Polynomials

CHAPTER 2

NCERT FOCUS

SOLUTIONS

EXERCISE - 2.1

1. (i) The given graph is parallel to x-axis, it does not intersect the x-axis.

$\therefore$ It has no zero.

(ii) The given graph intersects the x-axis at one point only.

$\therefore$ It has one zero.

(iii) The given graph intersects the x-axis at three points.

$\therefore$ It has three zeroes.

(iv) The given graph intersects the x-axis at two points.

$\therefore$ It has two zeroes.

(v) The given graph intersects the x-axis at four points.

$\therefore$ It has four zeroes.

(vi) The given graph meets the x-axis at three points.

$\therefore$ It has three zeroes.

EXERCISE - 2.2

1. (i) We have, $p(x) = x^2 - 2x - 8$

$= x^2 + 2x - 4x - 8 = x(x + 2) - 4(x + 2) = (x - 4)(x + 2)$

For $p(x) = 0$, we must have $(x - 4)(x + 2) = 0$

Either $x - 4 = 0 \Rightarrow x = 4$ or $x + 2 = 0 \Rightarrow x = -2$

$\therefore$ The zeroes of $x^2 - 2x - 8$ are 4 and -2

Now, sum of the zeroes $= 4 + (-2) = 2 = \frac{-(-2)}{1}$

$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= 4 \times (-2) = \frac{-8}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Thus, the relationship between the zeroes and the coefficients in the polynomial $x^2 - 2x - 8$ is verified.

(ii) We have, $p(s) = 4s^2 - 4s + 1$

$= 4s^2 - 2s - 2s + 1 = 2s(2s - 1) - 1(2s - 1)$

$= (2s - 1)(2s - 1)$

For $p(s) = 0$, we have, $(2s - 1) = 0 \Rightarrow s = \frac{1}{2}$

$\therefore$ The zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$

Sum of the zeroes $= \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2}$

and product of zeroes $= \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$

Thus, the relationship between the zeroes and coefficients in the polynomial $4s^2 - 4s + 1$ is verified.

(iii) We have, $p(x) = 6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3)$

$= (3x + 1)(2x - 3)$

For $p(x) = 0$, we have,

Either $(3x + 1) = 0 \Rightarrow x = -\frac{1}{3}$

or $(2x - 3) = 0 \Rightarrow x = \frac{3}{2}$

Thus, the zeroes of $6x^2 - 3 - 7x$ are $-\frac{1}{3}$ and $\frac{3}{2}$.

Now, sum of the zeroes $= -\frac{1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6}$

$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

and product of zeroes $= \left(-\frac{1}{3}\right) \times \frac{3}{2} = \frac{-3}{6}$

$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Thus, the relationship between the zeroes and coefficients in the polynomial $6x^2 - 3 - 7x$ is verified.

(iv) We have, $p(u) = 4u^2 + 8u = 4u(u + 2)$

For $p(u) = 0$, we have

Either $4u = 0 \Rightarrow u = 0$

or $u + 2 = 0 \Rightarrow u = -2$

$\therefore$ The zeroes of $4u^2 + 8u$ are 0 and -2.

Now, $4u^2 + 8u$ can be written as $4u^2 + 8u + 0$.

Sum of the zeroes $= 0 + (-2) = -2 = \frac{-(8)}{4}$

$= \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

and the product of zeroes $= 0 \times (-2) = 0 = \frac{0}{4}$

$= \frac{\text{Constant term}}{\text{Coefficient of } u^2}$

Thus, the relationship between the zeroes and the coefficients in the polynomial $4u^2 + 8u$ is verified.

(v) We have, $p(t) = t^2 - 15 = (t^2) - (\sqrt{15})^2$

$= (t + \sqrt{15})(t - \sqrt{15})$ $[\because a^2 - b^2 = (a + b)(a - b)]$

For $p(t) = 0$, we have

Either $(t + \sqrt{15}) = 0 \Rightarrow t = -\sqrt{15}$

or $t - \sqrt{15} = 0 \Rightarrow t = \sqrt{15}$

$\therefore$ The zeroes of $t^2 - 15$ are $-\sqrt{15}$ and $\sqrt{15}$

Now, we can write $t^2 - 15$ as $t^2 + 0t - 15$.

$\therefore$ Sum of the zeroes $= -\sqrt{15} + \sqrt{15} = 0 = \frac{-(0)}{1}$

$= \frac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2}$

Product of zeroes = $(-\sqrt{15})\times(\sqrt{15}) = \frac{-(15)}{1}$

$= \frac{\text{Constant term}}{\text{Coefficient of } t^2}$

Thus, the relationship between zeroes and the coefficients in the polynomial $t^2 - 15$ is verified.

(vi) We have, $p(x) = 3x^2 - x - 4$

$= 3x^2 + 3x - 4x - 4 = 3x(x + 1) - 4(x + 1) = (x + 1)(3x - 4)$

For $p(x) = 0$, we have

Either $(x + 1) = 0 \Rightarrow x = -1$

or $3x - 4 = 0 \Rightarrow x = 4/3$

$\therefore$ The zeroes of $3x^2 - x - 4$ are -1 and $4/3$

Now, sum of the zeroes = $(-1)+\frac{4}{3} = \frac{1}{3} = \frac{-(-1)}{3}$

$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

and product of zeroes = $(-1)\times\frac{4}{3} = \frac{(-4)}{3}$

$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Thus, the relationship between the zeroes and coefficients in the polynomial $3x^2 - x - 4$ is verified.

2. (i) Since, sum of the zeroes, $(\alpha+\beta) = \frac{1}{4}$

Product of the zeroes, $\alpha\beta = -1$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - \left(\frac{1}{4}\right)x + (-1) = x^2 - \frac{1}{4}x - 1 = \frac{1}{4}(4x^2 - x - 4)$

Since, $\frac{1}{4}(4x^2 - x - 4)$ and $(4x^2 - x - 4)$ have same zeroes, therefore $(4x^2 - x - 4)$ is the required quadratic polynomial.

(ii) Since, sum of the zeroes, $(\alpha+\beta) = \sqrt{2}$

Product of zeroes, $\alpha\beta = \frac{1}{3}$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (\sqrt{2})x + \left(\frac{1}{3}\right) = \frac{1}{3}(3x^2 - 3\sqrt{2}x + 1)$

Since, $\frac{1}{3}(3x^2 - 3\sqrt{2}x + 1)$ and $(3x^2 - 3\sqrt{2}x + 1)$ have same zeroes, therefore

$(3x^2 - 3\sqrt{2}x + 1)$ is the required quadratic polynomial.

(iii) Since, sum of zeroes, $(\alpha + \beta) = 0$

Product of zeroes, $\alpha\beta = \sqrt{5}$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (0)x + \sqrt{5} = x^2 + \sqrt{5}$

(iv) Since, sum of zeroes, $(\alpha + \beta) = 1$

Product of zeroes, $\alpha\beta = 1$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (1)x + 1 = x^2 - x + 1$

(v) Since, sum of the zeroes, $(\alpha+\beta) = -\frac{1}{4}$

Product of zeroes, $\alpha\beta = 1/4$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4} = x^2 + \frac{x}{4} + \frac{1}{4} = \frac{1}{4}(4x^2 + x + 1)$

Since, $\frac{1}{4}(4x^2 + x + 1)$ and $(4x^2 + x + 1)$ have same zeroes, therefore, the required quadratic polynomial is $(4x^2 + x + 1)$.

(vi) Since, sum of zeroes, $(\alpha + \beta) = 4$ and product of zeroes, $\alpha\beta = 1$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 4x + 1$.

EXERCISE - 2.3

1. (i) Here, dividend $p(x) = x^3 - 3x^2 + 5x - 3$,

and divisor $g(x) = x^2 - 2$

$\therefore$ We have

$$\begin{array}{r|l} & x - 3 \\ x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\ & x^3 \qquad\quad - 2x \\ & (-) \qquad\quad (+) \\ \hline & \quad - 3x^2 + 7x - 3 \\ & \quad - 3x^2 \qquad + 6 \\ & \quad (+) \qquad\quad (-) \\ \hline & \qquad\qquad 7x - 9 \end{array}$$

Thus, the quotient = $(x - 3)$ and remainder = $(7x - 9)$.

(ii) Here, dividend $p(x) = x^4 - 3x^2 + 4x + 5$

and divisor $g(x) = x^2 + 1 - x = x^2 - x + 1$

$\therefore$ We have

$$\begin{array}{r|l} & x^2 + x - 3 \\ x^2 - x + 1 & x^4 + 0x^3 - 3x^2 + 4x + 5 \\ & x^4 - x^3 \quad + x^2 \\ & (-) \ (+) \quad (-) \\ \hline & \quad x^3 - 4x^2 + 4x + 5 \\ & \quad x^3 - x^2 \quad + x \\ & \quad (-) \ (+) \quad (-) \\ \hline & \qquad - 3x^2 + 3x + 5 \\ & \qquad - 3x^2 + 3x - 3 \\ & \qquad (+) \quad (-) \quad (+) \\ \hline & \qquad\qquad\qquad 8 \end{array}$$

Thus, the quotient = $(x^2 + x - 3)$ and remainder = 8.

(iii) Here, dividend, $p(x) = x^4 - 5x + 6$
and divisor, $g(x) = 2 - x^2 = -x^2 + 2$

$\therefore$ We have

$$\begin{array}{r|l} & -x^2 - 2 \\ \hline -x^2 + 2 & x^4 - 5x + 6 \\ & x^4 \qquad - 2x^2 \\ & (-) \qquad (+) \\ \hline & 2x^2 - 5x + 6 \\ & 2x^2 \qquad - 4 \\ & (-) \qquad (+) \\ \hline & -5x + 10 \\ \hline \end{array}$$

Thus, the quotient $= -x^2 - 2$ and remainder $= -5x + 10$.

2. (i) Dividing $2t^4 + 3t^3 - 2t^2 - 9t - 12$ by $t^2 - 3$, we have

$$\begin{array}{r|l} & 2t^2 + 3t + 4 \\ \hline t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\ & 2t^4 \qquad - 6t^2 \\ & (-) \qquad (+) \\ \hline & 3t^3 + 4t^2 - 9t - 12 \\ & 3t^3 \qquad - 9t \\ & (-) \qquad (+) \\ \hline & 4t^2 - 12 \\ & 4t^2 - 12 \\ & (-) \quad (+) \\ \hline & 0 \\ \hline \end{array}$$

$\because$ Remainder = 0

$\therefore$ $(t^2 - 3)$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) Dividing $3x^4 + 5x^3 - 7x^2 + 2x + 2$ by $x^2 + 3x + 1$, we have

$$\begin{array}{r|l} & 3x^2 - 4x + 2 \\ \hline x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\ & 3x^4 + 9x^3 + 3x^2 \\ & (-) \quad (-) \quad (-) \\ \hline & -4x^3 - 10x^2 + 2x + 2 \\ & -4x^3 - 12x^2 - 4x \\ & (+) \quad (+) \quad (+) \\ \hline & 2x^2 + 6x + 2 \\ & 2x^2 + 6x + 2 \\ & (-) \quad (-) \quad (-) \\ \hline & 0 \\ \hline \end{array}$$

$\because$ Remainder = 0

$\therefore$ $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) Dividing $x^5 - 4x^3 + x^2 + 3x + 1$ by $x^3 - 3x + 1$, we get

$$\begin{array}{r|l} & x^2 - 1 \\ \hline x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\ & x^5 - 3x^3 + x^2 \\ & (-) \quad (+) \quad (-) \\ \hline & -x^3 + 3x + 1 \\ & -x^3 + 3x - 1 \\ & (+) \quad (-) \quad (+) \\ \hline & 2 \\ \hline \end{array}$$

$\because$ Remainder = 2, *i.e.*, remainder $\neq 0$

$\therefore$ $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

3. We have $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Given $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are zeroes of $p(x)$.

$\therefore$ $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)$ is a factor of $p(x)$.

Now, let us divide $3x^4 + 6x^3 - 2x^2 - 10x - 5$ by

$$\left[\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)\right] = \left(x^2 - \frac{5}{3}\right)$$

$$\begin{array}{r|l} & 3x^2 + 6x + 3 \\ \hline x^2 - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\ & 3x^4 \qquad - 5x^2 \\ & (-) \qquad (+) \\ \hline & 6x^3 + 3x^2 - 10x - 5 \\ & 6x^3 \qquad - 10x \\ & (-) \qquad (+) \\ \hline & 3x^2 - 5 \\ & 3x^2 - 5 \\ & (-) \quad (+) \\ \hline & 0 \\ \hline \end{array}$$

$\therefore$ $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 + 6x + 3)(x^2 - 5/3)$

$= 3(x^2 + 2x + 1)\left(x^2 - \frac{5}{3}\right) = 3(x + 1)^2 (x^2 - 5/3)$

Thus, the other zeroes of the given polynomial are -1 and -1.

4. Here, dividend, $p(x) = x^3 - 3x^2 + x + 2$,
divisor $= g(x)$, quotient $= (x - 2)$ and remainder $= (-2x + 4)$

$\because$ (Divisor × Quotient) + Remainder = Dividend

$\therefore$ $[g(x) \times (x - 2)] + [(-2x + 4)] = x^3 - 3x^2 + x + 2$

$\Rightarrow$ $g(x) \times (x - 2) = x^3 - 3x^2 + x + 2 - (-2x + 4)$

$= x^3 - 3x^2 + x + 2 + 2x - 4 = x^3 - 3x^2 + 3x - 2$

$\therefore$ $g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{x - 2}$

Now, dividing $x^3 - 3x^2 + 3x - 2$ by $x - 2$, we have

$$\begin{array}{r|l} x-2 & x^3-3x^2+3x-2 \quad (x^2-x+1 \\ & x^3-2x^2 \\ & (-)\ (+) \\ \hline & -x^2+3x-2 \\ & -x^2+2x \\ & (+)\ (-) \\ \hline & x-2 \\ & x-2 \\ & (-)\ (+) \\ \hline & 0 \end{array}$$

$\therefore \quad g(x) = \dfrac{x^3-3x^2+3x-2}{x-2} = x^2 - x + 1.$

Thus, the required divisor, $g(x) = x^2 - x + 1$.

5. (i) Let $p(x) = 3x^2 - 6x + 27$, $g(x) = 3$ and $q(x) = x^2 - 2x + 9$.

Then, deg $p(x)$ = deg $q(x)$ and $r(x) = 0$

Also, $p(x) = g(x) \times q(x) + r(x)$

(ii) Let $p(x) = 2x^3 - 2x^2 + 2x + 3$, $g(x) = 2x^2 - 1$, $q(x) = x - 1$ and $r(x) = 3x + 2$. Then, deg $q(x)$ = deg $r(x)$

Also, $p(x) = g(x) \times q(x) + r(x)$

(iii) Let $p(x) = 2x^3 - 4x^2 + x + 4$, $g(x) = 2x^2 + 1$, $q(x) = x - 2$ and $r(x) = 6$,

Then, deg $r(x) = 0$

Also, $p(x) = g(x) \times q(x) + r(x)$

EXERCISE - 2.4

1. (i) $\because p(x) = 2x^3 + x^2 - 5x + 2$

$$\therefore \quad p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + \frac{2}{1} = \frac{1+1-10+8}{4} = 0$$

$\Rightarrow \quad \dfrac{1}{2}$ is a zero of $p(x)$.

Again, $p(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 2 + 1 - 5 + 2 = 0$

$\Rightarrow$ 1 is a zero of $p(x)$.

Also, $p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$

$= -16 + 4 + 10 + 2 = -16 + 16 = 0$

$\Rightarrow$ – 2 is a zero of $p(x)$.

Now, $p(x) = 2x^3 + x^2 - 5x + 2$

$\therefore$ Comparing it with $ax^3 + bx^2 + cx + d$, we have $a = 2$, $b = 1$, $c = -5$ and $d = 2$

Also, $\dfrac{1}{2}$, 1 and – 2 are the zeroes of $p(x)$.

Let $\alpha = \dfrac{1}{2}, \beta = 1$ and $\gamma = -2$

$$\therefore \quad \alpha+\beta+\gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

Again, $\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{1}{2}(1) + 1(-2) + (-2)\left(\dfrac{1}{2}\right)$

$$= \frac{1}{2} - 2 - 1 = \frac{-5}{2} = \frac{c}{a}$$

and product of zeroes = $\alpha\beta\gamma$

$$= \frac{1}{2} \times 1 \times (-2) = -\frac{2}{2} = \frac{-d}{a}$$

Thus, the relationship between the coefficients and the zeroes of $p(x)$ is verified.

(ii) Here, $p(x) = x^3 - 4x^2 + 5x - 2$

$\therefore$ $p(2) = (2)^3 - 4(2)^2 + 5(2) - 2$

$= 8 - 16 + 10 - 2 = 18 - 18 = 0$

$\Rightarrow$ 2 is a zero of $p(x)$.

Again, $p(1) = (1)^3 - 4(1)^2 + 5(1) - 2$

$= 1 - 4 + 5 - 2 = 6 - 6 = 0$

$\Rightarrow$ 1 is a zero of $p(x)$.

Now, comparing $p(x) = x^3 - 4x^2 + 5x - 2$ with $ax^3 + bx^2 + cx + d = 0$, we have

$a = 1$, $b = -4$, $c = 5$ and $d = -2$

Also, 2, 1 and 1 are the zeroes of $p(x)$.

Let $\alpha = 2, \beta = 1, \gamma = 1$

Now, sum of zeroes = $\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = -b/a$

Again, $\alpha\beta + \beta\gamma + \gamma\alpha = 2(1) + 1(1) + 1(2)$

$= 2 + 1 + 2 = 5 = \dfrac{c}{a}$

and product of zeroes = $\alpha\beta\gamma = (2)(1)(1) = 2 = -d/a$

Thus, the relationship between the zeroes and the coefficients of $p(x)$ is verified.

2. Let the required cubic polynomial be $ax^3 + bx^2 + cx + d = 0$ and its zeroes be α, β and γ.

$$\therefore \quad \alpha+\beta+\gamma = 2 = \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3} = \frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = -7 = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a}$$

$$\alpha\beta\gamma = -14 = \frac{-(\text{Constant term})}{\text{Coefficient of } x^3} = \frac{-d}{a}$$

If $a = 1$, then $\dfrac{-b}{a} = \dfrac{-b}{1} = 2 \Rightarrow b = -2$,

$\dfrac{c}{a} = \dfrac{c}{1} = -7 \Rightarrow c = -7$

and $\dfrac{-d}{a} = -\dfrac{d}{1} = -14 \Rightarrow d = 14$

$\therefore$ The required cubic polynomial is

$1x^3 + (-2)x^2 + (-7)x + 14 = 0$

$= x^3 - 2x^2 - 7x + 14 = 0$

3. We have, $p(x) = x^3 - 3x^2 + x + 1$.

Comparing it with $Ax^3 + Bx^2 + Cx + D$,

We have $A = 1$, $B = -3$, $C = 1$ and $D = 1$

$\because$ It is given that $(a - b)$, a and $(a + b)$ are the zeroes of the polynomial.

$\therefore$ Let $\alpha = (a - b)$, $\beta = a$ and $\gamma = (a + b)$

$$\therefore \quad \alpha+\beta+\gamma = -\frac{B}{A} = -\frac{(-3)}{1} = 3$$

$\Rightarrow$ $(a - b) + a + (a + b) = 3 \Rightarrow 3a = 3 \Rightarrow a = 1$

Again, $\alpha\beta\gamma = \frac{-D}{A} = -1$

$\Rightarrow$ $(a - b) \times a \times (a + b) = -1$

$\Rightarrow$ $(1 - b) \times 1 \times (1 + b) = -1 \Rightarrow 1 - b^2 = -1$

$\Rightarrow$ $b^2 = 1 + 1 = 2 \Rightarrow b = \pm\sqrt{2}$

Thus, $a = 1$ and $b = \pm\sqrt{2}$

4. Here, $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

$\because$ Two of the zeroes of $p(x)$ are : $2 \pm \sqrt{3}$

$\therefore$ $[x - (2 + \sqrt{3})][x - (2 - \sqrt{3})]$

$= [(x - 2) - \sqrt{3}][(x - 2) + \sqrt{3}]$

$= (x - 2)^2 - (\sqrt{3})^2$

$= (x^2 + 4 - 4x) - 3 = x^2 - 4x + 1$

So, $x^2 - 4x + 1$ is a factor of $p(x)$.

Now, dividing $p(x)$ by $x^2 - 4x + 1$, we have

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & (-)\quad (+)\quad (-) \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & (+)\quad (-)\quad (+) \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & (+)\quad (-)\quad (+) \\
\hline
 & 0
\end{array}
$$

$\therefore$ $(x^2 - 4x + 1)(x^2 - 2x - 35) = p(x)$

$\Rightarrow$ $(x^2 - 4x + 1)\ (x^2 - 7x + 5x - 35) = p(x)$

$\Rightarrow$ $(x^2 - 4x + 1)\ [x(x - 7) + 5(x - 7)] = p(x)$

$\Rightarrow$ $(x^2 - 4x + 1)(x - 7)(x + 5) = p(x)$

i.e., $(x - 7)$ and $(x + 5)$ are other factors of $p(x)$.

$\therefore$ 7 and – 5 are other zeroes of the given polynomial.

5. Applying the division algorithm to the polynomials $x^4 - 6x^3 + 16x^2 - 25x + 10$ and $x^2 - 2x + k$, we have

$$
\begin{array}{r|l}
 & x^2 - 4x + (8 - k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\
 & x^4 - 2x^3 + kx^2 \\
 & (-)\quad (+)\quad (-) \\
\hline
 & -4x^3 + (16 - k)x^2 - 25x + 10 \\
 & -4x^3 + 8x^2 \qquad -4kx \\
 & (+)\quad (-)\qquad (+) \\
\hline
 & (8 - k)x^2 + (4k - 25)x + 10 \\
 & (8 - k)x^2 - 2(8 - k)x + k(8 - k) \\
 & (-)\qquad (+)\qquad (-) \\
\hline
 & (-9 + 2k)x - k(8 - k) + 10 \\
\hline
\end{array}
$$

$\therefore$ Remainder $= (2k - 9)x - k(8 - k) + 10$

But the remainder $= x + a$ (Given)

Therefore, comparing them, we have

$2k - 9 = 1 \Rightarrow 2k = 1 + 9 = 10 \Rightarrow k = 5$

and $a = -k(8 - k) + 10$

$= -5(8 - 5) + 10$

$= -5(3) + 10 = -15 + 10 = -5$

Thus, $k = 5$ and $a = -5$