 \text{ m}^3 = 6 \times \frac{15}{10} \times 5000 \text{ m}^3 = 45000 \text{ m}^3$

Since, the above amount (volume) of water is spread in the form of a cuboid of height as 8 cm $\left(= \frac{8}{100}\text{m}\right)$. Let the area of the cuboid = a

$\therefore$ Volume of the cuboid = Area × Height $= a \times \frac{8}{100} \text{m}^3$

Thus, $a \times \frac{8}{100} = 45000 \Rightarrow a = \frac{45000 \times 100}{8} = \frac{4500000}{8} \text{m}^2$

= 562500 m^2 = 56.25 hectares

Thus, the required area = 56.25 hectares.

9. Diameter of the pipe = 20 cm

$\Rightarrow$ Radius of the pipe $(r) = \frac{20}{2}$ cm = 10 cm

Since, the water flows through the pipe at 3 km/hr.

$\therefore$ Length of water column per hour(h) = 3 km

= 3 × 1000 m = 3000 × 100 cm = 300000 cm.

$\therefore$ Volume of water flows in 1 hour

$= \pi r^2 h = \pi \times 10^2 \times 300000 \text{ cm}^3$

$= \pi \times 30000000 \text{ cm}^3$

Now, for the cylindrical tank,

Diameter = 10 m ⇒ Radius (R) = $\frac{10}{2}$ m
= 5 × 100 cm = 500 cm
Height (H) = 2 m = 2 × 100 cm = 200 cm
∴ Volume of the cylindrical tank = $\pi R^2 H$
$= \pi \times (500)^2 \times 200 \text{ cm}^3$
Now, time required to fill the tank

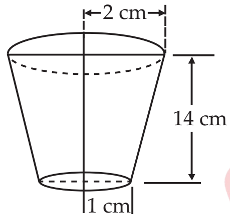
$$= \frac{\text{Volume of the tank}}{\text{Volume of water flows in 1 hour}}$$

$$= \frac{\pi \times 500 \times 500 \times 200}{\pi \times 30000000} \text{hrs} = \frac{5 \times 5 \times 2}{30} \text{hrs} = \frac{5}{3} \text{hrs}$$

$$= \frac{5}{3} \times 60 \text{ minutes} = 100 \text{ minutes.}$$

EXERCISE - 13.4

1. We have, $r_1 = 4/2 = 2$ cm, $r_2 = 2/2 = 1$ cm and $h = 14$ cm



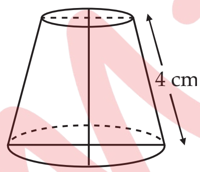
Volume of the glass

$$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14[2^2 + 1^2 + 2 \times 1]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14[4+1+2] = \frac{22}{3} \times 2 \times 7 = \frac{308}{3} = 102\frac{2}{3} \text{ cm}^3.$$

2. We have, slant height (l) = 4 cm



$2\pi r_1 = 18$ cm and $2\pi r_2 = 6$ cm

$$\Rightarrow \quad \pi r_1 = \frac{18}{2} = 9 \text{ cm}$$

and $\pi r_2 = \frac{6}{2} = 3$ cm

∴ Curved surface area of the frustum of the cone
$= \pi(r_1 + r_2)\, l = (\pi r_1 + \pi r_2)\, l = (9 + 3) \times 4$
$= 12 \times 4 \text{ cm}^2 = 48 \text{ cm}^2$.

3. Here, the radius of the open side (r_1) = 10 cm
The radius of the upper base (r_2) = 4 cm
Slant height (l) = 15 cm
∴ Area of the material required
= Curved surface area of the frustum
+ Area of the top end
$= \pi(r_1 + r_2)\, l + \pi r_2^2$

$$= \frac{22}{7} \times (10+4) \times 15 + \frac{22}{7} \times 4 \times 4 \text{ cm}^2$$

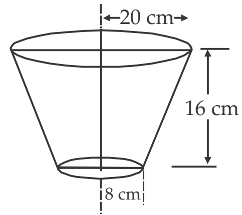
$$= \frac{22}{7} \times 14 \times 15 + \frac{22}{7} \times 16 = 660 + \frac{352}{7}$$

$$= \frac{4620 + 352}{7} = \frac{4972}{7} = 710\frac{2}{7} \text{cm}^2.$$

4. We have, $r_1 = 20$ cm, $r_2 = 8$ cm and $h = 16$ cm

∴ Volume of the frustum $= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$

$$= \frac{1}{3} \times \frac{314}{100} \times 16[20^2 + 8^2 + 20 \times 8]$$



$$= \frac{1}{3} \times \frac{314}{100} \times 16[400 + 64 + 160]$$

$$= \frac{1}{3} \times \frac{314}{100} \times 16 \times 624$$

$$= \left[\frac{314}{100} \times 16 \times 208\right] \text{cm}^3$$

$$= \left[\frac{314}{100} \times 16 \times 208\right] \div 1000 \text{ litres} = \frac{314 \times 16 \times 208}{100000} \text{ litres}$$

$$\therefore \quad \text{Cost of milk} = ₹\left(20 \times \frac{314 \times 16 \times 208}{100000}\right)$$

= ₹ 208.998 ≈ ₹ 209

Now, slant height of the given frustum

$$l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{16^2 + (20-8)^2} = \sqrt{16^2 + 12^2}$$

$$= \sqrt{256 + 144} = \sqrt{400} = 20 \text{ cm}$$

∴ Curved surface area = $\pi(r_1 + r_2)l$

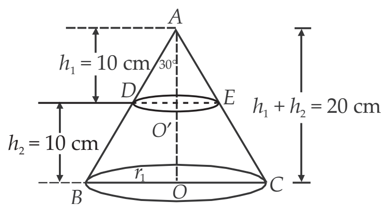
$$= \frac{314}{100}(20+8) \times 20 = \frac{314}{100} \times 28 \times 20 = 1758.4 \text{ cm}^2$$

Area of the bottom $= \pi r^2 = \frac{314}{100} \times 8 \times 8 = 200.96 \text{ cm}^2$

∴ Total area of metal required
= 1758.4 + 200.96 = 1959.36 cm²
Cost of metal sheet required

$$= \left(₹\frac{8}{100} \times 1959.36\right) = ₹\ 156.75.$$

5. Let us consider the frustum $DECB$ of the metallic cone ABC



Here, $r_1 = BO$ and $r_2 = DO'$

In ΔAOB, $\frac{r_1}{(h_1 + h_2)} = \tan 30° = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad r_1 = (h_1 + h_2) \times \frac{1}{\sqrt{3}} = 20 \times \frac{1}{\sqrt{3}} = \frac{20}{\sqrt{3}} \text{ cm}$$

In $\Delta ADO'$, $\frac{r_2}{h_1} = \tan 30°$

$$\Rightarrow \quad r_2 = h_1 \times \frac{1}{\sqrt{3}} = 10 \times \frac{1}{\sqrt{3}} = \frac{10}{\sqrt{3}} \text{ cm}$$

Now, the volume of the frustum $DBCE$

$$= \frac{1}{3}\pi h_2[r_1^2 + r_2^2 + r_1 r_2]$$

$$= \frac{1}{3} \times \pi \times 10\left[\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \frac{20}{\sqrt{3}} \times \frac{10}{\sqrt{3}}\right]$$

$$= \frac{\pi}{3} \times 10\left[\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right] = \frac{\pi}{3} \times 10\left[\frac{700}{3}\right] \text{cm}^3$$

Let l be the length and D be diameter of the wire drawn from the frustum. Since, the wire is in the form of a cylinder.

$\therefore$ Volume of the wire $= \pi r^2 h = \pi\left(\frac{D}{2}\right)^2 \times h$

$= \frac{\pi D^2 h}{4} = \frac{\pi h}{4\times 16\times 16}$ $\qquad \left[\because D = \frac{1}{16}\right]$

$\because$ Volume of the frustum = Volume of the wire

$\therefore \quad \frac{\pi}{3}\times 10\times \frac{700}{3} = \frac{\pi h}{4\times 16\times 16}$

$\Rightarrow \quad h = \frac{10\times 700}{3\times 3}\times 4\times 16\times 16 = \frac{7168000}{9\times 100}\text{ m} = 7964.44\text{ m}$

Thus, the required length of the wire = 7964.44 m

EXERCISE - 13.5

1. Since, diameter of the cylinder = 10 cm

$\therefore$ Radius of the cylinder (r) = 10/2 cm = 5 cm

$\Rightarrow$ Length of wire in one round = $2\pi r$

= 2 × 3.14 × 5 cm = 31.4 cm

$\because$ Diameter of wire = 3 mm = 3/10 cm

$\therefore$ The thickness of cylinder covered in one round = 3/10 cm

$\Rightarrow$ Number of rounds (turns) of the wire to cover

$12\text{ cm} = \frac{12}{3/10} = 12\times\frac{10}{3} = 40$

$\therefore$ Length of wire required to cover the whole surface = Length of wire required to complete 40 rounds

l = 40 × 31.4 cm = 1256 cm

Now, radius of the wire $= \frac{3}{2}\text{mm} = \frac{3}{20}\text{cm}$

$\therefore$ Volume of wire $= \pi r^2 l = 3.14\times\frac{3}{20}\times\frac{3}{20}\times 1256\text{ cm}^3$

$\because$ Density of wire = 8.88 g/cm^3

$\therefore$ Weight of the wire = [Volume of the wire] × density

$= \left[3.14\times\frac{3}{20}\times\frac{3}{20}\times 1256\right]\times 8.88\text{ g}$

$= 3.14\times\frac{3}{20}\times\frac{3}{20}\times 1256\times\frac{888}{100}\text{g}$

= 787.97 g = 788 g (approx.)

2. Let us consider the right ΔBAC, right angled at A such that AB = 3 cm and AC = 4 cm.

$\therefore$ Hypotenuse $BC = \sqrt{3^2+4^2} = 5\text{ cm}$

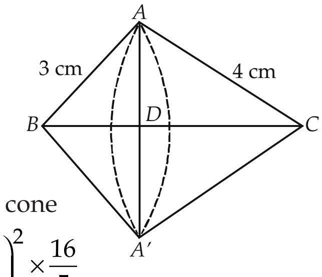
Obviously, we have obtained two cones on the same base AA' such that radius = DA or DA'

Now, $\frac{AD}{CA} = \frac{AB}{CB}$ $\qquad [\because ADB \sim \Delta CAB]$

$\Rightarrow \frac{AD}{4} = \frac{3}{5} \Rightarrow AD = \frac{3}{5}\times 4 = \frac{12}{5}\text{cm}$

Also, $\frac{DB}{AB} = \frac{AB}{CB} \Rightarrow \frac{DB}{3} = \frac{3}{5}$

$\Rightarrow \quad DB = \frac{3\times 3}{5} = \frac{9}{5}\text{cm}$



Since, $CD = BC - DB$

$\Rightarrow \quad CD = 5 - \frac{9}{5} = \frac{16}{5}\text{cm}$

Now, volume of the double cone

$= \frac{1}{3}\pi\times\left(\frac{12}{5}\right)^2\times\frac{9}{5} + \frac{1}{3}\pi\times\left(\frac{12}{5}\right)^2\times\frac{16}{5}$

$= \frac{1}{3}\pi\times\left(\frac{12}{5}\right)^2\left[\frac{9}{5}+\frac{16}{5}\right] = \frac{1}{3}\times\frac{314}{100}\times\frac{144}{25}\times 5 = 30.14\text{ cm}^3$

Surface area of the double cone = $\pi r l_1 + \pi r l_2$

$= \left(\pi\times\frac{12}{5}\times 3\right) + \left(\pi\times\frac{12}{5}\times 4\right) = \pi\times\frac{12}{5}[3+4]$

$= \frac{314}{100}\times\frac{12}{5}\times 7 = 52.75\text{ cm}^2$

3. $\because$ Dimensions of the cistern are 150 cm, 120 cm and 110 cm.

$\therefore$ Volume of the cistern = 150 × 120 × 110 = 1980000 cm^3

Volume of water contained in the cistern = 129600 cm^3

$\therefore$ Free space (volume) which is not filled with water = 1980000 – 129600 = 1850400 cm^3

Now, volume of one brick = 22.5 × 7.5 × 6.5 = 1096.875 cm^3

$\therefore$ Volume of water absorbed by one brick

$= \frac{1}{17}\times 1096.875\text{ cm}^3$

Let n bricks can be put in the cistern.

$\therefore$ Volume of water absorbed by n bricks

$= \frac{n}{17}\times 1096.875\text{cm}^3$

$\therefore$ Volume occupied by n bricks = Free space in the cistern + Volume of water absorbed by n-bricks

$\Rightarrow \quad n\times(1096.875) = 1850400 + \frac{n}{17}(1096.875)$

$\Rightarrow \quad 1096.875\,n - \frac{n}{17}(1096.875) = 1850400$

$\Rightarrow \quad \left(n - \frac{n}{17}\right)\times 1096.875 = 1850400$

$\Rightarrow \quad \frac{16}{17}n = \frac{1850400}{1096.875} \Rightarrow n = \frac{1850400}{1096.875}\times\frac{17}{16}$

$\Rightarrow \quad n = 1792.4102 \approx 1792$

Thus, 1792 bricks can be put in the cistern.

4. Volume of three rivers

= 3 {(Surface area of a river) × Depth}

$= 3\left\{\left(1072\text{ km}\times\frac{75}{1000}\text{km}\right)\times\frac{3}{1000}\text{km}\right\}$

$= 3\left\{\frac{241200}{1000000}\text{km}^3\right\} = 0.7236\text{ km}^3$

Volume of rainfall

= (Surface area of valley) × (Height of rainfall)

$= 97280 \times \frac{10}{100 \times 1000}$ $\left[\because 10 \text{ cm} = \frac{10}{100 \times 1000} \text{km}\right]$

$= \frac{9728}{1000} \text{km}^3 = 9.728 \text{ km}^3$

Thus, amount of rainfall in 1 fortnight *i.e.*, 14 days is 9.728 km^3.

$\therefore$ Amount of rainfall in 1 day = 9.728/14 = 0.6949 km^3

Since, $0.6949 \text{ km}^3 \approx 0.7236 \text{ km}^3$

$\therefore$ The additional water in the three rivers is equivalent to the total rainfall.

5. For the cylindrical part, we have

Diameter = 8 cm $\Rightarrow$ Radius (r) = 4 cm

Height (h) = 10 cm

Curved surface area = $2\pi rh$

$= 2 \times \frac{22}{7} \times 4 \times 10 = \frac{22}{7} \times 80 \text{ cm}^2$

For the frustum : $r_2 = \frac{18}{2} = 9$ cm, $r_1 = \frac{8}{2} = 4$ cm

Height (H) = 22 – 10 = 12 cm

$\therefore$ Slant height (l) = $\sqrt{H^2 + (r_2 - r_1)^2}$

$= \sqrt{12^2 + (9-4)^2} = \sqrt{144 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \text{cm}$

$\therefore$ Curved surface area = $\pi(r_2 + r_1)l$

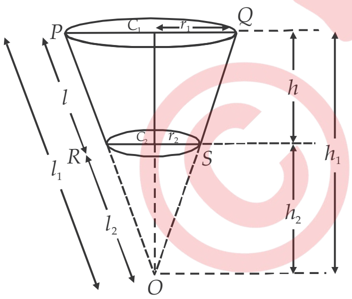
$= \frac{22}{7} \times (9+4) \times 13 = \frac{22}{7} \times 169 \text{ cm}^2$

Area of tin required = [Curved surface area of the frustum] + [Curved surface area of cylindrical portion]

$= \frac{22}{7} \times 169 + \frac{22}{7} \times 80 = \frac{22}{7}(169 + 80)$

$= \frac{22}{7}(249) = \frac{5478}{7} = 782\frac{4}{7} \text{ cm}^2$

6.



We have, curved surface area of the frustum $PQSR$

$= \begin{bmatrix} \text{curved surface area} \\ \text{of the right circular} \\ \text{cone } OPQ \end{bmatrix} - \begin{bmatrix} \text{curved surface area} \\ \text{of the right circular} \\ \text{cone } ORS \end{bmatrix}$

$= \pi r_1 l_1 - \pi r_2 l_2$ ...(i)

Now, $\Delta OC_1Q \sim \Delta OC_2S$ [By AA similarity Criterion]

$\therefore \frac{OQ}{OS} = \frac{QC_1}{SC_2} = \frac{OC_1}{OC_2}$

$\Rightarrow \frac{l_1}{l_2} = \frac{r_1}{r_2} = \frac{h_1}{h_2} \Rightarrow l_1 = \left(\frac{r_1}{r_2}\right) l_2$ and $\frac{l + l_2}{l_2} = \frac{r_1}{r_2}$

$(\because l_1 = l + l_2)$

$\Rightarrow \frac{l}{l_2} + 1 = \frac{r_1}{r_2} \Rightarrow \frac{l}{l_2} = \frac{r_1}{r_2} - 1 \therefore l = \left(\frac{r_1 - r_2}{r_2}\right) l_2$ ...(ii)

Now, from (i), we get

Curved surface area of the frustum

$= \pi r_1 \left(\frac{r_1}{r_2} l_2\right) - \pi r_2 l_2 = \pi l_2 \left[\frac{r_1^2}{r_2} - r_2\right] = \pi l_2 \left(\frac{r_1^2 - r_2^2}{r_2}\right)$

$= \pi l_2 \left[\frac{(r_1 + r_2)(r_1 - r_2)}{r_2}\right] = \pi \left(\frac{r_1 - r_2}{r_2}\right) l_2 \times (r_1 + r_2)$

$= \pi l\,(r_1 + r_2)$ [From (ii)]

Now, the total surface area of the frustum

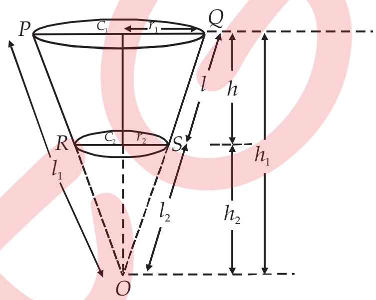
= (curved surface area) + (base surface area)

+ (top surface area)

$= \pi l\,(r_1 + r_2) + \pi r_2^2 + \pi r_1^2 = \pi\,(r_1 + r_2)\,l + \pi\,(r_1^2 + r_2^2)$

$= \pi\,[(r_1 + r_2)\,l + r_1^2 + r_2^2]$

7.



We have, volume of the frustum $RPQS$

= [volume of right circular cone OPQ]

– [volume of right circular cone ORS]

$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3}\pi[r_1^2 h_1 - r_2^2 h_2]$ ...(i)

Since, $\Delta OC_1Q \sim \Delta OC_2S$ [By AA similarity Criterion]

$\therefore \frac{OQ}{OS} = \frac{QC_1}{SC_2} = \frac{OC_1}{OC_2}$

$\Rightarrow \frac{l_1}{l_2} = \frac{r_1}{r_2} = \frac{h_1}{h_2} \Rightarrow h_1 = \left(\frac{r_1}{r_2} \times h_2\right)$ ...(ii)

$\Rightarrow \frac{r_1}{r_2} = \frac{h + h_2}{h_2} \Rightarrow \frac{r_1}{r_2} = \frac{h}{h_2} + 1$

$\Rightarrow \frac{h}{h_2} = \frac{r_1}{r_2} - 1 \Rightarrow h = \left[\frac{r_1}{r_2} - 1\right] \times h_2$

$\Rightarrow h = (r_1 - r_2)\frac{h_2}{r_2}$ ...(iii)

From (i) and (ii), we have

Volume of the frustum $RPQS$

$= \frac{1}{3}\pi\left[r_1^2 \times \frac{r_1}{r_2} h_2 - r_2^2 h_2\right]$

$= \frac{1}{3}\pi\left[\frac{r_1^3}{r_2} - r_2^2\right] h_2 = \frac{1}{3}\pi[r_1^3 - r_2^3]\frac{h_2}{r_2}$

$= \frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2)\left[(r_1 - r_2)\frac{h_2}{r_2}\right]$

$= \frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2)h$ [From (iii)]