# Real Numbers

CHAPTER 1

NCERT FOCUS

## SOLUTIONS

### EXERCISE - 1.1

**1.** (i) Here, 225 > 135

Applying Euclid's division lemma to 225 and 135, we get

225 = (135 × 1) + 90

Since, 90 ≠ 0, therefore, applying Euclid's division lemma to 135 and 90, we get

135 = (90 × 1) + 45

Since, 45 ≠ 0

∴ Applying Euclid's division lemma to 90 and 45, we get 90 = (45 × 2) + 0

Here, remainder, $r = 0$, when divisor is 45.

∴ HCF of 225 and 135 is 45.

(ii) Here, 38220 > 196

∴ Applying Euclid's division lemma, we get

38220 = (196 × 195) + 0

Here, $r = 0$, when divisor is 196.

∴ HCF of 38220 and 196 is 196.

(iii) Here, 867 > 255

∴ Applying Euclid's division lemma, we get

867 = (255 × 3) + 102,

255 = (102 × 2) + 51,

102 = (51× 2) + 0

Here remainder = 0, when divisor is 51.

∴ HCF of 867 and 255 is 51.

**2.** Let us consider a positive odd integer as '$a$'.

On dividing '$a$' by 6, let '$q$' be the quotient and '$r$' be the remainder.

∴ Using Euclid's division lemma, we get

$a = 6q + r$, where $0 \le r < 6$

$a = 6q + 0 = 6q$ or $a = 6q + 1$

or $a = 6q + 2$ or $a = 6q + 3$

or $a = 6q + 4$ or $a = 6q + 5$

But, $a = 6q$, $a = 6q + 2$, $a = 6q + 4$ are even values of '$a$'.

[∵ $6q = 2(3q) = 2m_1$, $6q + 2 = 2(3q + 1) = 2m_2$,
$6q + 4 = 2(3q + 2) = 2m_3$]

Being an odd integer, we have

$a = 6q + 1$ or $a = 6q + 3$ or $a = 6q + 5$

**3.** Total number of members = 616

∴ The total number of members are to march behind an army band of 32 members is HCF of 616 and 32.

*i.e.*, HCF of 616 and 32 is equal to the maximum number of columns such that the two groups can march in the same number of columns.

Applying Euclid's division lemma to 616 and 32, we get

616 = (32 × 19) + 8,

32 = (8 × 4) + 0

Here, remainder = 0, when divisor is 8.

∴ HCF of 616 and 32 is 8.

Hence, the required number of maximum columns = 8.

**4.** Let us consider an arbitrary positive integer as '$x$'. On dividing $x$ by 3 and applying Euclid's division lemma, we get that $x$ is of the form,

$3q$, $(3q + 1)$ or $(3q + 2)$

For $x = 3q$, we have

$x^2 = (3q)^2$

$\Rightarrow x^2 = 9q^2 = 3(3q^2) = 3m$ ...(i)

where $m = 3q^2$ is an integer.

For $x = 3q + 1$, we have

$x^2 = (3q + 1)^2 = 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1 = 3m + 1$ ...(ii)

where $m = 3q^2 + 2q$ is an integer.

For $x = 3q + 2$, we have

$x^2 = (3q + 2)^2$

$= 9q^2 + 12q + 4 = (9q^2 + 12q + 3) + 1$

$= 3(3q^2 + 4q + 1) + 1 = 3m + 1$ ...(iii)

where $m = 3q^2 + 4q + 1$ is an integer.

From (i), (ii) and (iii), we get $x^2 = 3m$ or $3m + 1$

Thus, the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer $m$.

**5.** Let us consider an arbitrary positive integer $x$ such that it is of the form $3q$, $(3q + 1)$ or $(3q + 2)$.

For $x = 3q$

$x^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$ ...(i)

where $3q^3 = m$ is an integer.

For $x = 3q + 1$

$x^3 = (3q + 1)^3 = 27q^3 + 27q^2 + 9q + 1$

$= 9(3q^3 + 3q^2 + q) + 1 = 9m + 1$ ...(ii)

where $3q^3 + 3q^2 + q = m$ is an integer.

For $x = 3q + 2$,

$x^3 = (3q + 2)^3 = 27q^3 + 54q^2 + 36q + 8$

$= 9(3q^3 + 6q^2 + 4q) + 8 = 9m + 8$ ...(iii)

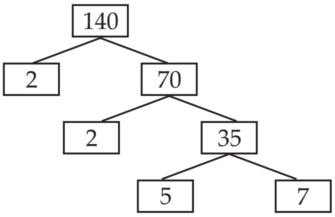
where $3q^3 + 6q^2 + 4q = m$ is an integer.

From (i), (ii) and (iii), we get $x^3 = 9m$, $(9m + 1)$ or $(9m + 8)$

Thus, cube of any positive integer can be in the form $9m$, $(9m + 1)$ or $(9m + 8)$ for some integer $m$.

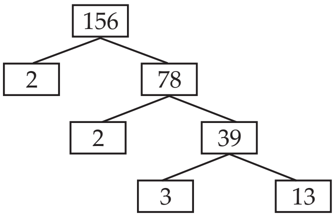
## EXERCISE - 1.2

**1.** (i) Using factor tree method, we have



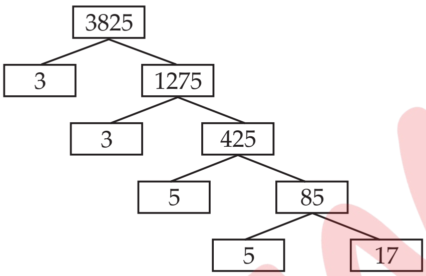

$\therefore$ $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) Using factor tree method, we have



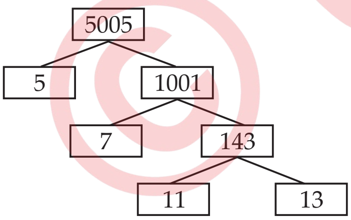

$\therefore$ $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii) Using factor tree method, we have



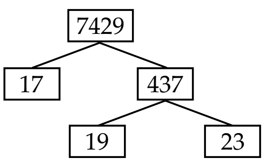

$\therefore$ $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

(iv) Using factor tree method, we have




$\therefore$ $5005 = 5 \times 7 \times 11 \times 13$

(v) Using factor tree method, we have




$\therefore$ $7429 = 17 \times 19 \times 23$

**2.** (i) The prime factorisation of 26 and 91 is,
$26 = 2 \times 13$ and $91 = 7 \times 13$

$\therefore$ LCM (26, 91) $= 2 \times 7 \times 13 = 182$
HCF (26, 91) = 13

Now, LCM × HCF $= 182 \times 13 = 2366$ and $26 \times 91 = 2366$
*i.e.*, LCM × HCF = Product of two numbers.

(ii) The prime factorisation of 510 and 92 is,
$510 = 2 \times 3 \times 5 \times 17$ and $92 = 2 \times 2 \times 23$

$\therefore$ LCM (510, 92) $= 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$
HCF (510, 92) = 2
Now, LCM × HCF $= 23460 \times 2 = 46920$
and $510 \times 92 = 46920$
*i.e.*, LCM × HCF = Product of two numbers.

(iii) The prime factorisation of 336 and 54 is,
$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$ and $54 = 2 \times 3 \times 3 \times 3$

$\therefore$ LCM (336, 54) $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 3024$
and HCF(336, 54) $= 2 \times 3 = 6$
Now, LCM × HCF $= 3024 \times 6 = 18144$
Also, $336 \times 54 = 18144$
Thus, LCM × HCF = Product of two numbers.

**3.** (i) The prime factorisation of 12, 15 and 21 is,
$12 = 2 \times 2 \times 3$, $15 = 3 \times 5$ and $21 = 3 \times 7$

$\therefore$ HCF (12, 15, 21) = 3
LCM (12, 15, 21) $= 2 \times 2 \times 3 \times 5 \times 7 = 420$

(ii) We have, $17 = 1 \times 17$, $23 = 1 \times 23$, $29 = 1 \times 29$

$\Rightarrow$ HCF (17, 23, 29) = 1
LCM (17, 23, 29) $= 17 \times 23 \times 29 = 11339$

(iii) The prime factorisation of 8, 9 and 25 is,
$8 = 2 \times 2 \times 2$, $9 = 3 \times 3$ and $25 = 5 \times 5$

$\therefore$ HCF (8, 9, 25) = 1
LCM (8, 9, 25) $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

**4.** Since, LCM × HCF = Product of the numbers

$\therefore$ LCM × 9 = 306× 657

$$\Rightarrow \text{LCM} = \frac{306 \times 657}{9} = 22338$$

Thus, LCM of 306 and 657 is 22338.

**5.** Here, $n$ is a natural number and let $6^n$ ends with digit 0.

$\therefore$ $6^n$ is divisible by 5.
But the prime factors of 6 are 2 and 3. *i.e.*, $6 = 2 \times 3$

$\Rightarrow$ $6^n = (2 \times 3)^n$
*i.e.*, In the prime factorisation of $6^n$, there is no factor 5.
So, by the fundamental theorem of Arithmetic, every composite number can be expressed as a product of primes and this factorisation is unique apart from the order in which the prime factorisation occurs.

$\therefore$ Our assumption that $6^n$ ends with digit 0, is wrong.
Thus, there does not exist any natural number $n$ for which $6^n$ ends with zero.

**6.** We have
$7 \times 11 \times 13 + 13 = 13((7 \times 11) + 1) = 13(78)$, which cannot be a prime number because it has factors 13 and 78.
Also, $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$
$= 5[7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1]$,
which is also not a prime number because it has a factor 5
Thus, $7 \times 11 \times 13 + 13$ and
$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

**7.** Time taken by Sonia to drive one round of the field = 18 minutes

Time taken by Ravi to drive one round of the field = 12 minutes

LCM of 18 and 12 gives the exact number of minutes after which they meet again at the starting point.

Now, $18 = 2 \times 3 \times 3$ and $12 = 2 \times 2 \times 3$

$\therefore$ LCM of 18 and 12 $= 2 \times 2 \times 3 \times 3 = 36$

Thus, they will meet again at the starting point after 36 minutes.

## EXERCISE - 1.3

**1.** Let $\sqrt{5}$ be a rational number.

So, we can find integers $a$ and $b$ ($b \neq 0$ and $a$, $b$ are co-prime) such that $\sqrt{5} = \frac{a}{b}$

$\Rightarrow \sqrt{5} \cdot b = a$ ...(i)

Squaring both sides, we get

$5b^2 = a^2$

$\Rightarrow$ 5 divides $a^2 \Rightarrow$ 5 divides $a$ ...(ii)

So, we can write $a = 5m$, where $m$ is an integer.

$\therefore$ Putting $a = 5m$ in (i), we get

$\sqrt{5}b = 5m$

$\Rightarrow 5b^2 = 25m^2$ [Squaring both sides]

$\Rightarrow b^2 = 5m^2$

$\Rightarrow$ 5 divides $b^2 \Rightarrow$ 5 divides $b$ ...(iii)

From (ii) and (iii), we have, $a$ and $b$ have 5 as a common factor which contradicts the fact that $a$ and $b$ are co-prime.

$\therefore$ Our supposition that $\sqrt{5}$ is rational, is wrong.

Hence, $\sqrt{5}$ is irrational.

**2.** Let $3 + 2\sqrt{5}$ be a rational number.

$\therefore$ We can find two co-prime integers $a$ and $b$ such that

$3 + 2\sqrt{5} = \frac{a}{b}$, where $b \neq 0$

$\therefore \frac{a}{b} - 3 = 2\sqrt{5} \Rightarrow \frac{a - 3b}{b} = 2\sqrt{5} \Rightarrow \frac{a - 3b}{2b} = \sqrt{5}$ ... (i)

$\because$ $a$ and $b$ are integers,

$\therefore \frac{a - 3b}{2b}$ is rational

So, $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

$\therefore$ Our supposition is wrong.

Hence, $3 + 2\sqrt{5}$ is irrational.

**3.** (i) We have $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2} \cdot \sqrt{2}$

Let $\frac{1}{\sqrt{2}}$ be rational,

$\therefore \frac{1}{2}(\sqrt{2})$ is rational

Let $\frac{1}{2}(\sqrt{2}) = \frac{a}{b}$, such that $a$ and $b$ are co-prime integers and $b \neq 0$.

$\therefore \sqrt{2} = \frac{2a}{b}$ ... (i)

Since, the division of two integers is rational.

$\therefore \frac{2a}{b}$ is rational.

From (i), $\sqrt{2}$ is rational number, which contradicts the fact that $\sqrt{2}$ is irrational.

$\therefore$ Our assumption is wrong.

Thus, $\frac{1}{\sqrt{2}}$ is irrational.

(ii) Let $7\sqrt{5}$ is rational.

$\therefore$ We can find two co-prime integers $a$ and $b$ such that

$7\sqrt{5} = \frac{a}{b}$, where $b \neq 0$

Now, $7\sqrt{5} = \frac{a}{b} \Rightarrow \sqrt{5} = \frac{a}{7b}$, which is a rational number.

[$\because$ $a$ and $b$ are integers.]

$\Rightarrow \sqrt{5}$ is a rational number.

This contradicts the fact that $\sqrt{5}$ is an irrational number.

$\therefore$ Our assumption is wrong.

Thus, we conclude that $7\sqrt{5}$ is irrational.

(iii) Let $6 + \sqrt{2}$ is rational.

$\therefore$ We can find two co-prime integers $a$ and $b$ such that

$6 + \sqrt{2} = \frac{a}{b}$, where $b \neq 0$

$\therefore \frac{a}{b} - 6 = \sqrt{2} \Rightarrow \sqrt{2} = \frac{a - 6b}{b}$, which is rational

$\Rightarrow \sqrt{2}$ is rational which contradicts the fact that $\sqrt{2}$ is an irrational number.

$\therefore$ Our supposition is wrong.

Hence, $6 + \sqrt{2}$ is an irrational number.

## EXERCISE - 1.4

**1.** (i) $\because 3125 = 5 \times 5 \times 5 \times 5 \times 5 = 1 \times 5^5 = 2^0 \times 5^5$, which is of the form $2^m \times 5^n$

$\therefore$ 13/3125 will have a terminating decimal expansion.

(ii) $\because 8 = 2 \times 2 \times 2 = 2^3 = 1 \times 2^3$
$= 5^0 \times 2^3$, which is of the form $2^m \times 5^n$

$\therefore$ 17/8 will have a terminating decimal expansion.

(iii) $\because 455 = 5 \times 7 \times 13$, which is not of the form $2^m \times 5^n$

$\therefore$ 64/455 will have a non-terminating repeating decimal expansion.

(iv) $\because 1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$
$= 2^6 \times 5^2$, which is of the form $2^m \times 5^n$

$\therefore \frac{15}{1600}$ will have a terminating decimal expansion.

(v) $\because 343 = 7 \times 7 \times 7 = 7^3$, which is not of the form $2^m \times 5^n$

$\therefore$ 29/343 will have a non-terminating repeating decimal expansion.

(vi) Here, denominator = $2^3 \times 5^2$, which is of the form $2^m \times 5^n$

$\therefore$ $\frac{23}{2^3 5^2}$ will have a terminating decimal expansion.

(vii) Here, denominator = $2^2 \cdot 5^7 \cdot 7^5$, which is not of the form $2^m \times 5^n$.

$\therefore$ $\frac{129}{2^2 5^7 7^5}$ will have a non-terminating repeating decimal expansion.

(viii) $\because$ $\frac{6}{15} = \frac{3 \times 2}{3 \times 5} = \frac{2}{5}$ and $5 = 2^0 \times 5$, which is of the form $2^m \times 5^n$.

$\therefore$ 6/15 will have a terminating decimal expansion.

(ix) $\because$ $50 = 2 \times 5 \times 5 = 2^1 \times 5^2$, which is of the form $2^m \times 5^n$.

$\therefore$ 35/50 will have a terminating decimal expansion.

(x) $\because$ $210 = 2 \times 3 \times 5 \times 7 = 2^1 \cdot 3^1 \cdot 5^1 \cdot 7^1$, which is not of the form $2^m \times 5^n$.

$\therefore$ 77/210 will have a non-terminating repeating decimal expansion.

**2.** (i) We have, $\frac{13}{3125} = \frac{13}{5 \times 5 \times 5 \times 5 \times 5} = \frac{13}{5^5}$

Multiplying and dividing by $2^5$, we have

$$\frac{13}{3125} = \frac{13}{5^5} \times \frac{2^5}{2^5} = \frac{13 \times 32}{(10)^5} = \frac{416}{100000} = 0.00416$$

(ii) We have, $\frac{17}{8} = \frac{17}{2 \times 2 \times 2} = \frac{17}{2^3}$

Multiplying and dividing by $5^3$, we have

$$\frac{17}{8} = \frac{17}{2^3} \times \frac{5^3}{5^3} = \frac{17 \times 125}{(10)^3} = \frac{2125}{1000} = 2.125$$

(iii) 64/455 represents non-terminating repeating decimal expansion.

(iv) $\frac{15}{1600} = \frac{15}{2 \times 2 \times 2 \times 2 \times 10 \times 10} = \frac{15}{2^4 \times 10^2}$

Multiplying and dividing by $5^4$, we have

$$\frac{15}{1600} = \frac{15 \times 5^4}{2^4 \times 5^4 \times 10^2} = \frac{15 \times 625}{10^4 \times 10^2} = \frac{9375}{10^6} = 0.009375$$

(v) $\frac{29}{343}$ represents non-terminating repeating decimal expansion.

(vi) $\frac{23}{2^3 \cdot 5^2}$

Multiplying and dividing by 5, we have

$$\frac{23}{2^3 \cdot 5^2} = \frac{23 \times 5}{2^3 \times 5^2 \times 5} = \frac{115}{2^3 \times 5^3} = \frac{115}{(10)^3} = 0.115$$

(vii) $\frac{129}{2^2 5^7 7^5}$ represents non-terminating repeating decimal expansion.

(viii) 6/15 = 2/5

Multiplying and dividing by 2, we have

$$\frac{6}{15} = \frac{2}{5} \times \frac{2}{2} = \frac{4}{10} = 0.4$$

(ix) $\frac{35}{50} = \frac{7}{10} = 0.7$

(x) 77/210 represents non-terminating repeating decimal expansion.

**3.** (i) 43.123456789

$\because$ The given decimal expansion terminates.

$\therefore$ It is a rational number of the form $p/q$

$\Rightarrow$ $\frac{p}{q} = 43.123456789$

$$= \frac{43123456789}{1000000000} = \frac{43123456789}{10^9} = \frac{43123456789}{5^9 \times 2^9}$$

Hence, $p = 43123456789$ and $q = 2^9 \times 5^9$

$\therefore$ Prime factors of $q$ are of the form $2^m \times 5^n$

(ii) 0.120120012000120000 ...

$\because$ The given decimal expansion is neither terminating nor repeating.

$\therefore$ It is an irrational number, hence cannot be written in $p/q$ form.

(iii) $43.\overline{123456789}$

$\because$ The given decimal expansion is non-terminating repeating.

$\therefore$ It is a rational number.

Let $x = 43.123456789\ldots$ ... (i)

Multiplying both sides by 1000000000, we have

$1000000000x = 43123456789.123456789...$ ...(ii)

Subtracting (i) from (ii), we have

$1000000000x - x$

$= (43123456789.123456789.......) - (43.123456789...)$

$\Rightarrow$ $999999999x = 43123456746$

$\Rightarrow$ $x = \frac{43123456746}{999999999} = \frac{4791495194}{111111111}$

Here, $p = 4791495194$ and $q = 111111111$, which is not of the form $2^m \times 5^n$ *i.e.*, the prime factors of $q$ are not of the form $2^m \times 5^n$.