Pair of Linear Equations in Two Variables

CHAPTER 3

NCERT FOCUS

SOLUTIONS

EXERCISE - 3.1

1. At present : Let Aftab's age = x years
His daughter's age = y years
Seven years ago : Aftab's age = $(x - 7)$ years
His daughter's age = $(y - 7)$ years
According to the condition-I, we have $(x - 7) = 7(y - 7)$
$\Rightarrow \quad x - 7 = 7y - 49 \Rightarrow x - 7y + 42 = 0$ .....(i)
After three years : Aftab's age = $(x + 3)$ years
His daughter's age = $(y + 3)$ years
According to the condition-II, we have
$(x + 3) = 3(y + 3)$
$\Rightarrow \quad x + 3 = 3y + 9 \Rightarrow x - 3y - 6 = 0$ .... (ii)
Hence, algebraic representation of given situation is
$x - 7y + 42 = 0$ and $x - 3y - 6 = 0$
Graphical representation of (i) and (ii) :
From equation (i), we have :

$l_1 : x - 7y + 42 = 0 \Rightarrow y = \frac{x+42}{7}$

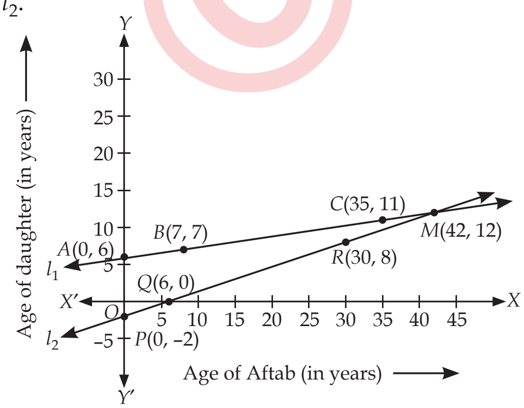
x	0	7	35
y	6	7	11

From equation (ii), we have

$l_2 : x - 3y - 6 = 0 \Rightarrow y = \frac{x-6}{3}$

x	0	6	30
y	–2	0	8

Plotting the points $A(0, 6)$, $B(7, 7)$ and $C(35, 11)$ on the graph paper and joining them, we get the line l_1.
Similarly, plotting the points $P(0, –2)$, $Q(6, 0)$ and $R(30, 8)$ on the graph paper and joining them, we get the line l_2.



Clearly, the lines l_1 and l_2 intersect each other at $M(42, 12)$.

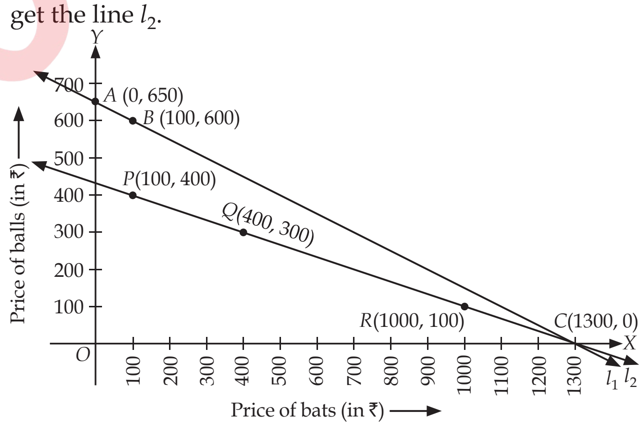
2. Let the cost of a bat = ₹ x and the cost of a ball = ₹ y
Algebraic representation :
Cost of 3 bats + Cost of 6 balls = ₹ 3900
$\Rightarrow \quad 3x + 6y = 3900 \Rightarrow x + 2y = 1300$ ..... (i)
Also, cost of 1 bat + cost of 3 balls = ₹ 1300
$\Rightarrow \quad x + 3y = 1300$ ..... (ii)
Thus, (i) and (ii) are the algebraic representations of the given situation.
Geometrical representation :
We have for equation (i), $l_1 : x + 2y = 1300 \Rightarrow y = \frac{1300 - x}{2}$

x	0	100	1300
y	650	600	0

For equation (ii), $l_2 : x + 3y = 1300 \Rightarrow y = \frac{1300 - x}{3}$

x	100	400	1000
y	400	300	100

Now, plotting the points $A(0, 650)$, $B(100, 600)$ and $C(1300, 0)$ on the graph paper and joining them, we get the line l_1.
Similarly, plotting the points $P(100, 400)$, $Q(400, 300)$ and $R(1000, 100)$, on the graph paper and joining them, we get the line l_2.



We also see from the graph that the straight lines representing the two equations intersect each other at $C(1300, 0)$.

3. Let the cost of 1 kg of apples = ₹ x
And the cost of 1 kg of grapes = ₹ y
Algebraic representation :
$2x + y = 160$ ...(i)
and $4x + 2y = 300 \Rightarrow 2x + y = 150$ ...(ii)
Geometrical representation :
We have, for equation (i), $l_1 : 2x + y = 160 \Rightarrow y = 160 - 2x$

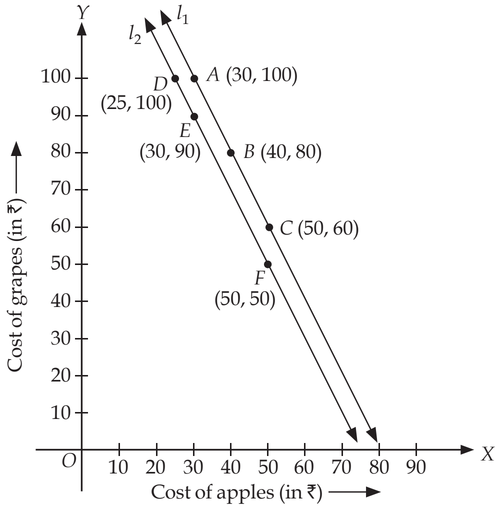
x	50	40	30
y	60	80	100

From equation (ii), we have

$l_2 : 2x + y = 150 \Rightarrow y = 150 - 2x$

x	50	30	25
y	50	90	100

Plotting the points $A(30, 100)$, $B(40, 80)$ and $C(50, 60)$ on the graph paper and joining them, we get the line l_1.
Similarly, plotting the points $D(25, 100)$, $E(30, 90)$ and $F(50, 50)$ on the graph paper and joining them, we get the line l_2.
The lines l_1 and l_2 are parallel.



EXERCISE - 3.2

1. (i) Let the number of boys be x and number of girls be y.

$\therefore \quad x + y = 10$ ..... (1)

$\because$ Number of girls = [Number of boys] + 4

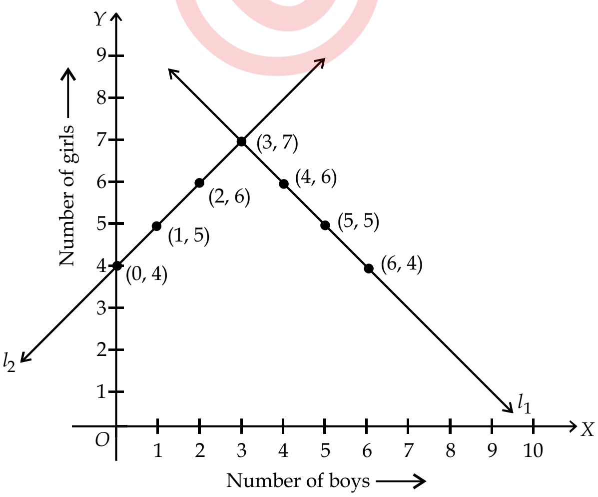
$\therefore \quad y = x + 4$ ..... (2)

Now, from equation (1), we have $l_1 : y = 10 - x$

x	6	4	5
y	4	6	5

And from equation (2), we have $l_2 : y = x + 4$

x	0	1	2
y	4	5	6



Since, l_1 and l_2 intersect at the point (3, 7).

$\therefore$ The solution of the given pair of linear equations is $x = 3, y = 7$

$\therefore$ Required number of boys and girls are 3 and 7 respectively.

(ii) Let the cost of a pencil is ₹ x and cost of a pen is ₹ y.

Since, cost of 5 pencils + Cost of 7 pens = ₹ 50

$\Rightarrow \quad 5x + 7y = 50$ ...(1)

Also, cost of 7 pencils + cost of 5 pens = ₹ 46

$\Rightarrow \quad 7x + 5y = 46$ ...(2)

Now, from equation (1), we have

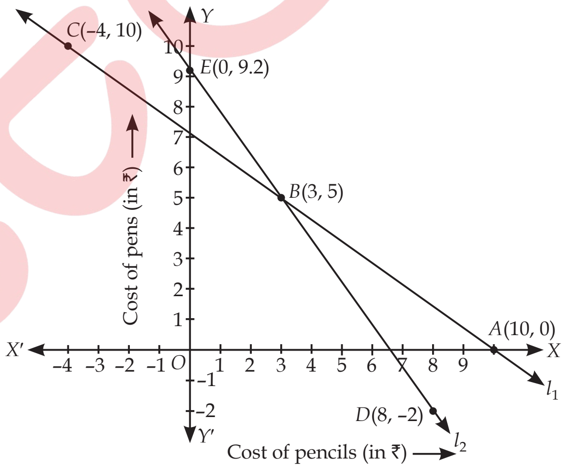
$$l_1 : y = \frac{50 - 5x}{7}$$

x	10	3	–4
y	0	5	10

And from equation (2), we have

$$l_2 : y = \frac{46 - 7x}{5}$$

x	8	3	0
y	–2	5	9.2



Since, l_1 and l_2 intersect at $B(3, 5)$.

$\therefore$ Cost of a pencil is ₹ 3 and cost of a pen is ₹ 5.

2. Comparing the given equations with
$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, we have

(i) For, $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$

$a_1 = 5, b_1 = -4, c_1 = 8$ and $a_2 = 7, b_2 = 6, c_2 = -9$

$$\because \frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the lines are intersecting, *i.e.*, they intersect at a unique point.

(ii) For, $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$, we have
$a_1 = 9, b_1 = 3, c_1 = 12$ and $a_2 = 18, b_2 = 6, c_2 = 24$

$$\because \frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, the lines are coincident.

(iii) For, $6x - 3y + 10 = 0$, $2x - y + 9 = 0$, we have
$a_1 = 6, b_1 = -3, c_1 = 10$ and $a_2 = 2, b_2 = -1, c_2 = 9$

$\because \quad \frac{a_1}{a_2}=\frac{6}{2}=3, \frac{b_1}{b_2}=\frac{-3}{-1}=3, \frac{c_1}{c_2}=\frac{10}{9} \quad \Rightarrow \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

So, the lines are parallel.

3. (i) For, $3x + 2y = 5$, $2x - 3y = 7$, we have

$a_1 = 3, b_1 = 2, c_1 = -5$ and $a_2 = 2, b_2 = -3, c_2 = -7$

$\therefore \quad \frac{a_1}{a_2}=\frac{3}{2}, \frac{b_1}{b_2}=\frac{2}{-3}$ and $\frac{c_1}{c_2}=\frac{-5}{-7}=\frac{5}{7}$

$\because \quad \frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

So, lines are intersecting *i.e.*, they intersect at a unique point.

$\therefore$ It is consistent pair of equations.

(ii) For, $2x - 3y = 8$, $4x - 6y = 9$, we have

$a_1 = 2, b_1 = -3, c_1 = -8$ and $a_2 = 4, b_2 = -6, c_2 = -9$

$\therefore \quad \frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{-3}{-6}=\frac{1}{2}$ and $\frac{c_1}{c_2}=\frac{-8}{-9}=\frac{8}{9}$

Here, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

So, lines are parallel *i.e.*, the given pair of linear equations has no solution.

$\therefore$ It is inconsistent pair of equations.

(iii) For, $\frac{3}{2}x+\frac{5}{3}y=7$, $9x - 10y = 14$, we have

$a_1=\frac{3}{2}, b_1=\frac{5}{3}, c_1=-7$ and $a_2 = 9, b_2 = -10, c_2 = -14$

$\therefore \quad \frac{a_1}{a_2}=\frac{3/2}{9}=\frac{3}{2}\times\frac{1}{9}=\frac{1}{6}$,

$\frac{b_1}{b_2}=\frac{5/3}{-10}=\frac{5}{3}\times\frac{1}{-10}=-\frac{1}{6}$ and $\frac{c_1}{c_2}=\frac{-7}{-14}=\frac{1}{2}$

Here, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$. So lines are intersecting.

So, the given pair of linear equations has a unique solution.

$\therefore$ It is a consistent pair of equations.

(iv) For, $5x - 3y = 11$, $-10x + 6y = -22$, we have

$a_1 = 5, b_1 = -3, c_1 = -11$ and $a_2 = -10, b_2 = 6, c_2 = 22$

$\therefore \quad \frac{a_1}{a_2}=\frac{5}{-10}=-\frac{1}{2}, \frac{b_1}{b_2}=\frac{-3}{6}=-\frac{1}{2}$ and $\frac{c_1}{c_2}=\frac{-11}{22}=-\frac{1}{2}$

Here, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

So, lines are coincident.

$\therefore$ The given pair of linear equations has infinitely many solutions.

Thus, they are consistent.

(v) For, $\frac{4}{3}x+2y=8$, $2x + 3y = 12$, we have

$a_1=\frac{4}{3}, b_1=2, c_1=-8$ and $a_2 = 2, b_2 = 3, c_2 = -12$

$\therefore \quad \frac{a_1}{a_2}=\frac{4/3}{2}=\frac{4}{3}\times\frac{1}{2}=\frac{2}{3}, \frac{b_1}{b_2}=\frac{2}{3}$ and $\frac{c_1}{c_2}=\frac{-8}{-12}=\frac{2}{3}$

Since, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

So, the lines are coincident *i.e.*, they have infinitely many solutions.

$\therefore$ The given pair of linear equations are consistent.

4. (i) For, $x + y = 5$, $2x + 2y = 10$, we have

$a_1 = 1, b_1 = 1, c_1 = -5$ and $a_2 = 2, b_2 = 2, c_2 = -10$

$\therefore \quad \frac{a_1}{a_2}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{1}{2}$ and $\frac{c_1}{c_2}=\frac{-5}{-10}=\frac{1}{2}$

$\Rightarrow \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

So, lines are coincident.

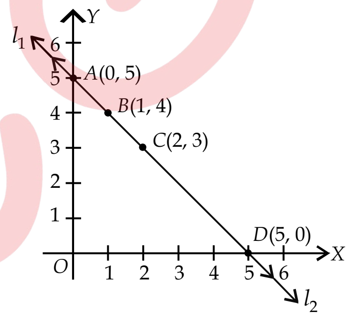
$\therefore$ The given pair of linear equations are consistent.

$l_1 : x + y = 5 \Rightarrow y = 5 - x$

x	0	5	1
y	5	0	4

$l_2 : 2x + 2y = 10 \Rightarrow x + y = 5 \Rightarrow y = 5 - x$

x	2	0	5
y	3	5	0



From graph, it is clear that lines l_1 and l_2 are coincident.

$\therefore$ They have infinitely many solutions.

(ii) For, $x - y = 8$, $3x - 3y = 16$

$\because \quad a_1 = 1, b_1 = -1, c_1 = -8$ and $a_2 = 3, b_2 = -3, c_2 = -16$

$\therefore \quad \frac{a_1}{a_2}=\frac{1}{3}, \frac{b_1}{b_2}=\frac{-1}{-3}=\frac{1}{3}$ and $\frac{c_1}{c_2}=\frac{-8}{-16}=\frac{1}{2} \quad \because \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\therefore$ The pair of linear equations is inconsistent and lines are parallel.

$\therefore$ The given system of equations has no solution.

(iii) For, $2x + y - 6 = 0$, $4x - 2y - 4 = 0$

$a_1 = 2, b_1 = 1, c_1 = -6$ and $a_2 = 4, b_2 = -2, c_2 = -4$

$\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{1}{-2}; \frac{c_1}{c_2}=\frac{-6}{-4}=\frac{3}{2}$

Here, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

$\therefore$ Lines are intersecting.

So, it is a consistent pair of linear equations.

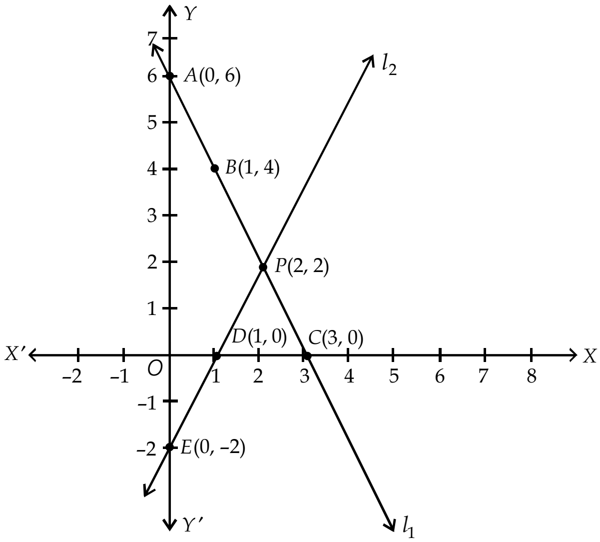
$\therefore$ It has a unique solution.

$l_1 : y = 6 - 2x$

x	0	3	1
y	6	0	4

and $l_2 : y=\frac{4x-4}{2}$

x	0	1	2
y	−2	0	2



$\because$ l_1 and l_2 intersect each other at $P(2, 2)$

$\therefore$ $x = 2$ and $y = 2$

(iv) For, $2x - 2y - 2 = 0$, $4x - 4y - 5 = 0$

$\because$ $a_1 = 2, b_1 = -2, c_1 = -2$ and $a_2 = 4, b_2 = -4, c_2 = -5$

$\therefore$ $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$; $\frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$

$\Rightarrow$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given pair of linear equations is inconsistent and lines are parallel.

Thus, the given system of equations has no solution.

5. Let the width of the garden be x m and the length of the garden be y m.

According to question, $4 + x = y$ ...(i)

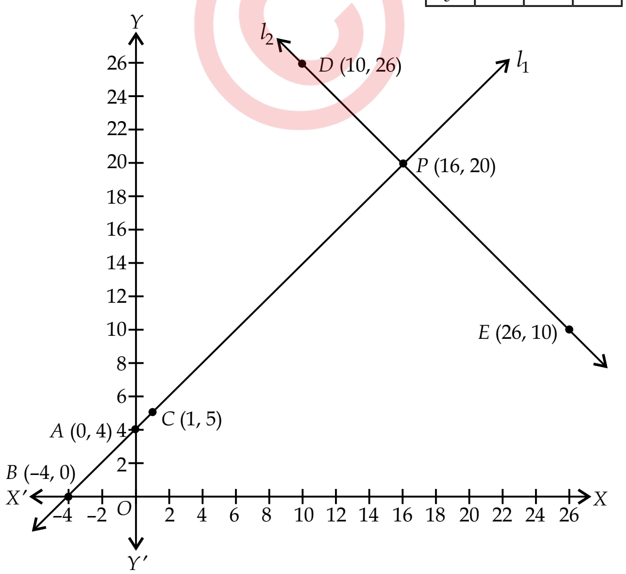
Also, $\frac{1}{2}$(perimeter) $= 36 \Rightarrow y + x = 36$ ...(ii)

From (i), $l_1 : y = x + 4$,

x	0	–4	1
y	4	0	5

From (ii), $l_2 : x + y = 36 \Rightarrow y = 36 - x$,

x	10	26	16
y	26	10	20



The lines l_1 and l_2 intersect each other at $P(16, 20)$.

$\therefore$ $x = 16$ and $y = 20$

So, Length = 20 m and width = 16 m

6. (i) Let the pair of linear equations be $2x + 3y - 8 = 0$, where $a_1 = 2$, $b_1 = 3$ and $c_1 = -8$ and $a_2x + b_2y + c_2 = 0$.

For intersecting lines, we have

$$\frac{2}{a_2} \neq \frac{3}{b_2} \neq \frac{-8}{c_2}$$

$\therefore$ We can have $a_2 = 3$, $b_2 = 2$ and $c_2 = -7$

$\therefore$ The required equation will be

$3x + 2y - 7 = 0$

(ii) For parallel lines, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore$ Line parallel to $2x + 3y - 8 = 0$, can be taken as $2x + 3y - 12 = 0$

(iii) For coincident lines, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\therefore$ Line coincident to $2x + 3y - 8 = 0$ can be taken as $2(2x + 3y - 8 = 0)$

$\Rightarrow$ $4x + 6y - 16 = 0$

7. We have, $x - y + 1 = 0$ ...(i)

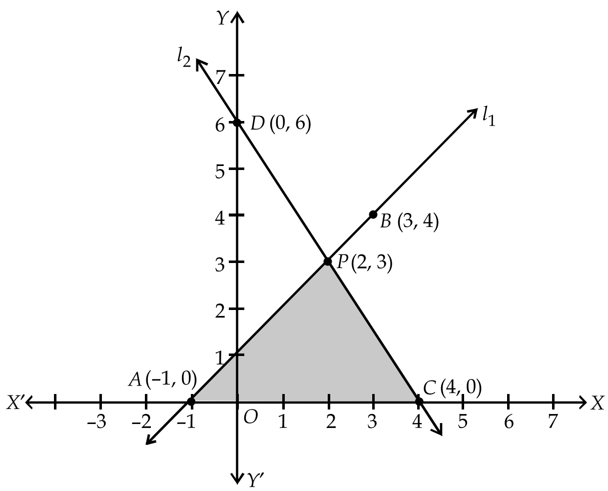
and $3x + 2y - 12 = 0$ ...(ii)

From (i), $l_1 : x - y + 1 = 0 \Rightarrow y = x + 1$

x	2	–1	3
y	3	0	4

From (ii), $l_2 : 3x + 2y - 12 = 0 \Rightarrow y = \frac{12 - 3x}{2}$

x	2	4	0
y	3	0	6



The lines l_1 and l_2 intersect at $P(2, 3)$. Thus, co-ordinates of the vertices of the shaded triangular region are $C(4, 0)$, $A(-1, 0)$ and $P(2, 3)$.

EXERCISE - 3.3

1. (i) We have, $x + y = 14$ ...(1)
and $x - y = 4$ ...(2)
From equation (1), we have $x = (14 - y)$
Substituting this value of x in (2), we get
$(14 - y) - y = 4 \Rightarrow 14 - 2y = 4$
$\Rightarrow -2y = -10 \Rightarrow y = 5$
Now, substituting $y = 5$ in (1), we get
$x + 5 = 14 \Rightarrow x = 9$
Hence, $x = 9$, $y = 5$ is the required solution.

(ii) We have, $s - t = 3$ ...(1)
and $\frac{s}{3} + \frac{t}{2} = 6$ ...(2)
From (1), we have $s = (3 + t)$ ...(3)
Substituting this value of s in (2), we get
$\frac{(3+t)}{3} + \frac{t}{2} = 6 \Rightarrow 2(3 + t) + 3(t) = 6 \times 6$
$\Rightarrow 6 + 2t + 3t = 36 \Rightarrow 5t = 30 \Rightarrow t = \frac{30}{5} = 6$
Substituting $t = 6$ in (3) we get, $s = 3 + 6 = 9$
Thus, $s = 9$, $t = 6$ is the required solution.

(iii) We have, $3x - y = 3$ ...(1)
$9x - 3y = 9$ ...(2)
From (1), $y = (3x - 3)$ ...(3)
Substituting this value of y in (2), we get
$9x - 3(3x - 3) = 9$
$\Rightarrow 9x - 9x + 9 = 9 \Rightarrow 9 = 9$, which is true statement.
$\therefore$ The equations (1) and (2) have infinitely many solutions.
To find these solutions, we put $x = k$ (any real constant) in (3), we get
$y = 3k - 3$
$\therefore$ $x = k$, $y = 3k - 3$ is the required solution, where k is any real number.

(iv) We have, $0.2x + 0.3y = 1.3$ ...(1)
and $0.4x + 0.5y = 2.3$ ...(2)
From (1), we have
$y = \frac{1.3 - 0.2x}{0.3}$ ...(3)
Substituting the value of y in (2), we get
$0.4x + 0.5\left[\frac{1.3 - 0.2x}{0.3}\right] = 2.3 \Rightarrow 0.4x + \left[\frac{0.65 - 0.1x}{0.3}\right] = 2.3$
$\Rightarrow 0.3 \times 0.4x + 0.65 - 0.1x = 0.3 \times 2.3$
$\Rightarrow 0.12x + 0.65 - 0.1x = 0.69$
$\Rightarrow 0.02x = 0.69 - 0.65 = 0.04 \Rightarrow x = \frac{0.04}{0.02} = 2$
From (3), $y = \frac{1.3 - 0.2(2)}{0.3} \Rightarrow y = \frac{1.3 - 0.4}{0.3} = \frac{0.9}{0.3} = 3$
Thus, $x = 2$ and $y = 3$ is the required solution.

(v) We have, $\sqrt{2}x + \sqrt{3}y = 0$ ...(1)
and $\sqrt{3}x - \sqrt{8}y = 0$ ...(2)
From (2), we have
$\sqrt{3}x = \sqrt{8}y \Rightarrow x = \frac{\sqrt{8}}{\sqrt{3}}y$ ...(3)
Substituting the value of x in (1), we get
$\sqrt{2}\left[\frac{\sqrt{8}}{\sqrt{3}}y\right] + \sqrt{3}y = 0$
$\Rightarrow \frac{\sqrt{16}}{\sqrt{3}}y + \sqrt{3}y = 0 \Rightarrow \frac{4}{\sqrt{3}}y + \sqrt{3}y = 0$
$\Rightarrow \left[\frac{4}{\sqrt{3}} + \sqrt{3}\right]y = 0 \Rightarrow y = 0$
Substituting $y = 0$ in (3), we get $x = 0$
Thus, $x = 0$ and $y = 0$ is the required solution.

(vi) We have, $\frac{3x}{2} - \frac{5y}{3} = -2$ ...(1) and $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ ...(2)
From (2), we have $\frac{x}{3} = \frac{13}{6} - \frac{y}{2}$
$\Rightarrow x = 3 \times \left(\frac{13}{6} - \frac{y}{2}\right) \Rightarrow x = \left[\frac{13}{2} - \frac{3}{2}y\right]$ ...(3)
Substituting the value of x in (1), we get
$\frac{3}{2}\left[\frac{13}{2} - \frac{3}{2}y\right] - \frac{5y}{3} = -2$
$\Rightarrow \frac{39}{4} - \frac{9y}{4} - \frac{5y}{3} = -2 \Rightarrow \frac{117 - 27y - 20y}{12} = -2$
$\Rightarrow 117 - 47y = -24 \Rightarrow -47y = -24 - 117 = -141$
$\Rightarrow y = \frac{-141}{-47} = 3$
Now, substituting $y = 3$ in (3), we get
$x = \frac{13}{2} - \frac{3}{2}(3) \Rightarrow x = \frac{13}{2} - \frac{9}{2} = \frac{4}{2} = 2$
Thus, $x = 2$ and $y = 3$ is the required solution.

2. We have, $2x + 3y = 11$ ...(1)
and $2x - 4y = -24$ ...(2)
From (1), we have
$2x = 11 - 3y$
$\Rightarrow x = \left[\frac{11 - 3y}{2}\right]$ ...(3)
Substituting this value of x in (2), we get
$2\left[\frac{11 - 3y}{2}\right] - 4y = -24 \Rightarrow 11 - 3y - 4y = -24$
$\Rightarrow -7y = -24 - 11 = -35 \Rightarrow y = \frac{-35}{-7} = 5$
Substituting $y = 5$ in (3), we get
$x = \frac{11 - 3(5)}{2} \Rightarrow x = \frac{11 - 15}{2} \Rightarrow x = \frac{-4}{2} = -2$
Thus, $x = -2$ and $y = 5$ is the required solution.
Also, $y = mx + 3 \Rightarrow 5 = m(-2) + 3$
$\Rightarrow -2m = 5 - 3 \Rightarrow -2m = 2 \Rightarrow m = -1$.

3. (i) Let the two numbers be x and y such that $x > y$.
$\because$ Difference between two numbers = 26
$\Rightarrow x - y = 26$ ...(1)
Again, one number = 3[the other number]
$\Rightarrow x = 3y$ $[\because x > y]$ ...(2)
Substituting $x = 3y$ in (1), we get
$3y - y = 26 \Rightarrow 2y = 26 \Rightarrow y = \frac{26}{2} = 13$

Now, substituting $y = 13$ in (2), we get

$x = 3(13) \Rightarrow x = 39$

Thus, two numbers are 39 and 13.

(ii) Let the two angles be x and y such that $x > y$.

$\because$ The larger angle exceeds the smaller by 18°

$\therefore$ $x = y + 18°$ ...(1)

Also, sum of two supplementary angles = 180°

$\therefore$ $x + y = 180°$ ...(2)

Substituting the value of x from (1) in (2), we get

$(18° + y) + y = 180°$

$\Rightarrow 2y = 180° - 18° = 162° \Rightarrow y = \frac{162°}{2} = 81°$

Substituting $y = 81°$ in (1), we get

$x = 18° + 81° = 99°$

Thus, $x = 99°$ and $y = 81°$ is the required solution.

(iii) Let the cost of a bat = ₹ x and the cost of a ball = ₹ y.

$\because$ [cost of 7 bats] + [cost of 6 balls] = ₹ 3800

$\Rightarrow 7x + 6y = 3800$ ...(1)

Again, [cost of 3 bats] + [cost of 5 balls] = ₹ 1750

$\Rightarrow 3x + 5y = 1750$ ...(2)

From (2), we have

$y = \left[\frac{1750 - 3x}{5}\right]$ ...(3)

Substituting this value of y in (1), we get

$7x + 6\left[\frac{1750 - 3x}{5}\right] = 3800$

$\Rightarrow 35x + 10500 - 18x = 19000$

$\Rightarrow 17x = 19000 - 10500 \Rightarrow x = \frac{8500}{17} = 500$

Substituting $x = 500$ in (3), we get

$y = \frac{1750 - 3(500)}{5} \Rightarrow y = \frac{1750 - 1500}{5} = \frac{250}{5} = 50$

Thus, $x = 500$ and $y = 50$

$\therefore$ Cost of a bat = ₹ 500 and cost of a ball = ₹ 50

(iv) Let fixed charges be ₹ x and charges per km be ₹ y.

$\because$ Charges for the journey of 10 km = ₹ 105

$\therefore$ $x + 10y = 105$ ...(1)

and charges for the journey of 15 km = ₹ 155

$\therefore$ $x + 15y = 155$ ...(2)

From (1), we have

$x = 105 - 10y$ ...(3)

Substituting the value of x in (2), we get

$(105 - 10y) + 15y = 155$

$\Rightarrow 5y = 155 - 105 = 50 \Rightarrow y = 10$

Substituting $y = 10$ in (3), we get

$x = 105 - 10(10) \Rightarrow x = 105 - 100 = 5$

Thus, $x = 5$ and $y = 10$

So, fixed charges = ₹ 5 and charges per km = ₹ 10.

Now, charges for 25 km = $x + 25y = 5 + 25(10)$

$= 5 + 250 =$ ₹ 255

$\therefore$ The charges for 25 km journey = ₹ 255

(v) Let the numerator = x and the denominator = y

$\therefore$ Fraction = x/y

Case I : $\frac{x+2}{y+2} = \frac{9}{11} \Rightarrow 11(x + 2) = 9(y + 2)$

$\Rightarrow 11x + 22 = 9y + 18$

$\Rightarrow 11x - 9y + 4 = 0$ ...(1)

Case II : $\frac{x+3}{y+3} = \frac{5}{6} \Rightarrow 6(x + 3) = 5(y + 3)$

$\Rightarrow 6x + 18 = 5y + 15$

$\Rightarrow 6x - 5y + 3 = 0$ ...(2)

Now, from (2), $x = \left[\frac{5y - 3}{6}\right]$ ...(3)

Substituting this value of x in (1), we get

$11\left[\frac{5y-3}{6}\right] - 9y + 4 = 0$

$\Rightarrow 55y - 33 - 54y + 24 = 0 \Rightarrow y - 9 = 0 \Rightarrow y = 9$

Now, substituting $y = 9$ in (3), we get

$x = \frac{5(9) - 3}{6} \Rightarrow x = \frac{45 - 3}{6} = \frac{42}{6} = 7$

$\therefore$ $x = 7$ and $y = 9 \Rightarrow$ Fraction = 7/9.

(vi) Let the present age of Jacob = x years and the present age of his son = y years.

$\therefore$ 5 years hence, age of Jacob = $(x + 5)$ years and age of his son = $(y + 5)$ years

Given, [Age of Jacob after 5 years] = 3[Age of his son after 5 years]

$\therefore$ $x + 5 = 3(y + 5) \Rightarrow x + 5 = 3y + 15$

$\Rightarrow x - 3y - 10 = 0$ ...(1)

5 years ago, age of Jacob = $(x - 5)$ years and age of his son = $(y - 5)$ years

Also, five years ago [Age of Jacob] = 7[Age of his son]

$\therefore$ $(x - 5) = 7(y - 5) \Rightarrow x - 5 = 7y - 35$

$\Rightarrow x - 7y + 30 = 0$ ...(2)

From (i), $x = 10 + 3y$ ...(3)

Substituting this value of x in (2), we get

$(10 + 3y) - 7y + 30 = 0 \Rightarrow -4y = -40 \Rightarrow y = 10$

Now, substituting $y = 10$ in (3), we get

$x = 10 + 3(10) \Rightarrow x = 10 + 30 = 40$

Thus, $x = 40$ and $y = 10$

$\therefore$ Present age of Jacob = 40 years and present age of his son = 10 years

EXERCISE - 3.4

1. (i) Elimination method : $x + y = 5$ ...(1)

$2x - 3y = 4$ ...(2)

Multiplying (1) by 3, we get

$3x + 3y = 15$ ...(3)

Adding (2) & (3), we get $5x = 19 \Rightarrow x = \frac{19}{5}$

Now, putting $x = \frac{19}{5}$ in (1), we get

$\frac{19}{5} + y = 5 \Rightarrow y = 5 - \frac{19}{5} = \frac{25 - 19}{5} = \frac{6}{5}$

Thus, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

Substitution Method :

We have, $x + y = 5 \Rightarrow y = 5 - x$ ...(1)

and $2x - 3y = 4$ ...(2)

Put $y = 5 - x$ in (2), we get

$2x - 3(5 - x) = 4 \Rightarrow 2x - 15 + 3x = 4$

$\Rightarrow \quad 5x = 19 \Rightarrow \quad x = \frac{19}{5}$

From. (1), $y = 5 - \frac{19}{5} = \frac{25-19}{5} = \frac{6}{5}$

Hence, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

(ii) Elimination method :

$3x + 4y = 10 \quad ...(1)$

$2x - 2y = 2 \quad ...(2)$

Multiplying (2) by 2, we get

$4x - 4y = 4 \quad ...(3)$

Adding (1) and (3), we get

$\therefore \quad 7x = 14 \quad \Rightarrow \quad x = \frac{14}{7} = 2$

Putting $x = 2$ in (1), we get

$3(2) + 4y = 10$

$\Rightarrow \quad 4y = 10 - 6 = 4 \Rightarrow \quad y = \frac{4}{4} = 1$

Thus, $x = 2$ and $y = 1$

Substitution Method :

$3x + 4y = 10 \quad \Rightarrow \quad y = \frac{10-3x}{4} \quad ...(1)$

$2x - 2y = 2 \Rightarrow x - y = 1 \quad ...(2)$

Putting $y = \frac{10-3x}{4}$ in (2), we get

$x - \left(\frac{10-3x}{4}\right) = 1 \quad \Rightarrow 4x - 10 + 3x = 4$

$\Rightarrow \quad 7x = 14 \quad \Rightarrow \quad x = \frac{14}{7} = 2$

Putting $x = 2$ in (1), we get

$y = \frac{10-3\times 2}{4} = \frac{10-6}{4} = \frac{4}{4} = 1$

Hence, $x = 2$ and $y = 1$

(iii) Elimination method :

$3x - 5y - 4 = 0 \quad ...(1)$

$9x = 2y + 7$ or $9x - 2y - 7 = 0 \quad ...(2)$

Multiplying (1) by 3, we get

$9x - 15y - 12 = 0 \quad ...(3)$

Subtracting (2) from (3), we get

$\therefore \quad 13y + 5 = 0 \quad \Rightarrow \quad y = \frac{-5}{13}$

Substituting the value of y in (1), we get

$3x - 5\left(\frac{-5}{13}\right) - 4 = 0 \quad \Rightarrow \quad 3x + \frac{25}{13} - 4 = 0$

$\Rightarrow \quad 3x = \frac{-25+52}{13} = \frac{27}{13} \Rightarrow x = \frac{27}{13} \times \frac{1}{3} = \frac{9}{13}$

Thus, $x = \frac{9}{13}$ and $y = -\frac{5}{13}$

Substitution Method :

$3x - 5y - 4 = 0 \quad \Rightarrow \quad y = \frac{3x-4}{5} \quad ...(1)$

$9x - 2y - 7 = 0 \quad ...(2)$

Putting $y = \frac{3x-4}{5}$ in (2), we get

$9x - 2\left(\frac{3x-4}{5}\right) - 7 = 0 \quad \Rightarrow 45x - 6x + 8 - 35 = 0$

$\Rightarrow \quad 39x = 27 \Rightarrow \quad x = \frac{27}{39} = \frac{9}{13}$

Putting $x = \frac{9}{13}$ in (1), we get

$y = \frac{3\times\frac{9}{13} - 4}{5} = \frac{27-52}{65} = \frac{-25}{65} \quad \Rightarrow \quad y = \frac{-5}{13}$

Hence, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

(iv) Elimination method :

$\frac{x}{2} + \frac{2y}{3} = -1 \quad ...(1)$

$x - \frac{y}{3} = 3 \quad ...(2)$

Multiplying (2) by 2, we get

$2x - \frac{2y}{3} = 6 \quad ...(3)$

Adding (1) and (3), we have

$\frac{x}{2} + 2x = 5 \quad \Rightarrow \quad \frac{5}{2}x = 5 \Rightarrow x = 5 \times \frac{2}{5} = 2$

Putting $x = 2$ in (1), we get

$\frac{2}{2} + \frac{2y}{3} = -1 \quad \Rightarrow 1 + \frac{2y}{3} = -1$

$\Rightarrow \quad \frac{2y}{3} = -1 - 1 = -2 \Rightarrow y = -2 \times \frac{3}{2} = -3$

Thus, $x = 2$ and $y = -3$

Substitution Method :

$\frac{x}{2} + \frac{2y}{3} = -1 \quad ...(1)$

$x - \frac{y}{3} = 3 \Rightarrow y = 3(x-3) \quad ...(2)$

Putting $y = 3(x - 3)$ in (1) from (2), we get

$\frac{x}{2} + \frac{2}{3} \times 3(x-3) = -1 \Rightarrow \frac{x}{2} + 2x - 6 = -1$

$\Rightarrow \quad \frac{5x}{2} = 5 \Rightarrow x = 5 \times \frac{2}{5} = 2$

Putting $x = 2$ in (2), we get

$y = 3(2 - 3) = 3(-1) = -3$

Hence, $x = 2$ and $y = -3$.

2. (i) Let the numerator = x and the denominator = y

$\therefore$ Fraction = x/y

Case I : $\frac{x+1}{y-1} = 1 \Rightarrow x + 1 = y - 1$

$\Rightarrow \quad x - y = -2 \quad ...(1)$

Case II : $\frac{x}{y+1} = \frac{1}{2} \quad \Rightarrow \quad x = \frac{1}{2}(y+1)$

$\Rightarrow \quad x - \frac{y}{2} = \frac{1}{2} \quad ...(2)$

Subtracting (2) from (1), we have

$-y + \frac{y}{2} = -2 - \frac{1}{2} \quad \Rightarrow \quad -\frac{1}{2}y = -\frac{5}{2} \Rightarrow \quad y = 5$

Now, putting $y = 5$ in (2), we have

$x - \frac{5}{2} = \frac{1}{2} \quad \Rightarrow \quad x = \frac{1}{2} + \frac{5}{2} = \frac{6}{2} = 3$

Thus, $x = 3$ and $y = 5$.

Hence, the required fraction = $3/5$

(ii) Let the present age of Nuri = x years and the present age of Sonu = y years

5 years ago :
Age of Nuri = $(x - 5)$ years and age of Sonu = $(y - 5)$ years
According to the question,
Age of Nuri = 3[Age of Sonu]
$\Rightarrow \quad x - 5 = 3[y - 5] \Rightarrow x - 5 = 3y - 15$
$\Rightarrow \quad x - 3y + 10 = 0$ ...(1)
10 years later :
Age of Nuri = $(x + 10)$ years and age of Sonu = $(y + 10)$ years
According to the question :
Age of Nuri = 2[Age of Sonu]
$\Rightarrow \quad x + 10 = 2(y + 10) \Rightarrow x + 10 = 2y + 20$
$\Rightarrow \quad x - 2y - 10 = 0$ ...(2)
Subtracting (1) from (2), $y - 20 = 0 \Rightarrow y = 20$
Putting $y = 20$ in (1), we get
$x - 3(20) + 10 = 0 \Rightarrow x - 50 = 0 \Rightarrow x = 50$
Thus, $x = 50$ and $y = 20$
$\therefore$ Age of Nuri = 50 years and age of Sonu = 20 years

(iii) Let the digit at unit's place = x and the digit at ten's place = y
$\therefore$ The number = $10y + x$
The number obtained by reversing the digits = $10x + y$
$\because$ 9[The number] = 2[Number obtained by reversing the digits]
$\therefore \quad 9[10y + x] = 2[10x + y] \Rightarrow 90y + 9x = 20x + 2y$
$\Rightarrow \quad 11x - 88y = 0$
$\Rightarrow \quad x - 8y = 0$ ...(1)
Also, sum of the digits = 9
$\therefore \quad x + y = 9$ ...(2)
Subtracting (1) from (2), we have
$9y = 9 \Rightarrow y = 1$
Putting $y = 1$ in (2), we get $x + 1 = 9 \Rightarrow x = 8$
Thus, $x = 8$ and $y = 1$
$\therefore$ The required number = $10y + x = (10 \times 1) + 8 = 10 + 8 = 18$

(iv) Let the number of 50 rupees notes = x
and the number of 100 rupees notes = y
According to the question,
Total number of notes = 25
$\therefore \quad x + y = 25$ ...(1)
$\because$ The value of all the notes = ₹ 2000
$\therefore \quad 50x + 100y = 2000$
$\Rightarrow \quad x + 2y = 40$ ...(2)
Subtracting (1) from (2), we get
$y = 15$
Putting $y = 15$ in (1), we get
$x + 15 = 25 \Rightarrow x = 25 - 15 = 10$
Thus, $x = 10$ and $y = 15$
$\therefore$ Number of 50 rupees notes = 10 and number of 100 rupees notes = 15.

(v) Let the fixed charge (for the three days) = ₹ x
and the additional charge for each extra day = ₹ y
$\because$ Charge for 7 days = ₹ 27
$\Rightarrow \quad x + 4y = 27$ ...(1) [$\because$ Extra days = 7 – 3 = 4]
Charge for 5 days = ₹ 21
$\Rightarrow \quad x + 2y = 21$ ...(2) [$\because$ Extra days = 5 – 3 = 2]
Subtracting (2) from (1), we get
$2y = 6 \Rightarrow y = 3$
Putting $y = 3$ in (2), we have
$x + 2(3) = 21 \Rightarrow x = 21 - 6 = 15$
So, $x = 15$ and $y = 3$
$\therefore$ Fixed charge = ₹ 15 and additional charge per day = ₹ 3.

EXERCISE - 3.5

1. Compare the given equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.
(i) For, $x - 3y - 3 = 0$, $3x - 9y - 2 = 0$
$a_1 = 1, b_1 = -3, c_1 = -3$ and $a_2 = 3, b_2 = -9, c_2 = -2$
Now, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
$\therefore$ The given system has no solution.
(ii) For $2x + y - 5 = 0, 3x + 2y - 8 = 0$
$a_1 = 2, b_1 = 1, c_1 = -5$ and $a_2 = 3, b_2 = 2, c_2 = -8$
Now, $\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\therefore$ The given system has a unique solution.
To solve the equation, we have

$$\frac{x}{\begin{array}{cc}1 & -5\\ 2 & -8\end{array}} = \frac{y}{\begin{array}{cc}-5 & 2\\ -8 & 3\end{array}} = \frac{1}{\begin{array}{cc}2 & 1\\ 3 & 2\end{array}}$$

$$\Rightarrow \frac{x}{(-8)-(-10)} = \frac{y}{(-15)-(-16)} = \frac{1}{4-3}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{1} = 1 \Rightarrow \frac{x}{2} = 1 \text{ and } \frac{y}{1} = 1 \Rightarrow x = 2 \text{ and } y = 1$$

(iii) For $3x - 5y - 20 = 0, 6x - 10y - 40 = 0$
$a_1 = 3, b_1 = -5, c_1 = -20$ and $a_2 = 6, b_2 = -10, c_2 = -40$
Since, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$
$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\therefore$ The given system of linear equations has infinitely many solutions.
(iv) For $x - 3y - 7 = 0, 3x - 3y - 15 = 0$
$a_1 = 1, b_1 = -3, c_1 = -7, \ a_2 = 3, b_2 = -3, c_2 = -15$
$\therefore \quad \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-3} = 1$ and $\frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$
Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ $\therefore$ The given system has unique solution.
Now, using cross multiplication method, we have

$$\frac{x}{\begin{array}{cc}-3 & -7\\ -3 & -15\end{array}} = \frac{y}{\begin{array}{cc}-7 & 1\\ -15 & 3\end{array}} = \frac{1}{\begin{array}{cc}1 & -3\\ 3 & -3\end{array}}$$

$$\Rightarrow \frac{x}{45-21} = \frac{y}{-21+15} = \frac{1}{-3+9}$$

$$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6} \Rightarrow x = \frac{24}{6} = 4, y = \frac{-6}{6} = -1$$

Thus, $x = 4$ and $y = -1$.

2. (i) For, $2x + 3y - 7 = 0$, $(a - b)x + (a + b)y - (3a + b - 2) = 0$
$a_1 = 2, b_1 = 3, c_1 = -7$ and
$a_2 = (a - b), b_2 = (a + b), c_2 = -(3a + b - 2)$

For an infinite number of solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{2}{(a-b)} = \frac{3}{(a+b)} = \frac{-7}{-(3a+b-2)}$

From the first two terms, we get

$\frac{2}{a-b} = \frac{3}{a+b}$

$\Rightarrow \quad 2a + 2b = 3a - 3b \Rightarrow 2a - 3a + 2b + 3b = 0$

$\Rightarrow \quad -a + 5b = 0 \Rightarrow a - 5b = 0$ ...(1)

From the last two terms, we get

$\frac{3}{a+b} = \frac{7}{3a+b-2}$

$\Rightarrow \quad 9a + 3b - 6 = 7a + 7b$

$\Rightarrow \quad 2a - 4b = 6 \Rightarrow a - 2b - 3 = 0$ ...(2)

Now, using cross-multiplication method, we have

$$\frac{a}{\begin{matrix}-5 & 0\\ -2 & -3\end{matrix}} = \frac{b}{\begin{matrix}0 & 1\\ -3 & 1\end{matrix}} = \frac{1}{\begin{matrix}1 & -5\\ 1 & -2\end{matrix}}$$

$\Rightarrow \quad \frac{a}{15-0} = \frac{b}{0+3} = \frac{1}{-2+5}$

$\Rightarrow \quad \frac{a}{15} = \frac{b}{3} = \frac{1}{3} \Rightarrow a = \frac{1}{3} \times 15 = 5, b = \frac{1}{3} \times 3 = 1$

Thus, $a = 5$ and $b = 1$.

(ii) For, $3x + y - 1 = 0$,

$(2k - 1)x + (k - 1)y - (2k + 1) = 0$

$a_1 = 3, b_1 = 1, c_1 = -1$ and $a_2 = 2k - 1, b_2 = k - 1,$ $c_2 = -(2k + 1)$

For no solution

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \Rightarrow \quad \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{-1}{-(2k+1)}$

Taking first two terms, we get

$3(k - 1) = 2k - 1 \Rightarrow 3k - 3 = 2k - 1$

$\Rightarrow \quad 3k - 2k = -1 + 3 \Rightarrow k = 2.$

3. Substitution method :

Given equations are

$8x + 5y = 9$ ...(1) and $3x + 2y = 4$ ...(2)

From (2), $y = \frac{4-3x}{2}$ ...(3)

Substituting this value of y in (1), we have

$8x + 5\left[\frac{4-3x}{2}\right] = 9$

$\Rightarrow \quad 16x + 20 - 15x = 18 \Rightarrow x = 18 - 20 = -2$

Substituting $x = -2$ in (3), we get

$y = \frac{4-3(-2)}{2} = \frac{4+6}{2} = \frac{10}{2} = 5$

Thus, $x = -2$ and $y = 5$

Cross-multiplication method :

For, $8x + 5y - 9 = 0, 3x + 2y - 4 = 0$

By cross-multiplication method, we have

$$\therefore \quad \frac{x}{\begin{matrix}5 & -9\\ 2 & -4\end{matrix}} = \frac{y}{\begin{matrix}-9 & 8\\ -4 & 3\end{matrix}} = \frac{1}{\begin{matrix}8 & 5\\ 3 & 2\end{matrix}}$$

$\Rightarrow \quad \frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15}$

$\Rightarrow \quad x = 1 \times (-2) = -2$ and $y = 1 \times 5 = 5$

Thus, $x = -2$ and $y = 5$

4. (i) Let the fixed charges = ₹ x and cost of food per day = ₹ y

For student A : Number of days = 20

$\therefore$ Cost of food for 20 days = ₹ $20y$

According to the question, $x + 20y = 1000$

$\Rightarrow \quad x + 20y - 1000 = 0$ ...(1)

For student B : Number of days = 26

$\therefore$ Cost of food for 26 days = ₹ $26y$

According to the question : $x + 26y = 1180$

$\Rightarrow \quad x + 26y - 1180 = 0$ ...(2)

Solving these by cross-multiplication, we get

$$\frac{x}{\begin{matrix}20 & -1000\\ 26 & -1180\end{matrix}} = \frac{y}{\begin{matrix}-1000 & 1\\ -1180 & 1\end{matrix}} = \frac{1}{\begin{matrix}1 & 20\\ 1 & 26\end{matrix}}$$

$\Rightarrow \quad \frac{x}{-23600+26000} = \frac{y}{-1000+1180} = \frac{1}{26-20}$

$\Rightarrow \quad \frac{x}{2400} = \frac{y}{180} = \frac{1}{6}$

$\therefore \quad x = \frac{1}{6} \times 2400 = 400, y = \frac{1}{6} \times 180 = 30$

Thus, $x = 400$ and $y = 30$

$\therefore$ Fixed charges = ₹ 400 and cost of food per day = ₹ 30

(ii) Let the numerator = x and the denominator = y

$\therefore$ Fraction = x/y

Condition-I : $\frac{x-1}{y} = \frac{1}{3}$

$\Rightarrow \quad 3x - 3 = y$

$\Rightarrow \quad 3x - y - 3 = 0$ ...(1)

Condition-II : $\frac{x}{y+8} = \frac{1}{4}$

$\Rightarrow \quad 4x = y + 8$

$\Rightarrow \quad 4x - y - 8 = 0$ ...(2)

From (1) and (2), we have

$a_1 = 3, b_1 = -1, c_1 = -3$ and $a_2 = 4, b_2 = -1, c_2 = -8$

Using cross-multiplication method, we get

$$\frac{x}{\begin{matrix}-1 & -3\\ -1 & -8\end{matrix}} = \frac{y}{\begin{matrix}-3 & 3\\ -8 & 4\end{matrix}} = \frac{1}{\begin{matrix}3 & -1\\ 4 & -1\end{matrix}}$$

$\Rightarrow \quad \frac{x}{8-3} = \frac{y}{-12+24} = \frac{1}{-3+4} \Rightarrow \frac{x}{5} = \frac{y}{12} = \frac{1}{1}$

$\therefore \quad x = \frac{1}{1} \times 5$ and $y = \frac{1}{1} \times 12 = 12$

Thus, $x = 5$ and $y = 12$ $\therefore$ Fraction = 5/12.

(iii) Let the number of correct answers = x and the number of wrong answers = y

Case-I : Marks for all correct answers = $(3 \times x) = 3x$

Marks for all wrong answers = $(1 \times y) = y$

$\therefore$ According to the condition :

$3x - y = 40$ ...(1)

Case-II : Marks for all correct answers = $(4 \times x) = 4x$

Marks for all wrong answers = $(2 \times y) = 2y$

$\therefore$ According to the condition :

$4x - 2y = 50 \Rightarrow 2x - y = 25$ ...(2)

From (1) and (2), we have

$a_1 = 3, b_1 = -1, c_1 = -40$ and

$a_2 = 2, b_2 = -1, c_2 = -25$

Using cross-multiplication method, we get

$$\frac{x}{\begin{matrix}-1 & -40\\ -1 & -25\end{matrix}} = \frac{y}{\begin{matrix}-40 & 3\\ -25 & 2\end{matrix}} = \frac{1}{\begin{matrix}3 & -1\\ 2 & -1\end{matrix}}$$

$$\Rightarrow \frac{x}{25-40} = \frac{y}{-80+75} = \frac{1}{-3+2} \Rightarrow \frac{x}{-15} = \frac{y}{-5} = \frac{1}{-1}$$

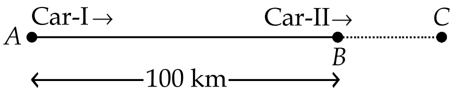
$$\Rightarrow x = \frac{1}{-1}\times(-15) = 15,\ y = \frac{1}{-1}\times(-5) = 5$$

$\therefore$ $x = 15$ and $y = 5$

Now, total number of questions = [Number of correct answers] + [Number of wrong answers] = 15 + 5 = 20

Thus, required number of questions = 20.

(iv) Let the speed of car-I be x km/hr and the speed of car-II be y km/hr.

Case-I : 

Distance travelled by car-I in 1 hour = AC

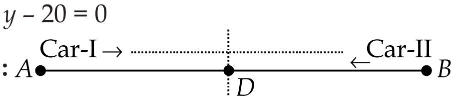
$\Rightarrow$ speed × time = 5 × x km, $AC = 5x$

Distance travelled by car-II, $BC = 5y$

Since $AB = AC - BC$, $100 = 5x - 5y$

$\Rightarrow$ $5x - 5y - 100 = 0$

$\Rightarrow$ $x - y - 20 = 0$ ...(1)

Case-II : 

Distance travelled by car-I in 1 hour = AD

$\therefore$ $AD = 1 \times x = x$

Distance travelled by car-II in 1 hour = BD

$\therefore$ $BD = 1 \times y = y$

Now, $AB = AD + DB \Rightarrow 100 = x + y$

$\Rightarrow$ $x + y = 100$ ...(2)

From (1) and (2), we have

$a_1 = 1, b_1 = -1, c_1 = -20$ and $a_2 = 1, b_2 = 1, c_2 = -100$,

Using cross-multiplication method, we get

$$\frac{x}{\begin{matrix}-1 & -20\\ 1 & -100\end{matrix}} = \frac{y}{\begin{matrix}-20 & 1\\ -100 & 1\end{matrix}} = \frac{1}{\begin{matrix}1 & -1\\ 1 & 1\end{matrix}}$$

$$\Rightarrow \frac{x}{100+20} = \frac{y}{-20+100} = \frac{1}{1+1} \Rightarrow \frac{x}{120} = \frac{y}{80} = \frac{1}{2}$$

$$\Rightarrow x = \frac{1}{2}\times 120 = 60,\ y = \frac{1}{2}\times 80 = 40$$

Thus, speed of car-I = 60 km/hr and speed of car-II = 40 km/hr.

(v) Let the length of the rectangle = x units and the breadth of the rectangle = y units

$\therefore$ Area of the rectangle = $x \times y = xy$

Condition-I :

(Length – 5) × (Breadth + 3) = Area – 9

$\Rightarrow$ $(x-5)(y+3) = xy - 9 \Rightarrow 3x - 5y - 15 = -9$

$\Rightarrow$ $3x - 5y - 6 = 0$ ...(1)

Condition-II :

(Length + 3) × (Breadth + 2) = Area + 67

$\Rightarrow$ $(x+3)(y+2) = xy + 67 \Rightarrow 2x + 3y + 6 = 67$

$\Rightarrow$ $2x + 3y - 61 = 0$ ...(2)

From (1) and (2), we have $a_1 = 3$, $b_1 = -5$, $c_1 = -6$ and $a_2 = 2$, $b_2 = 3$, $c_2 = -61$

Using cross-multiplication method, we get

$$\frac{x}{\begin{matrix}-5 & -6\\ 3 & -61\end{matrix}} = \frac{y}{\begin{matrix}-6 & 3\\ -61 & 2\end{matrix}} = \frac{1}{\begin{matrix}3 & -5\\ 2 & 3\end{matrix}}$$

$$\Rightarrow \frac{x}{305+18} = \frac{y}{-12+183} = \frac{1}{9+10} \Rightarrow \frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$$\therefore\ x = \frac{1}{19}\times 323 = 17,\ y = \frac{1}{19}\times 171 = 9$$

Thus, length of the rectangle = 17 units and breadth of the rectangle = 9 units.

EXERCISE - 3.6

1. (i) Put $\frac{1}{x} = u$ and $\frac{1}{y} = v$

$\therefore$ $\frac{1}{2x} + \frac{1}{3y} = 2 \Rightarrow \frac{u}{2} + \frac{v}{3} = 2$ ...(1)

And $\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6} \Rightarrow \frac{u}{3} + \frac{v}{2} = \frac{13}{6}$ ...(2)

Multiplying (1) by $\frac{1}{3}$ and (2) by $\frac{1}{2}$, we get

$\frac{1}{3}\left[\frac{u}{2} + \frac{v}{3} = 2\right] \Rightarrow \frac{u}{6} + \frac{v}{9} = \frac{2}{3}$ ...(3)

$\frac{1}{2}\left[\frac{u}{3} + \frac{v}{2} = \frac{13}{6}\right] \Rightarrow \frac{u}{6} + \frac{v}{4} = \frac{13}{12}$ ...(4)

Subtracting (3) from (4), we get

$\frac{v}{4} - \frac{v}{9} = \frac{13}{12} - \frac{2}{3} \Rightarrow \frac{9v - 4v}{36} = \frac{13-8}{12} \Rightarrow \frac{5}{36}v = \frac{5}{12}$

$\Rightarrow v = \frac{5}{12} \times \frac{36}{5} = 3$

Substituting $v = 3$ in (3), we get

$\frac{u}{6} + \frac{3}{9} = \frac{2}{3} \Rightarrow \frac{u}{6} = \frac{2}{3} - \frac{3}{9} = \frac{6-3}{9} = \frac{3}{9} \Rightarrow u = \frac{3}{9} \times 6 = 2$

Thus, $u = 2$ and $v = 3$

But, $u = \frac{1}{x} \Rightarrow \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$

And, $v = \frac{1}{y} \Rightarrow \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$

$\therefore$ The required solution is $x = \frac{1}{2}, y = \frac{1}{3}$.

(ii) We have $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$ ...(1)

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$ ...(2)

Putting $\frac{1}{\sqrt{x}} = u$ and $\frac{1}{\sqrt{y}} = v$, we get

$2u + 3v - 2 = 0$ ...(3)

$4u - 9v + 1 = 0$ ...(4)

Solving (3) and (4) by cross multiplication method, we have

$$\frac{u}{\begin{matrix}3 & -2\\ -9 & 1\end{matrix}} = \frac{v}{\begin{matrix}-2 & 2\\ 1 & 4\end{matrix}} = \frac{1}{\begin{matrix}2 & 3\\ 4 & -9\end{matrix}}$$

$$\Rightarrow \frac{u}{3-18} = \frac{v}{-8-2} = \frac{1}{-18-12} \Rightarrow \frac{u}{-15} = \frac{v}{-10} = \frac{1}{-30}$$

$$\Rightarrow \quad u = \frac{1}{-30} \times (-15) = \frac{1}{2},\ v = \frac{1}{-30} \times (-10) = \frac{1}{3}$$

But $u = \frac{1}{\sqrt{x}}$ and $v = \frac{1}{\sqrt{y}}$

$$\therefore \quad \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow \frac{1}{x} = \frac{1}{4} \quad \Rightarrow \ x = 4$$

and $\frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow \frac{1}{y} = \frac{1}{9} \quad \Rightarrow y = 9$

$\therefore$ The required solution is $x = 4,\ y = 9$.

(iii) We have $\frac{4}{x} + 3y = 14$ ...(1)

$\frac{3}{x} - 4y = 23$ ...(2)

Let $\frac{1}{x} = u$ in (1) and (2), we get

$4u + 3y - 14 = 0$ ...(3)

$3u - 4y - 23 = 0$ ...(4)

Solving (3) and (4) by cross multiplication method, we have

$$\frac{u}{\begin{matrix} 3 & -14 \\ -4 & -23 \end{matrix}} = \frac{y}{\begin{matrix} -14 & 4 \\ -23 & 3 \end{matrix}} = \frac{1}{\begin{matrix} 4 & 3 \\ 3 & -4 \end{matrix}}$$

$$\Rightarrow \quad \frac{u}{-69-56} = \frac{y}{-42+92} = \frac{1}{-16-9}$$

$$\Rightarrow \quad \frac{u}{-125} = \frac{y}{50} = \frac{1}{-25}$$

$$\Rightarrow u = \frac{1}{-25} \times (-125) = 5$$

And $y = \frac{1}{-25} \times 50 = -2$

Since, $u = \frac{1}{x} \quad \therefore \ \frac{1}{x} = 5 \Rightarrow x = \frac{1}{5}$

$\therefore$ The required solution is $x = \frac{1}{5},\ y = -2$.

(iv) We have, $\frac{5}{x-1} + \frac{1}{y-2} = 2$ ...(1)

$\frac{6}{x-1} - \frac{3}{y-2} = 1$ ...(2)

Let $\frac{1}{x-1} = u$ and $\frac{1}{y-2} = v$

$\therefore$ Equations (1) and (2) can be expressed as

$5u + v - 2 = 0$ ...(3)

$6u - 3v - 1 = 0$ ...(4)

Solving (3) and (4) by cross multiplication method, we have

$$\frac{u}{\begin{matrix} 1 & -2 \\ -3 & -1 \end{matrix}} = \frac{v}{\begin{matrix} -2 & 5 \\ -1 & 6 \end{matrix}} = \frac{1}{\begin{matrix} 5 & 1 \\ 6 & -3 \end{matrix}}$$

$$\Rightarrow \quad \frac{u}{-1-6} = \frac{v}{-12+5} = \frac{1}{-15-6} \Rightarrow \frac{u}{-7} = \frac{v}{-7} = \frac{1}{-21}$$

$$\therefore \quad u = \frac{1}{(-21)} \times (-7) = \frac{1}{3},\ v = \frac{1}{(-21)} \times (-7) = \frac{1}{3}$$

But, $u = \frac{1}{x-1} \quad \Rightarrow \ \frac{1}{x-1} = \frac{1}{3}$

$\Rightarrow \quad 3 = x - 1 \Rightarrow \ x = 4$

And, $v = \frac{1}{y-2} \Rightarrow \frac{1}{y-2} = \frac{1}{3}$

$\Rightarrow \quad 3 = y - 2 \Rightarrow \ y = 5$

Thus, the required solution is $x = 4,\ y = 5$.

(v) We have, $\frac{7x - 2y}{xy} = 5$ ...(1)

$\frac{8x + 7y}{xy} = 15$ ...(2)

From equation (1), $\frac{7x}{xy} - \frac{2y}{xy} = 5$

$\Rightarrow \quad \frac{7}{y} - \frac{2}{x} = 5$ ...(3)

From equation (2), $\frac{8x}{xy} + \frac{7y}{xy} = 15$

$\Rightarrow \quad \frac{8}{y} + \frac{7}{x} = 15$ ...(4)

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

$\therefore$ Equations (3) and (4) can be expressed as

$7v - 2u - 5 = 0$ ...(5)

$8v + 7u - 15 = 0$ ...(6)

Using cross multiplication method to solve (5) and (6), we get

$$\frac{v}{\begin{matrix} -2 & -5 \\ 7 & -15 \end{matrix}} = \frac{u}{\begin{matrix} -5 & 7 \\ -15 & 8 \end{matrix}} = \frac{1}{\begin{matrix} 7 & -2 \\ 8 & 7 \end{matrix}}$$

$$\Rightarrow \quad \frac{v}{30+35} = \frac{u}{-40+105} = \frac{1}{49+16} \Rightarrow \frac{v}{65} = \frac{u}{65} = \frac{1}{65}$$

$$\therefore \quad u = \frac{1}{65} \times 65 = 1,\ v = \frac{1}{65} \times 65 = 1$$

Since, $u = \frac{1}{x} \Rightarrow \ \frac{1}{x} = 1 \ \Rightarrow x = 1$

and $v = \frac{1}{y} \Rightarrow \ \frac{1}{y} = 1 \ \Rightarrow y = 1$

Thus, the required solution is $x = 1,\ y = 1$.

(vi) We have, $6x + 3y = 6xy$ ...(1)

$2x + 4y = 5xy$ ...(2)

From (1), we get

$\frac{6x}{xy} + \frac{3y}{xy} = \frac{6xy}{xy} \Rightarrow \frac{6}{y} + \frac{3}{x} = 6$ ...(3)

From (2), we get

$\frac{2x}{xy} + \frac{4y}{xy} = \frac{5xy}{xy} \Rightarrow \frac{2}{y} + \frac{4}{x} = 5$ ...(4)

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

$\therefore$ (3) and (4) can be expressed as

$6v + 3u = 6$ ...(5)

$2v + 4u = 5$ ...(6)

Multiplying (6) by 3, we get

$6v + 12u = 15$ ...(7)

Subtracting (5) from (7), we get $9u = 9 \Rightarrow u = 1$

Substituting $p = 1$ in (5), we get $6v + 3(1) = 6$

$\Rightarrow 6v = 6 - 3 = 3 \Rightarrow v = \frac{3}{6} = \frac{1}{2}$

Since, $u = \frac{1}{x} \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1$

and $v = \frac{1}{y} \Rightarrow \frac{1}{y} = \frac{1}{2} \Rightarrow y = 2$

Thus, the required solution is $x = 1, y = 2$.

(vii) We have, $\frac{10}{x+y} + \frac{2}{x-y} = 4$ ...(1)

and $\frac{15}{x+y} - \frac{5}{x-y} = -2$ ...(2)

Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$

∴ Equations (1) and (2) can be expressed as

$10u + 2v - 4 = 0$ ...(3)

$15u - 5v + 2 = 0$ ...(4)

Solving (3) and (4) by cross-multiplication method, we get

$$\frac{u}{\begin{matrix}2 & -4\\ -5 & 2\end{matrix}} = \frac{v}{\begin{matrix}-4 & 10\\ 2 & 15\end{matrix}} = \frac{1}{\begin{matrix}10 & 2\\ 15 & -5\end{matrix}}$$

$$\Rightarrow \frac{u}{4-20} = \frac{v}{-60-20} = \frac{1}{-50-30} \Rightarrow \frac{u}{-16} = \frac{v}{-80} = \frac{1}{-80}$$

$\therefore\ u = \frac{1}{(-80)} \times (-16) = \frac{1}{5}$ and $v = \frac{1}{(-80)} \times (-80) = 1$

But, $u = \frac{1}{x+y} = \frac{1}{5} \Rightarrow x + y = 5$ ...(5)

And $v = \frac{1}{x-y} = \frac{1}{1} \Rightarrow x - y = 1$ ...(6)

Adding (5) and (6), we have

$2x = 6 \Rightarrow x = 3$

From (5), $3 + y = 5 \Rightarrow y = 2$

Thus, the required solution is $x = 3, y = 2$.

(viii) We have, $\frac{1}{(3x+y)} + \frac{1}{(3x-y)} = \frac{3}{4}$ ...(1)

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$ ...(2)

Let $\frac{1}{(3x+y)} = u$ and $\frac{1}{(3x-y)} = v$

∴ (1) and (2) can be expressed as

$u + v = \frac{3}{4}$ ...(3)

$\frac{u}{2} - \frac{v}{2} = -\frac{1}{8}$ ...(4)

Multiplying equation (3) by 1/2, we get

$\frac{u}{2} + \frac{v}{2} = \frac{3}{8}$ ...(5)

Adding (4) and (5), we get

$\left(\frac{u}{2} + \frac{v}{2}\right) = \left(\frac{3}{8} - \frac{1}{8}\right) \Rightarrow u = \frac{2}{8} = \frac{1}{4}$

From (3), $\frac{1}{4} + v = \frac{3}{4} \Rightarrow v = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

But, $u = \frac{1}{3x+y} \Rightarrow \frac{1}{3x+y} = \frac{1}{4} \Rightarrow 3x + y = 4$ ...(6)

And $v = \frac{1}{3x-y} \Rightarrow \frac{1}{3x-y} = \frac{1}{2} \Rightarrow 3x - y = 2$ ...(7)

Adding (6) and (7), we get

$6x = 6 \Rightarrow x = 1$

Subtracting (7) from (6), we get

$2y = 2 \Rightarrow y = 1$

Thus, the required solution is $x = 1, y = 1$.

2. (i) Let the speed of rowing in still water = x km/hr and the speed of the water current = y km/hr.

∴ Downstream speed = $(x + y)$ km/hr

Upstream speed = $(x - y)$ km/hr

According to the question,

$2 = \frac{20}{x+y}$ $\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$

$\Rightarrow x + y = 10$ ...(1)

and $2 = \frac{4}{x-y}$

$\Rightarrow x - y = 2$ ...(2)

Adding (1) and (2), we get

$2x = 12 \Rightarrow x = \frac{12}{2} = 6$

From (1), $6 + y = 10$

$\Rightarrow y = 10 - 6 = 4$

Thus, speed of rowing in still water = 6 km/hr, speed of water current = 4 km/hr

(ii) Let the time taken to finish the task by one woman alone = x days and by one man alone = y days.

∴ one woman's 1 day work = $1/x$

one man's 1 day work = $1/y$

Since, [2 women + 5 men] finish the task in 4 days

$\therefore\ 4 \times \left[\frac{2}{x} + \frac{5}{y}\right] = 1$

$\Rightarrow \frac{2}{x} + \frac{5}{y} = \frac{1}{4}$ ...(1)

Again, 3 women + 6 men, finish the task in 3 days

$\therefore\ 3 \times \left[\frac{3}{x} + \frac{6}{y}\right] = 1 \Rightarrow \frac{3}{x} + \frac{6}{y} = \frac{1}{3}$ ...(2)

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

∴ Equations (1) and (2) can be expressed as

$2u + 5v = \frac{1}{4} \Rightarrow 8u + 20v - 1 = 0$

$3u + 6v = \frac{1}{3} \Rightarrow 9u + 18v - 1 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 8, b_1 = 20, c_1 = -1,$
$a_2 = 9, b_2 = 18, c_2 = -1$

Using cross multiplication method, we get

$$\frac{u}{\begin{matrix}20 & -1\\ 18 & -1\end{matrix}} = \frac{v}{\begin{matrix}-1 & 8\\ -1 & 9\end{matrix}} = \frac{1}{\begin{matrix}8 & 20\\ 9 & 18\end{matrix}}$$

$$\Rightarrow \frac{u}{-20+18} = \frac{v}{-9+8} = \frac{1}{144-180}$$

$$\Rightarrow \frac{u}{-2} = \frac{v}{-1} = \frac{1}{-36}$$

$$\therefore\ u = \frac{1}{-36} \times (-2) = \frac{1}{18},\ v = \frac{1}{-36} \times (-1) = \frac{1}{36}$$

Since, $u = \frac{1}{x} \Rightarrow \frac{1}{x} = \frac{1}{18} \Rightarrow x = 18$

and $v = \frac{1}{y} \Rightarrow \frac{1}{y} = \frac{1}{36} \Rightarrow y = 36$

$\therefore$ One man can finish the work in 36 days and one woman can finish the work in 18 days.

(iii) Let the speed of the train = x km/hr and the speed of the bus = y km/hr

Case I : Total journey = 300 km

$\therefore$ Journey travelled by train = 60 km

$\therefore$ Journey travelled by bus = (300 – 60) km = 240 km

$\because$ Total time taken = 4 hours

$$\therefore\ \frac{60}{x} + \frac{240}{y} = 4$$

$$\Rightarrow \frac{1}{x} + \frac{4}{y} = \frac{1}{15} \quad \text{...(1)}$$

Case II : Distance travelled by train = 100 km

Distance travelled by bus = (300 – 100) km = 200 km

Total time taken = 4 hrs 10 mins $= \left(4 + \frac{10}{60}\right)$ hrs $= \frac{25}{6}$ hrs

$$\therefore\ \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$$

$$\Rightarrow \frac{4}{x} + \frac{8}{y} = \frac{1}{6} \quad \text{...(2)}$$

Multiplying (1) by 4, we get

$$\frac{4}{x} + \frac{16}{y} = \frac{4}{15} \quad \text{...(3)}$$

Subtracting (3) from (2), we get

$$\frac{8}{y} - \frac{16}{y} = \frac{1}{6} - \frac{4}{15} \Rightarrow \frac{-8}{y} = \frac{5-8}{30} = \frac{-3}{30}$$

$$\Rightarrow \frac{1}{y} = \frac{-3}{30} \times \frac{1}{(-8)} = \frac{1}{80} \Rightarrow y = 80$$

From (1), $\frac{1}{x} + \frac{4}{80} = \frac{1}{15}$

$$\Rightarrow \frac{1}{x} = \frac{1}{15} - \frac{4}{80} = \frac{1}{15} - \frac{1}{20} \Rightarrow \frac{1}{x} = \frac{4-3}{60} = \frac{1}{60}$$

$\therefore$ $x = 60$

Thus, speed of the train = 60 km/hr and speed of the bus = 80 km/hr.

EXERCISE - 3.7

1. Let the age of Ani = x years

and the age of Biju = y years

Case I : $y > x$

According to 1st condition : $y - x = 3$ ...(1)

Now, [Age of Ani's father] = 2[Age of Ani] = $2x$ years

Also, [Age of Biju's sister] = $\frac{1}{2}$ [Age of Biju] $= \frac{1}{2}y$

According to 2nd condition : $2x - \frac{1}{2}y = 30$

$\Rightarrow$ $4x - y = 60$ ...(2)

Adding (1) and (2), we get

$y - x + 4x - y = 63$

$\Rightarrow$ $3x = 63 \Rightarrow x = \frac{63}{3} = 21$

Substituting the value of x in equation (1),

we get $y - 21 = 3 \Rightarrow y = 3 + 21 = 24$

$\therefore$ Age of Ani = 21 years

Age of Biju = 24 years

Case II : $x > y$

$\therefore$ $x - y = 3$ ...(1)

According to the condition : $2x - \frac{1}{2}y = 30$

$\Rightarrow$ $4x - y = 60$ ...(2)

Subtracting, (1) from (2), we get

$4x - y - x + y = 60 - 3$

$\Rightarrow$ $3x = 57 \Rightarrow x = \frac{57}{3} = 19$

Substituting the value of x in equation (1), we get

$19 - y = 3 \Rightarrow y = 16$

$\therefore$ Age of Ani = 19 years

Age of Biju = 16 years

2. Let the capital of 1st friend = ₹ x,

and the capital of 2nd friend = ₹ y

According to the condition,

$x + 100 = 2(y - 100)$

$\Rightarrow$ $x + 100 - 2y + 200 = 0 \Rightarrow x - 2y + 300 = 0$ ...(1)

Also, $6(x - 10) = y + 10 \Rightarrow 6x - y - 70 = 0$ ...(2)

From (1), $x = -300 + 2y$ ...(3)

Substituting the value of x in equation (2), we get

$6[-300 + 2y] - y - 70 = 0$

$\Rightarrow$ $-1870 + 11y = 0 \Rightarrow y = \frac{1870}{11} = 170$

Now, Substituting the value of y in equation (3), we get,

$x = -300 + 2y$

$= -300 + 2(170) = -300 + 340 = 40$

Thus, 1st friend has ₹ 40 and the 2nd friend has ₹ 170.

3. Let the actual speed of the train = x km/hr

and the actual time taken = y hours

$\because$ Distance = speed × time

According to 1st condition : $(x + 10) \times (y - 2) = xy$

$\Rightarrow$ $xy - 2x + 10y - 20 = xy$

$\Rightarrow \quad 2x - 10y + 20 = 0$ ...(1)

According to 2nd condition : $(x - 10) \times (y + 3) = xy$

$\Rightarrow \quad xy + 3x - 10y - 30 = xy$

$\Rightarrow \quad 3x - 10y - 30 = 0$ ...(2)

Using cross multiplication for solving (1) and (2), we get

$$\frac{x}{\begin{matrix}-10 & 20\\ -10 & -30\end{matrix}} = \frac{y}{\begin{matrix}20 & 2\\ -30 & 3\end{matrix}} = \frac{1}{\begin{matrix}2 & -10\\ 3 & -10\end{matrix}}$$

$$\Rightarrow \frac{x}{300+200} = \frac{y}{60+60} = \frac{1}{-20+30} \Rightarrow \frac{x}{500} = \frac{y}{120} = \frac{1}{10}$$

$$\Rightarrow x = \frac{1}{10} \times 500 = 50, \; y = \frac{1}{10} \times 120 = 12$$

Thus, the distance covered by the train

= 50 × 12 km = 600 km

4. Let the number of students = x

and the number of rows = y

$\therefore$ Number of students in each row

$$= \frac{\text{Number of students}}{\text{Number of rows}} = \frac{x}{y}$$

According to 1st condition : $\left(\frac{x}{y} + 3\right) \times (y - 1) = x$

[$\because$ Number of students in a row × Number of rows = Number of students]

$$\Rightarrow x - \frac{x}{y} + 3y - 3 = x$$

$$\Rightarrow \frac{x}{y} - 3y + 3 = 0 \quad \text{...(1)}$$

Also, according to 2nd condition :

$$\left(\frac{x}{y} - 3\right) \times (y + 2) = x$$

$$\Rightarrow x + \frac{2x}{y} - 3y - 6 = x$$

$$\Rightarrow \frac{2x}{y} - 3y - 6 = 0 \quad \text{...(2)}$$

Let $\frac{x}{y} = p$

$\therefore$ Equation (1) and (2) can be expressed as :

$p - 3y + 3 = 0$ ...(3)

and $2p - 3y - 6 = 0$ ...(4)

Subtracting (3) from (4), we get

$2p - 3y - 6 - p + 3y - 3 = 0$

$\Rightarrow \quad p - 9 = 0 \Rightarrow p = 9$

Substituting value of p in equation (3), we get,

$9 - 3y + 3 = 0 \Rightarrow -3y = -12$

$$\Rightarrow y = \frac{-12}{-3} = 4$$

We have, $\frac{x}{y} = 9$ [$\because p = 9$]

$$\therefore \frac{x}{4} = 9 \Rightarrow x = 4 \times 9 = 36$$

Thus, number of students in the class, $x = 36$

5. $\because$ Sum of angles of a triangle = 180°

$\therefore \quad \angle A + \angle B + \angle C = 180°$ ...(1)

$\because \quad \angle C = 3\angle B = 2(\angle A + \angle B)$ ...(2)

From (1) and (2), we have

$\angle A + \angle B + 2(\angle A + \angle B) = 180°$

$\Rightarrow \quad \angle A + \angle B + 2\angle A + 2\angle B = 180°$

$\Rightarrow \quad \angle A + \angle B = 60°$ ...(3)

Also, $\angle A + \angle B + 3\angle B = 180°$

$\Rightarrow \quad \angle A + 4\angle B = 180°$ ...(4)

Subtracting (3) from (4), we get

$\angle A + 4\angle B - \angle A - \angle B = 180° - 60°$

$$\Rightarrow 3\angle B = 120° \Rightarrow \angle B = \frac{120°}{3} = 40°$$

Substituting $\angle B = 40°$ in (4) we get,

$\angle A + 4(40°) = 180°$

$\Rightarrow \quad \angle A = 180° - 160° = 20°$

$\therefore \quad \angle C = 3\angle B = 3 \times 40° = 120°$

Thus, $\angle A = 20°$, $\angle B = 40°$ and $\angle C = 120°$.

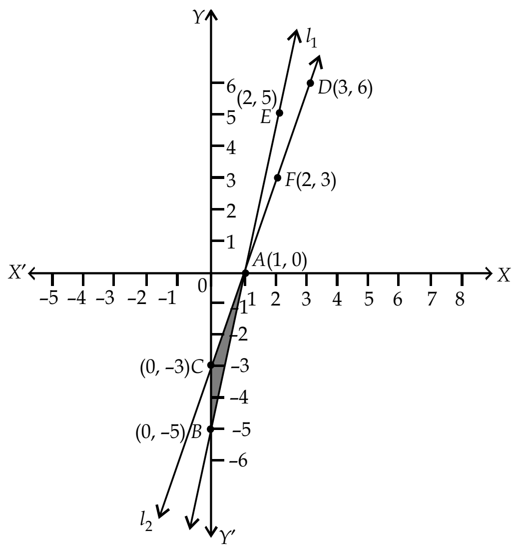
6. To draw the graph of $5x - y = 5$, we get

x	1	2	0
y	0	5	–5

and for equation $3x - y = 3$, we get

x	2	3	0
y	3	6	–3

Plotting the points (1, 0), (2, 5) and (0, –5), we get a straight line l_1. Plotting the points (2, 3), (3, 6) and (0, –3), we get a straight line l_2.



From the figure, obviously, the vertices of the triangle formed are $A(1, 0)$, $B(0, -5)$ and $C(0, -3)$.

7. (i) We have, $px + qy = p - q$ ...(1)

$qx - py = p + q$ ...(2)

Multiplying (1) by p and (2) by q, we get

$p^2x + qpy = p^2 - pq$ ...(3)

$q^2x - pqy = q^2 + pq$ ...(4)

Adding (3) and (4), we get

$p^2x + q^2x = p^2 + q^2$

$\Rightarrow \quad (p^2 + q^2)x = p^2 + q^2$

$\Rightarrow \quad x = \frac{p^2 + q^2}{p^2 + q^2} = 1$

Substituting $x = 1$ in (1) we get,

$p(1) + qy = p - q \Rightarrow p + qy = p - q$

$\Rightarrow \quad y = -1$

Thus, the required solution is $x = 1, y = -1$

(ii) We have, $ax + by = c \Rightarrow ax + by - c = 0$ ...(1)

$bx + ay = (1 + c) \Rightarrow bx + ay - (1 + c) = 0$ ...(2)

By cross multiplication, we have

$$\frac{x}{\begin{matrix} b & -c \\ a & -(1+c) \end{matrix}} = \frac{y}{\begin{matrix} -c & a \\ -(1+c) & b \end{matrix}} = \frac{1}{\begin{matrix} a & b \\ b & a \end{matrix}}$$

$$\Rightarrow \quad \frac{x}{-b - bc + ac} = \frac{y}{-bc + a + ac} = \frac{1}{a^2 - b^2}$$

$$\therefore \quad x = \frac{-b - bc + ac}{a^2 - b^2}, y = \frac{-bc + a + ac}{a^2 - b^2}$$

$$\Rightarrow \quad x = \frac{c(a - b) - b}{a^2 - b^2}, y = \frac{c(a - b) + a}{a^2 - b^2}$$

(iii) We have, $\frac{x}{a} - \frac{y}{b} = 0$ ...(1)

$ax + by = a^2 + b^2$ ...(2)

From (1), we have

$\frac{x}{a} = \frac{y}{b} \Rightarrow y = \left(\frac{x}{a} \times b\right)$ ...(3)

Substituting $y = \left(\frac{b}{a}x\right)$ in (2), we have

$$ax + b\left(\frac{b}{a}x\right) = a^2 + b^2$$

$$\Rightarrow \quad x\left[\frac{a^2 + b^2}{a}\right] = a^2 + b^2 \Rightarrow x = \frac{a^2 + b^2}{a^2 + b^2} \times a \Rightarrow x = a$$

Substituting $x = a$ in (3), we get

$y = \frac{a}{a} \times b \Rightarrow y = b$

Thus, the required solution is $x = a, y = b$.

(iv) We have,

$(a - b)x + (a + b)y = a^2 - 2ab - b^2$ ...(1)

$(a + b)(x + y) = a^2 + b^2$ ...(2)

From (2),

$(a + b)x + (a + b)y = a^2 + b^2$ ...(3)

Subtracting (3) from (1), we get

$x[(a - b) - (a + b)] = a^2 - 2ab - b^2 - a^2 - b^2$

$\Rightarrow \quad x[a - b - a - b] = -2ab - 2b^2$

$\Rightarrow \quad x(-2b) = -2b(a + b)$

$\Rightarrow \quad x = \frac{-2b(a + b)}{-2b} \Rightarrow \quad x = a + b$

Substituting $x = (a + b)$ in (1), we get

$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$

$\Rightarrow \quad (a + b)y = a^2 - 2ab - b^2 - a^2 + b^2$

$\Rightarrow \quad (a + b)y = -2ab \Rightarrow y = \frac{-2ab}{(a + b)}$

Thus, the required solution is

$x = a + b, y = -\frac{2ab}{a + b}$

(v) We have, $152x - 378y = -74$ ...(1)

$-378x + 152y = -604$ ...(2)

Adding (1) and (2), we have

$-226x - 226y = -678$

$\Rightarrow \quad x + y = 3$ ...(3)

Subtracting (1) from (2), we get

$-530x + 530y = -530$

$\Rightarrow \quad -x + y = -1$

$\Rightarrow \quad x - y = 1$ ...(4)

Adding (3) and (4), we get

$2x = 4 \Rightarrow x = 2$

Subtracting (3) from (4), we get

$-2y = -2 \Rightarrow y = \frac{-2}{-2} = 1$

Thus, the required solution is $x = 2$ and $y = 1$

8. $\because$ *ABCD* is a cyclic quadrilateral.

$\therefore \quad \angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$\Rightarrow \quad [4y + 20°] + [-4x] = 180°$

$\Rightarrow \quad 4y - 4x + 20° - 180° = 0 \Rightarrow 4y - 4x - 160° = 0$

$\Rightarrow \quad y - x - 40° = 0$ ...(1)

And $[3y - 5°] + [-7x + 5°] = 180°$

$\Rightarrow \quad 3y - 5° - 7x + 5° - 180° = 0$

$\Rightarrow \quad 3y - 7x - 180° = 0$ ...(2)

Multiplying (1) by 7, we get

$7y - 7x - 280° = 0$ ...(3)

Subtracting (3) from (2), we get

$3y - 7x - 180° - 7y + 7x + 280° = 0$

$\Rightarrow \quad -4y + 100° = 0 \Rightarrow y = \frac{-100°}{-4} = 25°$

Now, substituting $y = 25°$ in (1), we get

$-x = 40° - 25° = 15°$

$\Rightarrow \quad x = -15°$

$\therefore \quad \angle A = 4y + 20° = 4(25°) + 20° = 100° + 20° = 120°$

$\angle B = 3y - 5° = 3(25°) - 5° = 75° - 5° = 70°$

$\angle C = -4x = -4(-15°) = 60°,$

$\angle D = -7x + 5° = -7(-15°) + 5° = 105° + 5° = 110°$

Thus, $\angle A = 120°, \angle B = 70°, \angle C = 60°, \angle D = 110°$.