

# Quadratic Equations

## CHAPTER 4



### SOLUTIONS

#### EXERCISE - 4.1

1. (i) We have,  $(x+1)^2 = 2(x-3)$   
 $\Rightarrow x^2 + 2x + 1 = 2x - 6$   
 $\Rightarrow x^2 + 2x + 1 - 2x + 6 = 0 \Rightarrow x^2 + 7 = 0$   
 Since,  $x^2 + 7$  is a quadratic polynomial.  
 $\therefore (x+1)^2 = 2(x-3)$  is a quadratic equation.
- (ii) We have,  $x^2 - 2x = (-2)(3-x)$   
 $\Rightarrow x^2 - 2x = -6 + 2x \Rightarrow x^2 - 2x - 2x + 6 = 0$   
 $\Rightarrow x^2 - 4x + 6 = 0$   
 Since,  $x^2 - 4x + 6$  is a quadratic polynomial.  
 $\therefore x^2 - 2x = (-2)(3-x)$  is a quadratic equation.
- (iii) We have,  $(x-2)(x+1) = (x-1)(x+3)$   
 $\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$   
 $\Rightarrow x^2 - x - 2 - x^2 - 2x + 3 = 0 \Rightarrow -3x + 1 = 0$   
 Since,  $-3x + 1$  is a linear polynomial.  
 $\therefore (x-2)(x+1) = (x-1)(x+3)$  is not a quadratic equation.
- (iv) We have,  $(x-3)(2x+1) = x(x+5)$   
 $\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$   
 $\Rightarrow 2x^2 - 5x - 3 - x^2 - 5x = 0 \Rightarrow x^2 - 10x - 3 = 0$   
 Since,  $x^2 - 10x - 3$  is a quadratic polynomial.  
 $\therefore (x-3)(2x+1) = x(x+5)$  is a quadratic equation.
- (v) We have,  $(2x-1)(x-3) = (x+5)(x-1)$   
 $\Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$   
 $\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$   
 $\Rightarrow 2x^2 - 7x + 3 - x^2 - 4x + 5 = 0 \Rightarrow x^2 - 11x + 8 = 0$   
 Since,  $x^2 - 11x + 8$  is a quadratic polynomial.  
 $\therefore (2x-1)(x-3) = (x+5)(x-1)$  is a quadratic equation.
- (vi) We have,  $x^2 + 3x + 1 = (x-2)^2$   
 $\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$   
 $\Rightarrow x^2 + 3x + 1 - x^2 - 4x + 4 = 0 \Rightarrow 7x - 3 = 0$   
 Since,  $7x - 3$  is a linear polynomial.  
 $\therefore x^2 + 3x + 1 = (x-2)^2$  is not a quadratic equation.
- (vii) We have,  $(x+2)^3 = 2x(x^2-1)$   
 $\Rightarrow x^3 + 3x^2(2) + 3x(2)^2 + (2)^3 = 2x^3 - 2x$   
 $\Rightarrow x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$   
 $\Rightarrow x^3 + 6x^2 + 12x + 8 - 2x^3 + 2x = 0$   
 $\Rightarrow -x^3 + 6x^2 + 14x + 8 = 0$   
 Since,  $-x^3 + 6x^2 + 14x + 8$  is a cubic polynomial.  
 $\therefore (x+2)^3 = 2x(x^2-1)$  is not a quadratic equation.
- (viii) We have,  $x^3 - 4x^2 - x + 1 = (x-2)^3$   
 $\Rightarrow x^3 - 4x^2 - x + 1 = x^3 + 3x^2(-2) + 3x(-2)^2 + (-2)^3$   
 $\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$   
 $\Rightarrow x^3 - 4x^2 - x + 1 - x^3 + 6x^2 - 12x + 8 = 0$   
 $\Rightarrow 2x^2 - 13x + 9 = 0$   
 Since,  $2x^2 - 13x + 9$  is a quadratic polynomial.  
 $\therefore x^3 - 4x^2 - x + 1 = (x-2)^3$  is a quadratic equation.

2. (i) Let the breadth =  $x$  metres  
 $\therefore$  Length =  $2(\text{Breadth}) + 1$   
 $\therefore$  Length =  $(2x + 1)$  metres  
 Since, length  $\times$  breadth = Area  
 $\therefore (2x + 1) \times x = 528 \Rightarrow 2x^2 + x = 528$   
 $\Rightarrow 2x^2 + x - 528 = 0$   
 Thus, the required quadratic equation is  $2x^2 + x - 528 = 0$ .
- (ii) Let the two consecutive positive integers be  $x$  and  $(x+1)$ .  
 $\therefore$  Product of two consecutive positive integers = 306  
 $\therefore x(x+1) = 306 \Rightarrow x^2 + x = 306 \Rightarrow x^2 + x - 306 = 0$   
 Thus, the required quadratic equation is  $x^2 + x - 306 = 0$ .
- (iii) Let the present age of Rohan be  $x$  years.  
 $\therefore$  His mother's age =  $(x + 26)$  years  
 After 3 years, Rohan's age =  $(x + 3)$  years  
 After 3 years, his mother's age =  $[(x + 26) + 3]$  years  
 $= (x + 29)$  years  
 According to the condition,  $(x + 3) \times (x + 29) = 360$   
 $\Rightarrow x^2 + 29x + 3x + 87 = 360$   
 $\Rightarrow x^2 + 29x + 3x + 87 - 360 = 0 \Rightarrow x^2 + 32x - 273 = 0$   
 Thus, the required quadratic equation is  
 $x^2 + 32x - 273 = 0$ .
- (iv) Let the speed of the train =  $u$  km/hr  
 Distance covered = 480 km  
 Time taken =  $\frac{\text{Distance}}{\text{Speed}} = \frac{480}{u}$  hours  
 In other case, speed =  $(u - 8)$  km/hour  
 $\therefore$  Time taken =  $\frac{\text{Distance}}{\text{Speed}} = \frac{480}{(u - 8)}$  hours  
 According to the condition,  $\frac{480}{u - 8} - \frac{480}{u} = 3$   
 $\Rightarrow \frac{480u - 480(u - 8)}{(u - 8)u} = 3$   
 $\Rightarrow \frac{480u - 480u + 3840}{u^2 - 8u} = 3$   
 $\Rightarrow \frac{3840}{u^2 - 8u} = 3$   
 $\Rightarrow 3840 = 3(u^2 - 8u)$   
 $\Rightarrow 3840 = 3u^2 - 24u$   
 $\Rightarrow 3u^2 - 24u - 3840 = 0 \Rightarrow u^2 - 8u - 1280 = 0$   
 Thus, the required quadratic equation is  
 $u^2 - 8u - 1280 = 0$ .

#### EXERCISE - 4.2

1. (i) We have,  $x^2 - 3x - 10 = 0$   
 $\Rightarrow x^2 - 5x + 2x - 10 = 0 \Rightarrow x(x - 5) + 2(x - 5) = 0$   
 $\Rightarrow (x - 5)(x + 2) = 0$   
 $\Rightarrow x - 5 = 0$  or  $x + 2 = 0 \Rightarrow x = 5$  or  $x = -2$   
 Thus, the required roots are 5 and -2.
- (ii) We have,  $2x^2 + x - 6 = 0 \Rightarrow 2x^2 + 4x - 3x - 6 = 0$   
 $\Rightarrow 2x(x + 2) - 3(x + 2) = 0 \Rightarrow (x + 2)(2x - 3) = 0$   
 $\Rightarrow x + 2 = 0$  or  $2x - 3 = 0 \Rightarrow x = -2$  or  $x = 3/2$   
 Thus, the required roots are -2 and 3/2.
- (iii) We have,  $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$$\begin{aligned} \Rightarrow \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} &= 0 \\ \Rightarrow \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) &= 0 \\ \Rightarrow (x + \sqrt{2})(\sqrt{2}x + 5) &= 0 \\ \Rightarrow x + \sqrt{2} &= 0 \text{ or } \sqrt{2}x + 5 = 0 \\ \Rightarrow x = -\sqrt{2} \text{ or } x = \frac{-5}{\sqrt{2}} &= \frac{-5\sqrt{2}}{2} \end{aligned}$$

Thus, the required roots are  $-\sqrt{2}$  and  $\frac{-5\sqrt{2}}{2}$ .

(iv) We have,  $2x^2 - x + \frac{1}{8} = 0$

$$\begin{aligned} \Rightarrow 16x^2 - 8x + 1 &= 0 \Rightarrow 16x^2 - 4x - 4x + 1 = 0 \\ \Rightarrow 4x(4x - 1) - 1(4x - 1) &= 0 \Rightarrow (4x - 1)(4x - 1) = 0 \\ \Rightarrow 4x - 1 &= 0 \Rightarrow x = 1/4 \end{aligned}$$

Thus, the required roots are  $1/4$  and  $1/4$ .

(v) We have,  $100x^2 - 20x + 1 = 0$

$$\begin{aligned} \Rightarrow 100x^2 - 10x - 10x + 1 &= 0 \\ \Rightarrow 10x(10x - 1) - 1(10x - 1) &= 0 \Rightarrow (10x - 1)(10x - 1) = 0 \\ \Rightarrow (10x - 1) &= 0 \Rightarrow x = 1/10 \end{aligned}$$

Thus, the required roots are  $\frac{1}{10}$  and  $\frac{1}{10}$ .

2. (i) Let John had  $x$  marbles and Jivanti had  $(45 - x)$  marbles.

According to question,

$$\begin{aligned} (x - 5) \times (45 - x - 5) &= 124 \\ \Rightarrow (x - 5) \times (40 - x) &= 124 \Rightarrow x^2 - 45x + 324 = 0 \\ \Rightarrow x^2 - 9x - 36x + 324 &= 0 \Rightarrow x(x - 9) - 36(x - 9) = 0 \\ \Rightarrow (x - 9)(x - 36) &= 0 \\ \Rightarrow x - 9 &= 0 \text{ or } x - 36 = 0 \Rightarrow x = 9 \text{ or } x = 36 \end{aligned}$$

$\therefore$  If John had 9 marbles, then Jivanti had  $45 - 9 = 36$  marbles.

If John had 36 marbles, then Jivanti had  $45 - 36 = 9$  marbles.

(ii) Let the number of toys produced in a day be  $x$ .

Then, cost of 1 toy =  $\frac{750}{x}$

According to question,  $\frac{750}{x} = 55 - x$

$$\begin{aligned} \Rightarrow 750 &= 55x - x^2 \Rightarrow x^2 - 55x + 750 = 0 \\ \Rightarrow x^2 - 30x - 25x + 750 &= 0 \\ \Rightarrow x(x - 30) - 25(x - 30) &= 0 \Rightarrow (x - 30)(x - 25) = 0 \\ \Rightarrow x - 30 &= 0 \text{ or } x - 25 = 0 \Rightarrow x = 30 \text{ or } x = 25 \end{aligned}$$

Hence, number of toys produced on that day is either 30 or 25.

3. Let one of the numbers be  $x$ .

$\therefore$  Other number =  $27 - x$

According to the condition,

$$\begin{aligned} x(27 - x) &= 182 \Rightarrow 27x - x^2 = 182 \\ \Rightarrow x^2 - 27x + 182 &= 0 \Rightarrow x^2 - 13x - 14x + 182 = 0 \\ \Rightarrow x(x - 13) - 14(x - 13) &= 0 \Rightarrow (x - 13)(x - 14) = 0 \\ \Rightarrow x - 13 &= 0 \text{ or } x - 14 = 0 \Rightarrow x = 13 \text{ or } x = 14 \end{aligned}$$

Thus, the required numbers are 13 and 14.

4. Let the two consecutive positive integers be  $x$  and  $(x + 1)$ .

Since, the sum of the squares of the numbers is 365.

$$\begin{aligned} \therefore x^2 + (x + 1)^2 &= 365 \Rightarrow x^2 + x^2 + 2x + 1 = 365 \\ \Rightarrow 2x^2 + 2x - 364 &= 0 \Rightarrow x^2 + x - 182 = 0 \\ \Rightarrow x^2 + 14x - 13x - 182 &= 0 \\ \Rightarrow x(x + 14) - 13(x + 14) &= 0 \Rightarrow (x + 14)(x - 13) = 0 \\ \Rightarrow x + 14 &= 0 \text{ or } x - 13 = 0 \Rightarrow x = -14 \text{ or } x = 13 \end{aligned}$$

Since,  $x$  has to be a positive integer, so  $x = -14$  is rejected.

$$\therefore x = 13 \Rightarrow x + 1 = 13 + 1 = 14$$

Thus, the required consecutive positive integers are 13 and 14.

5. Let the base of the given right triangle be  $x$  cm.

$\therefore$  Its altitude =  $(x - 7)$  cm

$$\therefore \text{Hypotenuse} = \sqrt{(\text{Base})^2 + (\text{Altitude})^2}$$

[By Pythagoras theorem]

$$\therefore 13 = \sqrt{x^2 + (x - 7)^2}$$

On squaring both sides, we get,  $169 = x^2 + (x - 7)^2$

$$\begin{aligned} \Rightarrow 169 &= x^2 + x^2 - 14x + 49 \Rightarrow 2x^2 - 14x + 49 - 169 = 0 \\ \Rightarrow 2x^2 - 14x - 120 &= 0 \Rightarrow x^2 - 7x - 60 = 0 \\ \Rightarrow x^2 - 12x + 5x - 60 &= 0 \Rightarrow x(x - 12) + 5(x - 12) = 0 \\ \Rightarrow (x - 12)(x + 5) &= 0 \\ \Rightarrow x - 12 &= 0 \text{ or } x + 5 = 0 \Rightarrow x = 12 \text{ or } x = -5 \end{aligned}$$

But the sides of a triangle can never be negative, so,  $x = -5$  is rejected.

$$\therefore x = 12$$

$\therefore$  Length of base = 12 cm

$\Rightarrow$  Length of altitude =  $(12 - 7)$  cm = 5 cm

Thus, the required base is 12 cm and altitude is 5 cm.

6. Let the number of articles produced in a day =  $x$

$\therefore$  Cost of production of each article = ₹  $(2x + 3)$

Total cost = ₹ 90

$$\begin{aligned} \therefore x \times (2x + 3) &= 90 \Rightarrow 2x^2 + 3x = 90 \\ \Rightarrow 2x^2 + 3x - 90 &= 0 \Rightarrow 2x^2 - 12x + 15x - 90 = 0 \\ \Rightarrow 2x(x - 6) + 15(x - 6) &= 0 \Rightarrow (x - 6)(2x + 15) = 0 \\ \Rightarrow x - 6 &= 0 \text{ or } 2x + 15 = 0 \Rightarrow x = 6 \text{ or } x = \frac{-15}{2} \end{aligned}$$

But the number of articles produced can never be negative,

$$\text{so, } x = \frac{-15}{2} \text{ is rejected.}$$

$$\therefore x = 6$$

$\therefore$  Cost of production of each article = ₹  $(2 \times 6 + 3) = ₹ 15$

Thus, the required number of articles produced is 6 and the cost of each article is ₹ 15.

### EXERCISE - 4.3

1. (i) We have,  $2x^2 - 7x + 3 = 0$

Dividing both sides by 2, we get  $x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

$$\begin{aligned} \Rightarrow \left\{ x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 \right\} - \left(\frac{7}{4}\right)^2 + \frac{3}{2} &= 0 \\ \Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} &= 0 \Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{24}{16} = 0 \\ \Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{25}{16} &= 0 \Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16} = \left(\frac{5}{4}\right)^2 \end{aligned}$$

$$\Rightarrow x - \frac{7}{4} = \pm \frac{5}{4}$$

$$\text{Case I : When } x - \frac{7}{4} = \frac{5}{4} \Rightarrow x = \frac{5}{4} + \frac{7}{4} \Rightarrow x = \frac{12}{4} = 3$$

$$\text{Case II : When } x - \frac{7}{4} = -\frac{5}{4} \Rightarrow x = -\frac{5}{4} + \frac{7}{4} \Rightarrow x = \frac{2}{4} = \frac{1}{2}$$

Thus, the required roots are 3 and  $1/2$ .

(ii) We have,  $2x^2 + x - 4 = 0$

Dividing both sides by 2, we get  $x^2 + \frac{x}{2} - 2 = 0$

$$\Rightarrow \left\{ x^2 + \frac{x}{2} + \left( \frac{1}{4} \right)^2 \right\} - \left( \frac{1}{4} \right)^2 - 2 = 0$$

$$\Rightarrow \left( x + \frac{1}{4} \right)^2 - \frac{1}{16} - 2 = 0 \Rightarrow \left( x + \frac{1}{4} \right)^2 - \frac{33}{16} = 0$$

$$\Rightarrow \left( x + \frac{1}{4} \right)^2 = \frac{33}{16} \Rightarrow \left( x + \frac{1}{4} \right)^2 = \left( \frac{\sqrt{33}}{4} \right)^2$$

$$\Rightarrow x + \frac{1}{4} = \pm \frac{\sqrt{33}}{4}$$

$$\text{Case I : When } x + \frac{1}{4} = \frac{\sqrt{33}}{4}$$

$$\Rightarrow x = \frac{\sqrt{33}}{4} - \frac{1}{4} \Rightarrow x = \frac{\sqrt{33} - 1}{4}$$

$$\text{Case II : When } x + \frac{1}{4} = -\frac{\sqrt{33}}{4}$$

$$\Rightarrow x = -\frac{\sqrt{33}}{4} - \frac{1}{4} \Rightarrow x = \frac{-\sqrt{33} - 1}{4}$$

Thus, the required roots are  $\frac{\sqrt{33} - 1}{4}$  and  $\frac{-\sqrt{33} - 1}{4}$ .

(iii) We have,  $4x^2 + 4\sqrt{3}x + 3 = 0$

Dividing both sides by 4, we get  $x^2 + \sqrt{3}x + \frac{3}{4} = 0$

$$\Rightarrow \left\{ x^2 + \sqrt{3}x + \left( \frac{\sqrt{3}}{2} \right)^2 \right\} - \left( \frac{\sqrt{3}}{2} \right)^2 + \frac{3}{4} = 0$$

$$\Rightarrow \left( x + \frac{\sqrt{3}}{2} \right)^2 - \frac{3}{4} + \frac{3}{4} = 0 \Rightarrow \left( x + \frac{\sqrt{3}}{2} \right)^2 = 0$$

$$\Rightarrow \left( x + \frac{\sqrt{3}}{2} \right) \left( x + \frac{\sqrt{3}}{2} \right) = 0 \Rightarrow x = -\frac{\sqrt{3}}{2} \text{ and } x = -\frac{\sqrt{3}}{2}$$

Thus, the required roots are  $-\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$ .

(iv) We have,  $2x^2 + x + 4 = 0$

Dividing both sides by 2, we get  $x^2 + \frac{x}{2} + 2 = 0$

$$\Rightarrow \left\{ x^2 + \frac{x}{2} + \left( \frac{1}{4} \right)^2 \right\} - \left( \frac{1}{4} \right)^2 + 2 = 0$$

$$\Rightarrow \left( x + \frac{1}{4} \right)^2 - \frac{1}{16} + 2 = 0$$

$$\Rightarrow \left( x + \frac{1}{4} \right)^2 + \frac{31}{16} = 0 \Rightarrow \left( x + \frac{1}{4} \right)^2 = -\frac{31}{16}$$

Since, the square of a number cannot be negative.

$\therefore \left( x + \frac{1}{4} \right)$  cannot give a real value.

$\therefore$  There is no real value of  $x$  satisfying the given equation.

2. (i) Comparing the given equation with  $ax^2 + bx + c = 0$ , we get  $a = 2$ ,  $b = -7$  and  $c = 3$ .

$$\therefore b^2 - 4ac = (-7)^2 - 4(2)(3) = 49 - 24 = 25 > 0$$

Since  $b^2 - 4ac > 0$ , therefore the given equation has real roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-7) \pm \sqrt{25}}{2(2)} = \frac{7 \pm 5}{4}$$

Taking positive sign,  $x = \frac{7+5}{4} = \frac{12}{4} = 3$

Taking negative sign,  $x = \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}$

Thus, the roots of the given equation are 3 and  $1/2$ .

(ii) Comparing the given equation with  $ax^2 + bx + c = 0$ , we get  $a = 2$ ,  $b = 1$  and  $c = -4$ .

$$\therefore b^2 - 4ac = (1)^2 - 4(2)(-4) = 1 + 32 = 33 > 0$$

Since  $b^2 - 4ac > 0$ , therefore the given equation has real roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-1 \pm \sqrt{33}}{2(2)} = \frac{-1 \pm \sqrt{33}}{4}$$

Taking positive sign,  $x = \frac{-1 + \sqrt{33}}{4}$

Taking negative sign,  $x = \frac{-1 - \sqrt{33}}{4}$

Thus, the roots of the given equation are

$$\frac{-1 + \sqrt{33}}{4} \text{ and } \frac{-1 - \sqrt{33}}{4}$$

(iii) Comparing the given equation with  $ax^2 + bx + c = 0$ , we get  $a = 4$ ,  $b = 4\sqrt{3}$  and  $c = 3$ .

$$\therefore b^2 - 4ac = (4\sqrt{3})^2 - 4(4)(3) = (16 \times 3) - 48 = 48 - 48 = 0$$

Since  $b^2 - 4ac = 0$ , therefore the given equation has real

roots, which are given by  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{0}}{2(4)} = \frac{-4\sqrt{3}}{8} = \frac{-\sqrt{3}}{2}$$

$$\Rightarrow x = \frac{-\sqrt{3}}{2} \text{ and } x = \frac{-\sqrt{3}}{2}$$

Thus, the roots of the given equation are  $\frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$ .

(iv) Comparing the given equation with  $ax^2 + bx + c = 0$ , we get  $a = 2$ ,  $b = 1$  and  $c = 4$ .

$$\therefore b^2 - 4ac = (1)^2 - 4(2)(4) = 1 - 32 = -31 < 0$$

Since  $b^2 - 4ac < 0$ , therefore the given equation does not have real roots.

3. (i) We have,  $x - \frac{1}{x} = 3$

$$\Rightarrow x^2 - 1 = 3x \Rightarrow x^2 - 3x - 1 = 0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ , we get  $a = 1$ ,  $b = -3$  and  $c = -1$ .

$$\therefore b^2 - 4ac = (-3)^2 - 4(1)(-1) = 9 + 4 = 13 > 0$$

Since  $b^2 - 4ac > 0$ , therefore equation (1) has real roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-3) \pm \sqrt{13}}{2(1)} = \frac{3 \pm \sqrt{13}}{2}$$

Taking positive sign,  $x = \frac{3 + \sqrt{13}}{2}$

Taking negative sign,  $x = \frac{3 - \sqrt{13}}{2}$

Thus, the required roots of the given equation are  $\frac{3 + \sqrt{13}}{2}$  and  $\frac{3 - \sqrt{13}}{2}$ .

(ii) We have,  $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

$$\Rightarrow (x-7) - (x+4) = \frac{11}{30}(x+4)(x-7)$$

$$\Rightarrow x-7-x-4 = \frac{11}{30}(x^2-3x-28)$$

$$\Rightarrow -11 \times 30 = 11(x^2-3x-28)$$

$$\Rightarrow -30 = x^2-3x-28 \Rightarrow x^2-3x-28+30=0$$

$$\Rightarrow x^2-3x+2=0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ , we get  $a = 1, b = -3$  and  $c = 2$ .

$$\therefore b^2 - 4ac = (-3)^2 - 4(1)(2) = 9 - 8 = 1 > 0$$

Since  $b^2 - 4ac > 0$ , therefore equation (1) has real roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-3) \pm \sqrt{1}}{2(1)} = \frac{3 \pm 1}{2}$$

Taking positive sign,  $x = \frac{3+1}{2} = \frac{4}{2} = 2$

Taking negative sign,  $x = \frac{3-1}{2} = 1$

Thus, the required roots of the given equation are 2 and 1.

4. Let the present age of Rehman be  $x$  years.

3 years ago, Rehman's age =  $(x-3)$  years

5 years later, Rehman's age =  $(x+5)$  years

According to the condition,  $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$$\Rightarrow \frac{(x+5) + (x-3)}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow \frac{(x+5+x-3)}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow 3(x+5+x-3) = (x-3)(x+5)$$

$$\Rightarrow 3(2x+2) = x^2+2x-15 \Rightarrow 6x+6 = x^2+2x-15$$

$$\Rightarrow x^2+2x-6x-15-6=0 \Rightarrow x^2-4x-21=0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ , we get  $a = 1, b = -4$  and  $c = -21$ .

$$\therefore b^2 - 4ac = (-4)^2 - 4(1)(-21) = 16 + 84 = 100 > 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-4) \pm \sqrt{100}}{2(1)} = \frac{4 \pm 10}{2}$$

Taking positive sign,  $x = \frac{4+10}{2} = \frac{14}{2} = 7$

Taking negative sign,  $x = \frac{4-10}{2} = \frac{-6}{2} = -3$

Since, age cannot be negative, so  $x = -3$  is rejected.

So, the present age of Rehman = 7 years.

5. Let Shefali's marks in Mathematics =  $x$

$\therefore$  Marks in English =  $30 - x$

[ $\because$  Sum of the marks in English and Mathematics = 30]

According to the condition,  $(x+2) \times [(30-x)-3] = 210$

$$\Rightarrow (x+2) \times (30-x-3) = 210 \Rightarrow (x+2)(-x+27) = 210$$

$$\Rightarrow -x^2 + 25x + 54 = 210 \Rightarrow -x^2 + 25x + 54 - 210 = 0$$

$$\Rightarrow -x^2 + 25x - 156 = 0 \Rightarrow x^2 - 25x + 156 = 0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ , we get  $a = 1, b = -25$  and  $c = 156$ .

$$\therefore b^2 - 4ac = (-25)^2 - 4(1)(156) = 625 - 624 = 1 > 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-25) \pm \sqrt{1}}{2(1)} \Rightarrow x = \frac{25 \pm 1}{2}$$

Taking positive sign,  $x = \frac{25+1}{2} = \frac{26}{2} = 13$

Taking negative sign,  $x = \frac{25-1}{2} = \frac{24}{2} = 12$

When  $x = 13$ , then  $30 - x = 30 - 13 = 17$

When  $x = 12$ , then  $30 - x = 30 - 12 = 18$

Thus, marks in Mathematics = 13, marks in English = 17

or marks in Mathematics = 12, marks in English = 18.

6. Let the shorter side i.e., breadth =  $x$  metres

$\therefore$  The longer side i.e., length =  $(x+30)$  metres

and diagonal =  $(x+60)$  metres

In a rectangle,

$$(\text{diagonal})^2 = (\text{breadth})^2 + (\text{length})^2$$

$$\Rightarrow (x+60)^2 = x^2 + (x+30)^2$$

$$\Rightarrow x^2 + 120x + 3600 = x^2 + x^2 + 60x + 900$$

$$\Rightarrow x^2 + 120x + 3600 = 2x^2 + 60x + 900$$

$$\Rightarrow 2x^2 - x^2 + 60x - 120x + 900 - 3600 = 0$$

$$\Rightarrow x^2 - 60x - 2700 = 0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ , we get

$$a = 1, b = -60 \text{ and } c = -2700.$$

$$\therefore b^2 - 4ac = (-60)^2 - 4(1)(-2700)$$

$$\Rightarrow b^2 - 4ac = 3600 + 10800 = 14400 > 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-60) \pm \sqrt{14400}}{2(1)} \Rightarrow x = \frac{60 \pm 120}{2}$$

Taking positive sign,  $x = \frac{60+120}{2} = \frac{180}{2} = 90$

Taking negative sign,  $x = \frac{60-120}{2} = \frac{-60}{2} = -30$

Since breadth cannot be negative.

$$\therefore x \neq -30 \Rightarrow x = 90$$

$$\therefore x + 30 = 90 + 30 = 120$$

Thus, the shorter side is 90 metres and the longer side is 120 metres.

7. Let the larger number be  $x$ .

Since,  $(\text{smaller number})^2 = 8(\text{larger number})$

$$\Rightarrow (\text{smaller number})^2 = 8x$$

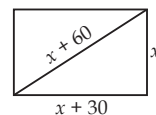
$$\Rightarrow \text{smaller number} = \sqrt{8x}$$

$$\text{According to the condition, } x^2 - (\sqrt{8x})^2 = 180$$

$$\Rightarrow x^2 - 8x = 180 \Rightarrow x^2 - 8x - 180 = 0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ , we get

$$a = 1, b = -8, c = -180$$



$$\therefore b^2 - 4ac = (-8)^2 - 4(1)(-180) = 64 + 720 = 784 > 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-8) \pm \sqrt{784}}{2(1)} \Rightarrow x = \frac{8 \pm 28}{2}$$

Taking positive sign,  $x = \frac{8+28}{2} = \frac{36}{2} = 18$

Taking negative sign,  $x = \frac{8-28}{2} = \frac{-20}{2} = -10$

But  $x = -10$  is not admissible.

$\therefore$  The larger number = 18

$\Rightarrow$  Smaller number =  $\sqrt{8 \times 18} = \sqrt{144} = \pm 12$

Thus, the smaller number = 12 or -12

Thus, the two numbers are 18 and 12 or 18 and -12.

**8.** Let the uniform speed of the train be  $x$  km/hr.

Since, time taken by the train =  $\frac{\text{Distance}}{\text{Speed}}$

$\Rightarrow$  Time taken =  $\frac{360}{x}$  hours

If, speed =  $(x + 5)$  km/hr, then

Time taken =  $\frac{360}{(x+5)}$  hours

$\therefore$  According to the condition,

$$\frac{360}{x+5} - \frac{360}{x} = -1 \Rightarrow 360 \left[ \frac{1}{x+5} - \frac{1}{x} \right] = -1$$

$$\Rightarrow \frac{1}{x+5} - \frac{1}{x} = \frac{-1}{360} \Rightarrow \frac{x - (x+5)}{x(x+5)} = \frac{-1}{360}$$

$$\Rightarrow x - x - 5 = \frac{-(x+5)x}{360} \Rightarrow -5 \times 360 = -(x^2 + 5x)$$

$$\Rightarrow -1800 = -x^2 - 5x \Rightarrow x^2 + 5x - 1800 = 0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ , we get  $a = 1$ ,  $b = 5$  and  $c = -1800$ .

$$\therefore b^2 - 4ac = (5)^2 - 4(1)(-1800) = 25 + 7200 = 7225 > 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-5 \pm \sqrt{7225}}{2(1)} \Rightarrow x = \frac{-5 \pm 85}{2}$$

Taking positive sign,  $x = \frac{-5+85}{2} = \frac{80}{2} = 40$

Taking negative sign,  $x = \frac{-5-85}{2} = \frac{-90}{2} = -45$

Since, the speed of a vehicle cannot be negative.

$$\therefore x \neq -45 \Rightarrow x = 40$$

Thus, speed of the train is 40 km/hr.

**9.** Let the smaller tap fills the tank in  $x$  hours.

$\therefore$  The larger tap fills the tank in  $(x - 10)$  hours.

Amount of water flowing through both the taps in one

$$\text{hour} = \frac{1}{x} + \frac{1}{x-10} = \frac{x-10+x}{x(x-10)} = \frac{2x-10}{x^2-10x}$$

According to the condition,  $\frac{8}{75} = \left( \frac{2x-10}{x^2-10x} \right)$

$$\Rightarrow \frac{75(2x-10)}{8(x^2-10x)} = 1 \Rightarrow \frac{150x-750}{8x^2-80x} = 1$$

$$\Rightarrow 8x^2 - 80x = 150x - 750 \Rightarrow 8x^2 - 80x - 150x + 750 = 0$$

$$\Rightarrow 8x^2 - 230x + 750 = 0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ , we get

$$a = 8, b = -230 \text{ and } c = 750.$$

$$\therefore b^2 - 4ac = (-230)^2 - 4(8)750$$

$$= 52900 - 24000 = 28900 > 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-230) \pm \sqrt{28900}}{2(8)} \Rightarrow x = \frac{230 \pm 170}{16}$$

Taking positive sign,  $x = \frac{230+170}{16} = \frac{400}{16} = 25$

Taking negative sign,  $x = \frac{230-170}{16} = \frac{60}{16} = \frac{15}{4}$

For  $x = \frac{15}{4}$ ,  $x - 10 = \frac{15}{4} - 10 = \frac{-25}{4}$ , which is not possible.

[ $\therefore$  Time cannot be negative]

$$\therefore x = 25 \Rightarrow x - 10 = 25 - 10 = 15$$

Thus, time taken to fill the tank by the smaller tap alone is 25 hours and by the larger tap alone is 15 hours.

**10.** Let the average speed of the passenger train be  $x$  km/hr.

$\therefore$  Average speed of the express train =  $(x + 11)$  km/hr

Total distance covered = 132 km

Also, Time =  $\frac{\text{Distance}}{\text{Speed}}$

Time taken by the passenger train =  $\frac{132}{x}$  hours

Time taken by the express train =  $\frac{132}{x+11}$  hours

According to the condition, we get  $\frac{132}{x+11} = \left( \frac{132}{x} \right) - 1$

$$\Rightarrow \frac{132}{x+11} - \frac{132}{x} = -1 \Rightarrow 132 \left[ \frac{1}{x+11} - \frac{1}{x} \right] = -1$$

$$\Rightarrow 132 \left[ \frac{x - x - 11}{x(x+11)} \right] = -1 \Rightarrow 132 \left[ \frac{-11}{x^2 + 11x} \right] = -1$$

$$\Rightarrow -11(132) = -1(x^2 + 11x) \Rightarrow 1452 = (x^2 + 11x)$$

$$\Rightarrow x^2 + 11x - 1452 = 0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ ,

we get  $a = 1$ ,  $b = 11$  and  $c = -1452$ .

$$\therefore b^2 - 4ac = (11)^2 - 4(1)(-1452) = 121 + 5808 = 5929 > 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-11 \pm \sqrt{5929}}{2(1)} = \frac{-11 \pm 77}{2}$$

Taking positive sign,  $x = \frac{-11+77}{2} = \frac{66}{2} = 33$

Taking negative sign,  $x = \frac{-11-77}{2} = \frac{-88}{2} = -44$

But average speed cannot be negative.

$$\therefore x \neq -44 \Rightarrow x = 33$$

$\therefore$  Average speed of the passenger train = 33 km/hr

And average speed of the express train

$$= (x + 11) = (33 + 11) = 44 \text{ km/hr}$$

**11.** Let the side of the smaller square be  $x$  m.

$\Rightarrow$  Perimeter of the smaller square =  $4x$  m

So, perimeter of the larger square =  $(4x + 24)$  m

$\Rightarrow$  Side of the larger square



$$= \frac{\text{Perimeter of larger square}}{4}$$

$$= \frac{(4x + 24)}{4} = \frac{4(x + 6)}{4} = (x + 6)m$$

Area of the smaller square =  $x^2 \text{ m}^2$

Area of the larger square =  $(x + 6)^2 \text{ m}^2$

According to the condition,  $x^2 + (x + 6)^2 = 468$

$$\Rightarrow x^2 + x^2 + 12x + 36 = 468 \Rightarrow 2x^2 + 12x - 432 = 0$$

$$\Rightarrow x^2 + 6x - 216 = 0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ , we get

$a = 1$ ,  $b = 6$  and  $c = -216$ .

$$\therefore b^2 - 4ac = (6)^2 - 4(1)(-216) = 36 + 864 = 900 > 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-6 \pm \sqrt{900}}{2(1)} = \frac{-6 \pm 30}{2}$$

$$\text{Taking positive sign, } x = \frac{-6 + 30}{2} = \frac{24}{2} = 12$$

$$\text{Taking negative sign, } x = \frac{-6 - 30}{2} = \frac{-36}{2} = -18$$

But the length of a square cannot be negative.

$$\therefore x \neq -18 \Rightarrow x = 12$$

Length of the smaller square = 12 m

and the length of the larger square =  $x + 6 = 12 + 6 = 18 \text{ m}$

#### EXERCISE - 4.4

1. Comparing the given quadratic equation with  $ax^2 + bx + c = 0$ , we get  $a = 2$ ,  $b = -3$  and  $c = 5$ .

$$\therefore D = b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31 < 0$$

$\therefore$  The given quadratic equation has no real roots.

(ii) Comparing the given quadratic equation with  $ax^2 + bx + c = 0$ , we get  $a = 3$ ,  $b = -4\sqrt{3}$  and  $c = 4$ .

$$\therefore D = b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4) = (16 \times 3) - 48$$

$$= 48 - 48 = 0$$

$\therefore$  The given quadratic equation has two real roots which are equal. Hence, the roots are

$$\frac{-b}{2a} \text{ and } \frac{-b}{2a} \text{ i.e., } \frac{-(-4\sqrt{3})}{2 \times 3} \text{ and } \frac{-(-4\sqrt{3})}{2 \times 3} \text{ i.e., } \frac{2}{\sqrt{3}} \text{ and } \frac{2}{\sqrt{3}}.$$

(iii) Comparing the given quadratic equation with  $ax^2 + bx + c = 0$ , we get  $a = 2$ ,  $b = -6$  and  $c = 3$ .

$$\therefore D = b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12 > 0$$

Since,  $b^2 - 4ac$  is positive.

$\therefore$  The given quadratic equation has two real and distinct roots, which are given by  $x = \frac{-b \pm \sqrt{D}}{2a}$

$$\Rightarrow x = \frac{-(-6) \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

$$\text{Thus, the roots are } \frac{3 + \sqrt{3}}{2} \text{ and } \frac{3 - \sqrt{3}}{2}.$$

2. (i) Comparing the given quadratic equation with  $ax^2 + bx + c = 0$ , we get  $a = 2$ ,  $b = k$  and  $c = 3$ .

$$\therefore D = b^2 - 4ac = (k)^2 - 4(2)(3) = k^2 - 24$$

$\therefore$  For a quadratic equation to have equal roots,  $D = 0$

$$\Rightarrow b^2 - 4ac = 0 \Rightarrow k^2 - 24 = 0 \Rightarrow k = \pm\sqrt{24} \Rightarrow k = \pm 2\sqrt{6}$$

Thus, the required values of  $k$  are  $2\sqrt{6}$  and  $-2\sqrt{6}$ .

$$(ii) \quad kx(x - 2) + 6 = 0 \Rightarrow kx^2 - 2kx + 6 = 0$$

Comparing  $kx^2 - 2kx + 6 = 0$  with  $ax^2 + bx + c = 0$ , we get  
 $a = k$ ,  $b = -2k$  and  $c = 6$ .

$$\therefore D = b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$$

Since, the roots are equal.

$$\therefore D = b^2 - 4ac = 0 \Rightarrow 4k^2 - 24k = 0$$

$$\Rightarrow 4k(k - 6) = 0 \Rightarrow 4k = 0 \text{ or } k - 6 = 0 \Rightarrow k = 0 \text{ or } k = 6$$

But  $k$  cannot be 0, otherwise, the given equation is not quadratic. Thus, the required value of  $k$  is 6.

3. Let the breadth be  $x \text{ m}$ .  $\therefore$  Length =  $2x \text{ m}$

Now, Area = Length  $\times$  Breadth =  $2x \times x = 2x^2 \text{ m}^2$

According to the given condition,  $2x^2 = 800$

$$\Rightarrow x^2 = \frac{800}{2} = 400 \Rightarrow x^2 - 400 = 0$$

Here,  $a = 1$ ,  $b = 0$  and  $c = -400$

$$\therefore D = b^2 - 4ac = 0 - 4(1)(-400) = 1600 > 0$$

So, the roots are real and distinct.

$$\therefore x = \frac{0 \pm \sqrt{1600}}{2(1)} = \pm \frac{40}{2} = \pm 20$$

Therefore,  $x = 20$  or  $x = -20$

But  $x = -20$  is not possible. [ $\therefore$  Breadth cannot be negative]

$$\therefore x = 20 \Rightarrow 2x = 2 \times 20 = 40$$

Thus, it is possible to design a rectangular mango grove with length = 40 m and breadth = 20 m.

4. Let the age of one friend =  $x$  years

$\therefore$  Age of other friend =  $(20 - x)$  years

Four years ago,

Age of one friend =  $(x - 4)$  years

Age of other friend =  $(20 - x - 4)$  years =  $(16 - x)$  years

According to the condition,  $(x - 4) \times (16 - x) = 48$

$$\Rightarrow 16x - 64 - x^2 + 4x = 48 \Rightarrow -x^2 + 20x - 64 - 48 = 0$$

$$\Rightarrow -x^2 + 20x - 112 = 0 \Rightarrow x^2 - 20x + 112 = 0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ , we get  
 $a = 1$ ,  $b = -20$  and  $c = 112$ .

$$\therefore D = b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48 < 0$$

$\therefore$  The quadratic equation (1) has no real roots.

Thus, the given situation is not possible.

5. Let the breadth of the rectangle =  $x \text{ m}$ .

Since, the perimeter of the rectangle = 80 m

$$\therefore 2(\text{Length} + \text{Breadth}) = 80 \Rightarrow 2(\text{Length} + x) = 80$$

$$\Rightarrow \text{Length} + x = 80/2 = 40 \Rightarrow \text{Length} = (40 - x) \text{ m}$$

$$\therefore \text{Area of the rectangle} = (40 - x) \times x = (40x - x^2) \text{ m}^2$$

According to the given condition,

Area of the rectangle =  $400 \text{ m}^2$

$$\Rightarrow 40x - x^2 = 400 \Rightarrow x^2 - 40x + 400 = 0 \quad \dots(1)$$

Comparing equation (1) with  $ax^2 + bx + c = 0$ , we get

$a = 1$ ,  $b = -40$  and  $c = 400$ .

$$\therefore D = b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$$

$\therefore$  Equation (1) has two equal and real roots. Hence,

$$\text{the roots are } \frac{-b}{2a} \text{ and } \frac{-b}{2a} \text{ i.e., } \frac{-(-40)}{2(1)} = \frac{40}{2} = 20$$

$$\therefore \text{Breadth} = x \text{ m} = 20 \text{ m, Length} = 40 - x = 40 - 20 = 20 \text{ m}$$

Thus, it is possible to design a rectangular park of given perimeter and area.

