Coordinate Geometry

CHAPTER 7

NCERT FOCUS

SOLUTIONS

EXERCISE - 7.1

1. (i) Here, $x_1 = 2, y_1 = 3$ and $x_2 = 4, y_2 = 1$

$\therefore$ The required distance

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4-2)^2 + (1-3)^2}$

$= \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

(ii) Here, $x_1 = -5, y_1 = 7$ and $x_2 = -1, y_2 = 3$

$\therefore$ The required distance

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{[-1-(-5)]^2 + (3-7)^2}$

$= \sqrt{(-1+5)^2 + (-4)^2} = \sqrt{16+16} = 4\sqrt{2}$ units

(iii) Here $x_1 = a, y_1 = b$ and $x_2 = -a, y_2 = -b$

$\therefore$ The required distance

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-a-a)^2 + (-b-b)^2}$

$= \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2} = \sqrt{4(a^2 + b^2)}$

$= 2\sqrt{(a^2 + b^2)}$ units

2. Part-I

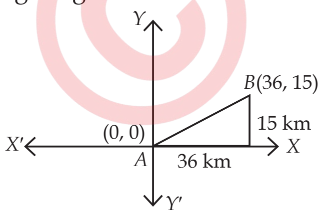
Let the given points be $A(0, 0)$ and $B(36, 15)$.

Then, $AB = \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{(36)^2 + (15)^2}$

$= \sqrt{1296 + 225} = \sqrt{1521} = \sqrt{39^2} = 39$ units

Part-II

The given situation can be represented graphically as shown in the figure given below.



We have $A(0, 0)$ and $B(36, 15)$ as the positions of two towns.

Now, $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(36-0)^2 + (15-0)^2} = 39$ km.

3. Let the given points be $A(1, 5)$, $B(2, 3)$ and $C(-2, -11)$.

Clearly, A, B and C will be collinear, if

$AB + BC = AC$ or $AC + CB = AB$ or $BA + AC = BC$

Here, $AB = \sqrt{(2-1)^2 + (3-5)^2}$

$= \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5} = 2.24$ units (Approx.)

$BC = \sqrt{(-2-2)^2 + (-11-3)^2}$

$= \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$

$= 14.56$ units (Approx.)

and, $AC = \sqrt{(-2-1)^2 + (-11-5)^2}$

$= \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$ units

$= 16.28$ units (Approx.)

Since, $AB + BC \neq AC$, $AC + CB \neq AB$ and $BA + AC \neq BC$

$\therefore$ A, B and C are not collinear.

4. Let the given points be $A(5, -2)$, $B(6, 4)$ and $C(7, -2)$.

Then, $AB = \sqrt{(6-5)^2 + [4-(-2)]^2}$

$= \sqrt{(1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$ units

$BC = \sqrt{(7-6)^2 + (-2-4)^2}$

$= \sqrt{(1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$ units

and $AC = \sqrt{(7-5)^2 + [-2-(-2)]^2}$

$= \sqrt{(2)^2 + (0)^2} = \sqrt{4+0} = 2$ units

Since, $AB = BC$

$\therefore$ ΔABC is an isosceles triangle.

5. The coordinates of given points are $A(3, 4)$, $B(6, 7)$, $C(9, 4)$ and $D(6, 1)$

$\therefore$ $AB = \sqrt{(6-3)^2 + (7-4)^2}$

$= \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$BC = \sqrt{(9-6)^2 + (4-7)^2}$

$= \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$CD = \sqrt{(6-9)^2 + (1-4)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$AD = \sqrt{(6-3)^2 + (1-4)^2}$

$= \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$AC = \sqrt{(9-3)^2 + (4-4)^2} = \sqrt{(6)^2 + (0)^2} = 6$ units

and $BD = \sqrt{(6-6)^2 + (1-7)^2} = \sqrt{(0)^2 + (-6)^2} = 6$ units

Since, $AB = BC = CD = AD$ *i.e.*, all the four sides are equal. and also, $BD = AC$ *i.e.*, both the diagonals are also equal.

$\therefore$ $ABCD$ is a square.

Thus, Champa is correct.

6. (i) Let the given points be $A(-1, -2)$, $B(1, 0)$, $C(-1, 2)$ and $D(-3, 0)$.

Now, $AB = \sqrt{(1+1)^2 + (0+2)^2}$

$= \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8}$ units

$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8}$ units

$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8}$ units

$DA = \sqrt{(-1+3)^2 + (-2-0)^2} = \sqrt{4+4} = \sqrt{8}$ units

$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+4^2} = 4$ units

$BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{(-4)^2} = 4$ units

Since, $AB = BC = CD = DA$ *i.e.*, all the sides are equal, and also, $AC = BD$ *i.e.*, the diagonals are also equal.

$\therefore$ $ABCD$ is a square.

(ii) Let the given points be $A(-3, 5)$, $B(3, 1)$, $C(0, 3)$ and $D(-1, -4)$.

Now, $AB = \sqrt{[3-(-3)]^2 + (1-5)^2}$

$= \sqrt{6^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

$BC = \sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$ units

$CD = \sqrt{(-1-0)^2 + (-4-3)^2} = \sqrt{(-1)^2 + (-7)^2}$

$= \sqrt{1+49} = \sqrt{50}$ units

$DA = \sqrt{[-3-(-1)]^2 + [5-(-4)]^2} = \sqrt{(-2)^2 + (9)^2}$

$= \sqrt{4+81} = \sqrt{85}$ units

$AC = \sqrt{[0-(-3)]^2 + (3-5)^2} = \sqrt{(3)^2 + (-2)^2}$

$= \sqrt{9+4} = \sqrt{13}$ units

and $BD = \sqrt{(-1-3)^2 + (-4-1)^2} = \sqrt{(-4)^2 + (-5)^2}$

$= \sqrt{16+25} = \sqrt{41}$ units

Here, we can see that $\left[\because \sqrt{13} + \sqrt{13} = 2\sqrt{13}\right]$

$AC + BC = AB$

$\Rightarrow$ A, B and C are collinear points. Hence, $ABCD$ is not a quadrilateral.

(iii) Let the given points be $A(4, 5)$, $B(7, 6)$, $C(4, 3)$ and $D(1, 2)$.

Now, $AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{3^2 + 1^2} = \sqrt{10}$ units

$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18}$ units

$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{10}$ units

$DA = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{9+9} = \sqrt{18}$ units

$AC = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{0 + (-2)^2} = 2$ units

and $BD = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{36+16} = \sqrt{52}$ units

Since, $AB = CD$, $BC = DA$ *i.e.*, opposite sides of the given quadrilateral are equal, and also, $AC \neq BD$, *i.e.*, diagonals are unequal.

$\therefore$ $ABCD$ is a parallelogram.

7. We know that any point on x-axis is of the form $(x, 0)$.

$\therefore$ Let the required point be $P(x, 0)$.

Also, let the given points be $A(2, -5)$ and $B(-2, 9)$.

Now, $AP = \sqrt{(x-2)^2 + [0-(-5)]^2}$

$= \sqrt{(x-2)^2 + 5^2} = \sqrt{x^2 - 4x + 4 + 25} = \sqrt{x^2 - 4x + 29}$

and $BP = \sqrt{[x-(-2)]^2 + (0-9)^2}$

$= \sqrt{(x+2)^2 + (-9)^2} = \sqrt{x^2 + 4x + 4 + 81} = \sqrt{x^2 + 4x + 85}$

Since, A and B are equidistant from P.

$\therefore$ $AP = BP$

$\Rightarrow$ $\sqrt{x^2 - 4x + 29} = \sqrt{x^2 + 4x + 85}$

$\Rightarrow$ $x^2 - 4x + 29 = x^2 + 4x + 85$

$\Rightarrow$ $-8x = 56$ $\Rightarrow$ $x = \dfrac{56}{-8} = -7$

$\therefore$ The required point is $(-7, 0)$.

8. The given points are $P(2, -3)$ and $Q(10, y)$.

$\therefore$ $PQ = \sqrt{(10-2)^2 + [y-(-3)]^2}$

$= \sqrt{8^2 + (y+3)^2} = \sqrt{64 + y^2 + 6y + 9} = \sqrt{y^2 + 6y + 73}$

But $PQ = 10$ [Given]

$\therefore$ $\sqrt{y^2 + 6y + 73} = 10$

On squaring both sides, we get $y^2 + 6y + 73 = 100$

$\Rightarrow$ $y^2 + 6y - 27 = 0$

$\Rightarrow$ $y^2 - 3y + 9y - 27 = 0$ $\Rightarrow (y-3)(y+9) = 0$

$\Rightarrow$ $y = 3$ or $y = -9$

$\therefore$ The required values of y are 3 and -9.

9. Here, $QP = \sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{5^2 + (-4)^2}$

$= \sqrt{25+16} = \sqrt{41}$ units

and $QR = \sqrt{(x-0)^2 + (6-1)^2}$

$= \sqrt{x^2 + 5^2} = \sqrt{x^2 + 25}$ units

$\because$ $QP = QR$

$\therefore$ $\sqrt{41} = \sqrt{x^2 + 25}$

On squaring both sides, we get $x^2 + 25 = 41$

$\Rightarrow$ $x^2 = 16$ $\Rightarrow$ $x = \pm 4$

Thus, the point R is $(4, 6)$ or $(-4, 6)$

Now, $QR = \sqrt{[(\pm 4) - (0)]^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$ units

and $PR = \sqrt{(4-5)^2 + (6+3)^2}$ or $\sqrt{(-4-5)^2 + (6+3)^2}$

$\Rightarrow$ $PR = \sqrt{1+81}$ or $\sqrt{81+81}$

$\Rightarrow$ $PR = \sqrt{82}$ units or $9\sqrt{2}$ units

10. Let $A(x, y)$, $B(3, 6)$ and $C(-3, 4)$ be the given points. Now let, the point $A(x, y)$ is equidistant from $B(3, 6)$ and $C(-3, 4)$.
Then, we get $AB = AC$

$$\Rightarrow \sqrt{(3-x)^2+(6-y)^2} = \sqrt{(-3-x)^2+(4-y)^2}$$

On squaring both sides, we get

$$(3-x)^2 + (6-y)^2 = (-3-x)^2 + (4-y)^2$$

$\Rightarrow$ $9 + x^2 - 6x + 36 + y^2 - 12y = 9 + x^2 + 6x + 16 + y^2 - 8y$

$\Rightarrow$ $-6x - 6x + 36 - 12y - 16 + 8y = 0$

$\Rightarrow$ $-12x - 4y + 20 = 0 \Rightarrow -3x - y + 5 = 0$

$\Rightarrow$ $3x + y - 5 = 0$, which is the required relation between x and y.

EXERCISE - 7.2

1. Let the required point be $P(x, y)$.
Here, the end points are $(-1, 7)$ and $(4, -3)$

$\because$ Ratio $= 2 : 3 = m_1 : m_2$

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$= \frac{(2\times 4) + 3\times(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

$$\text{and } y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2\times(-3) + (3\times 7)}{2+3}$$

$$= \frac{-6+21}{5} = \frac{15}{5} = 3$$

Thus, the required point is $(1, 3)$.

2. Let the given points be $A(4, -1)$ and $B(-2, -3)$.

Let the points P and Q trisect AB.
i.e., $AP = PQ = QB$
i.e., P divides AB in the ratio of $1 : 2$ and Q divides AB in the ratio of $2 : 1$.
Let the coordinates of P be (x, y).

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1(-2) + 2(4)}{1+2} = \frac{-2+8}{3} = 2 \text{ and}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1(-3) + 2\times(-1)}{1+2} = \frac{-3-2}{3} = \frac{-5}{3}$$

$\therefore$ The required coordinates of P are $\left(2, \frac{-5}{3}\right)$.

Let the coordinates of Q be (X, Y).

$$\therefore \quad X = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2(-2) + 1(4)}{2+1} = \frac{-4+4}{3} = 0$$

$$Y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2(-3) + 1(-1)}{2+1} = \frac{-6-1}{3} = \frac{-7}{3}$$

$\therefore$ The required coordinates of Q are $\left(0, \frac{-7}{3}\right)$.

3. Let us consider 'A' as origin, then AB is the x-axis and AD is the y-axis.

Now, the position of green flag-post is $\left(2, \frac{100}{4}\right)$ or $(2, 25)$.

and, the position of red flag-post is $\left(8, \frac{100}{5}\right)$ or $(8, 20)$.

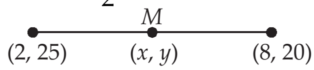
$\therefore$ Distance between both the flags

$$= \sqrt{(8-2)^2 + (20-25)^2}$$

$$= \sqrt{6^2 + (-5)^2} = \sqrt{36+25} = \sqrt{61} \text{ m}$$

Let the mid-point of the line segment joining the two flags be $M(x, y)$.

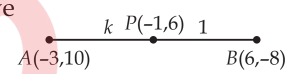
$$\therefore \quad x = \frac{2+8}{2} \text{ and } y = \frac{25+20}{2}$$



$\Rightarrow$ $x = 5$ and $y = 22.5$

Thus, the blue flag is on the 5^{th} line at a distance 22.5 m above AB.

4. Given, points are $A(-3, 10)$ and $B(6, -8)$
Let the point $P(-1, 6)$ divides AB in the ratio $k : 1$.
Using section formula, we have



$$(-1, 6) = \left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$$

$$\Rightarrow \quad \frac{6k-3}{k+1} = -1 \text{ and } \frac{-8k+10}{k+1} = 6$$

$\Rightarrow$ $6k - 3 = -k - 1$ and $-8k + 10 = 6k + 6$

$\Rightarrow$ $7k = 2$ and $14k = 4$

$$\Rightarrow \quad k = \frac{2}{7}$$

$\therefore$ Required ratio is $\frac{2}{7} : 1$ *i.e.*, $2 : 7$.

5. The given points are $A(1, -5)$ and $B(-4, 5)$.
Let the required ratio be $k : 1$ and the required point be $P(x, y)$.
Since the point P lies on x-axis,
$\therefore$ Its y-coordinate is 0.

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } 0 = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\Rightarrow \quad x = \frac{k(-4) + 1(1)}{k+1} \text{ and } 0 = \frac{5k-5}{k+1}$$

$$\Rightarrow \quad x = \frac{-4k+1}{k+1} \text{ and } 0 = \frac{5k-5}{k+1}$$

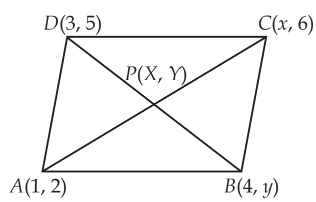
$\Rightarrow$ $x(k + 1) = -4k + 1$ and $5k - 5 = 0 \Rightarrow k = 1$

$\Rightarrow$ $x(1 + 1) = -4 + 1$ $\quad [\because k = 1]$

$$\Rightarrow \quad 2x = -3 \quad \Rightarrow \quad x = -\frac{3}{2}$$

$\therefore$ The required ratio is $1 : 1$ and coordinates of P are $\left(\frac{-3}{2}, 0\right)$.

6. Let the given points are $A(1, 2)$, $B(4, y)$, $C(x, 6)$ and $D(3, 5)$.
Since, the diagonals of a parallelogram bisect each other.



$\therefore$ The coordinates of P are

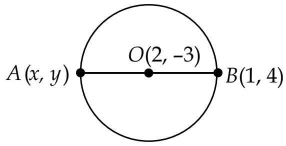
$$X = \frac{x+1}{2} = \frac{3+4}{2}$$

$\Rightarrow \quad x + 1 = 7 \Rightarrow x = 6$ and $Y = \frac{5+y}{2} = \frac{6+2}{2}$

$\Rightarrow \quad 5 + y = 8 \Rightarrow y = 3$

$\therefore$ The required values of x and y are 6 and 3 respectively.

7. Here, centre of the circle is $O(2, -3)$.



Let the end points of the diameter be $A(x, y)$ and $B(1, 4)$.

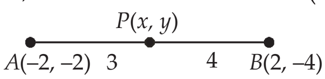
The centre of a circle bisects the diameter.

$\therefore \quad 2 = \frac{x+1}{2} \Rightarrow x + 1 = 4 \Rightarrow x = 3$

And, $-3 = \frac{y+4}{2} \Rightarrow y + 4 = -6 \Rightarrow y = -10$

Hence, the coordinates of A are $(3, -10)$.

8.



Here, the given points are $A(-2, -2)$ and $B(2, -4)$.

Let the coordinates of P are (x, y).

Since, the point P lies on AB such that

$AP = \frac{3}{7}AB \Rightarrow \frac{AP}{AB} = \frac{3}{7} \Rightarrow \frac{AB}{AP} = \frac{7}{3}$

$\Rightarrow \quad \frac{AP + BP}{AP} = \frac{7}{3}$ $\quad (\because AB = AP + BP)$

$\Rightarrow \quad 1 + \frac{BP}{AP} = \frac{3+4}{3} = 1 + \frac{4}{3} \quad \Rightarrow \quad \frac{BP}{AP} = \frac{4}{3}$

$\Rightarrow \quad AP : PB = 3 : 4$ *i.e.*, $P(x, y)$ divides AB in the ratio $3 : 4$.

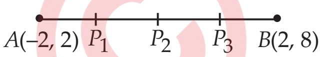
$\therefore \quad x = \frac{3 \times 2 + 4 \times (-2)}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$ and

$y = \frac{3 \times (-4) + 4 \times (-2)}{3+4} = \frac{-12-8}{7} = \frac{-20}{7}$

Thus, the coordinates of P are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

9. Here, the given points are $A(-2, 2)$ and $B(2, 8)$.

Let P_1, P_2 and P_3 divide AB in four equal parts.



Since, $AP_1 = P_1P_2 = P_2P_3 = P_3B$

$\therefore \quad P_2$ is the mid-point of AB

$\therefore \quad$ Coordinates of P_2 are $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = (0, 5)$

Again, P_1 is the mid-point of AP_2.

$\therefore \quad$ Coordinates of P_1 are

$\left(\frac{-2+0}{2}, \frac{2+5}{2}\right) = \left(-1, \frac{7}{2}\right)$

Also, P_3 is the mid-point of P_2B.

$\therefore \quad$ Coordinates of P_3 are

$\left(\frac{0+2}{2}, \frac{5+8}{2}\right) = \left(1, \frac{13}{2}\right)$

Thus, the coordinates of P_1, P_2 and P_3 are

$\left(-1, \frac{7}{2}\right)$, $(0, 5)$ and $\left(1, \frac{13}{2}\right)$ respectively.

10. Let the vertices of the given rhombus are $A(3, 0)$, $B(4, 5)$, $C(-1, 4)$ and $D(-2, -1)$.

$\because \quad AC$ and BD are the diagonals of rhombus $ABCD$.

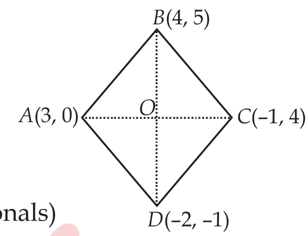
$AC = \sqrt{(-1-3)^2 + (4-0)^2}$

$= \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16} = 4\sqrt{2}$ units

$BD = \sqrt{(-2-4)^2 + (-1-5)^2}$

$= \sqrt{(-6)^2 + (-6)^2}$

$= \sqrt{36+36} = 6\sqrt{2}$ units



$\therefore \quad$ Area of a rhombus

$= \frac{1}{2} \times$ (Product of diagonals)

$= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 4 \times 6 = 24$ square units.

EXERCISE - 7.3

1. (i) Let the vertices of the triangles be $A(2, 3)$, $B(-1, 0)$ and $C(2, -4)$.

$\therefore \quad$ Area of a ΔABC

$= \frac{1}{2}[2\{0-(-4)\} + (-1)\{-4-(3)\} + 2\{3-0\}]$

$= \frac{1}{2}[2(0+4) + (-1)(-4-3) + 2(3)]$

$= \frac{1}{2}[8+7+6] = \frac{1}{2}[21] = \frac{21}{2}$ sq.units

(ii) Let the vertices of the triangles be $A(-5, -1)$, $B(3, -5)$ and $C(5, 2)$.

$\therefore \quad$ Area of a ΔABC

$= \frac{1}{2}[-5\{-5-2\} + 3\{2-(-1)\} + 5\{-1-(-5)\}]$

$= \frac{1}{2}[-5\{-7\} + 3\{2+1\} + 5\{-1+5\}]$

$= \frac{1}{2}[35+9+20] = \frac{1}{2} \times 64 = 32$ sq.units

2. The given three points will be collinear if the area of triangle formed by them is zero.

(i) Let $A(7, -2)$, $B(5, 1)$ and $C(3, k)$ be the vertices of a triangle.

$\therefore \quad$ The given points will be collinear, if ar $(\Delta ABC) = 0$

$\Rightarrow \quad \frac{1}{2}[7(1-k) + 5(k+2) + 3(-2-1)] = 0$

$\Rightarrow \quad 7 - 7k + 5k + 10 - 9 = 0$

$\Rightarrow \quad 8 - 2k = 0 \quad \Rightarrow \quad 2k = 8 \quad \Rightarrow \quad k = \frac{8}{2} = 4$

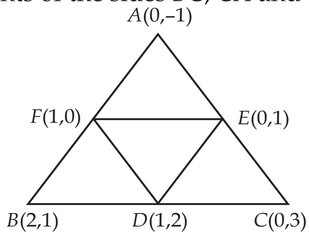
(ii) Let $A(8, 1)$, $B(k, -4)$ and $C(2, -5)$ be the vertices of a triangle.

The given points will be collinear, if ar$(\Delta ABC) = 0$

$\Rightarrow \quad \frac{1}{2}[8(-4+5) + k(-5-1) + 2(1+4)] = 0$

$\Rightarrow \quad 8 - 6k + 10 = 0 \Rightarrow 6k = 18 \Rightarrow k = 3.$

3. Let the vertices of the triangle be $A(0, -1)$, $B(2, 1)$ and $C(0, 3)$.
Let D, E and F be the mid-points of the sides BC, CA and AB respectively.



Then, coordinates of D are

$$\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1,2)$$

Coordinates of E are

$$\left(\frac{0+0}{2}, \frac{3+(-1)}{2}\right) = (0,1)$$

Coordinates of F are $\left(\frac{2+0}{2}, \frac{1+(-1)}{2}\right) = (1, 0)$

Now, $\text{ar}(\Delta ABC) = \frac{1}{2}[0(1-3) + 2\{3-(-1)\} + 0(-1-1)]$

$$= \frac{1}{2}[0+8+0] = \frac{1}{2} \times 8 = 4 \text{ sq.units}$$

Now, $\text{ar}(\Delta DEF) = \frac{1}{2}[1(1-0) + 0(0-2) + 1(2-1)]$

$$= \frac{1}{2}[1+0+1] = \frac{1}{2} \times 2 = 1 \text{ sq.unit}$$

$\therefore \frac{\text{ar}(\Delta DEF)}{\text{ar}(\Delta ABC)} = \frac{1}{4}$; $\therefore$ $\text{ar}(\Delta DEF) : \text{ar}(\Delta ABC) = 1 : 4$

4. Let $A(-4, -2)$, $B(-3, -5)$, $C(3, -2)$ and $D(2, 3)$ be the vertices of the quadrilateral.
Let us join diagonal BD.
Now, $\text{ar}(\Delta ABD)$

$$= \frac{1}{2}[(-4)\{-5-3\} + (-3)\{3-(-2)\} + 2\{(-2)-(-5)\}]$$

$$= \frac{1}{2}[(-4)(-8) + (-3)(5) + 2(-2+5)]$$

$$= \frac{1}{2}[32 + (-15) + 6] = \frac{1}{2}[23] = \frac{23}{2} \text{ sq.units}$$

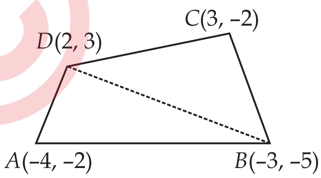
Also, $\text{ar}(\Delta CBD)$

$$= \frac{1}{2}[3(-5-3) + (-3)\{3-(-2)\} + 2\{(-2)-(-5)\}]$$

$$= \frac{1}{2}[3(-8) + (-3)(5) + 2(3)]$$

$$= \frac{1}{2}[-24 - 15 + 6]$$

$$= \frac{1}{2}[-33] = -\frac{33}{2}$$



$\therefore$ $\text{ar}(\Delta CBD) = \frac{33}{2}$ sq,. units

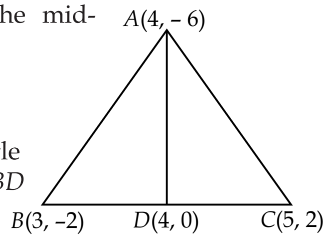
[$\because$ Area of triangle cannot be negative]

Since, ar(quad. $ABCD$) = $\text{ar}(\Delta ABD)$ + $\text{ar}(\Delta CBD)$

$\therefore$ ar(quad. $ABCD$) $= \left(\frac{23}{2} + \frac{33}{2}\right)$ sq. units

$$= \frac{56}{2} \text{ sq.units} = 28 \text{ sq.units}$$

5. Here, the vertices of the triangle are $A(4, -6)$, $B(3, -2)$ and $C(5, 2)$.
Let D be the mid-point of BC.



$\therefore$ The coordinates of the mid-point of D are

$$\left\{\frac{3+5}{2}, \frac{-2+2}{2}\right\} = (4, 0)$$

Since AD divides the triangle ABC into two parts *i.e.*, ΔABD and ΔADC,
Now, $\text{ar}(\Delta ABD)$

$$= \frac{1}{2}[4\{(-2)-0\} + 3(0+6) + 4(-6+2)]$$

$$= \frac{1}{2}[(-8) + 18 + (-16)] = \frac{1}{2}(-6) = -3$$

$\therefore$ $\text{ar}(\Delta ABD) = 3$ sq. units ...(i)

[$\because$ Area of triangle cannot be negative]

Also, $\text{ar}(\Delta ADC) = \frac{1}{2}[4(0-2) + 4(2+6) + 5(-6-0)]$

$$= \frac{1}{2}[-8 + 32 - 30] = \frac{1}{2}[-6] = -3$$

$\therefore$ $\text{ar}(\Delta ADC) = 3$ sq. units ...(ii)

[$\because$ Area of triangle cannot be negative]

From (i) and (ii), we have
$\text{ar}(\Delta ABD) = \text{ar}(\Delta ADC)$
Hence, a median divides the triangle into two triangles of equal areas.

EXERCISE - 7.4

1. Let the point C divides the line segment joining the points $A(2, -2)$ and $B(3, 7)$ in the ratio $k : 1$.
Using section formula, we have

Coordinates of C are $\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

Since, the point C lies on the given line $2x + y - 4 = 0$.

$\therefore$ We have $2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) - 4 = 0$

$\Rightarrow$ $2(3k + 2) + (7k - 2) - 4(k + 1) = 0$
$\Rightarrow$ $6k + 4 + 7k - 2 - 4k - 4 = 0$

$\Rightarrow$ $9k - 2 = 0 \Rightarrow k = \frac{2}{9}$

$\therefore$ The required ratio is $\frac{2}{9} : 1$ *i.e.*, $2 : 9$.

2. Let the given points be $A(x, y)$, $B(1, 2)$ and $C(7, 0)$.
Given, the points A, B and C will be collinear.

$\therefore$ Area of $\Delta ABC = 0 \Rightarrow \frac{1}{2}[x(2-0) + 1(0-y) + 7(y-2)] = 0$

$\Rightarrow$ $2x - y + 7y - 14 = 0$
$\Rightarrow$ $2x + 6y - 14 = 0 \Rightarrow x + 3y - 7 = 0$, which is the required relation between x and y.

3. Let $P(x, y)$ be the centre of the circle and the circle is passing through the points $A(6, -6)$, $B(3, -7)$ and $C(3, 3)$.
$\therefore$ $AP = BP = CP$
Taking $AP = BP$, we have $AP^2 = BP^2$
$\Rightarrow$ $(x-6)^2 + (y+6)^2 = (x-3)^2 + (y+7)^2$
$\Rightarrow$ $x^2 - 12x + 36 + y^2 + 12y + 36 = x^2 - 6x + 9 + y^2 + 14y + 49$
$\Rightarrow$ $-12x + 6x + 12y - 14y + 72 - 58 = 0$
$\Rightarrow$ $-6x - 2y + 14 = 0$

$\Rightarrow \quad 3x + y - 7 = 0$ ... (i)

Taking $BP = CP$, we have

$BP^2 = CP^2$

$\Rightarrow \quad (x-3)^2 + (y+7)^2 = (x-3)^2 + (y-3)^2$

$\Rightarrow \quad x^2 - 6x + 9 + y^2 + 14y + 49 = x^2 - 6x + 9 + y^2 - 6y + 9$

$\Rightarrow \quad 14y + 6y + 58 - 18 = 0 \Rightarrow 20y + 40 = 0$

$\Rightarrow \quad y = \frac{-40}{20} = -2$ ... (ii)

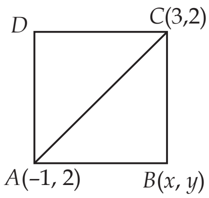
From (i) and (ii), we get

$3x - 2 - 7 = 0 \quad \Rightarrow \quad 3x = 9 \Rightarrow x = 3$

$\therefore \quad x = 3$ and $y = -2$

Hence, the required centre is (3, –2).

4. Let us have a square $ABCD$ such that $A(-1, 2)$ and $C(3, 2)$ are the opposite vertices.



Let $B(x, y)$ be an unknown vertex.

Since, all sides of a square are equal.

$\therefore \quad AB = CB \Rightarrow AB^2 = CB^2$

$\Rightarrow \quad (x+1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$

$\Rightarrow \quad x^2 + 1 + 2x + y^2 + 4 - 4y = x^2 + 9 - 6x + y^2 + 4 - 4y$

$\Rightarrow \quad 2x + 1 = -6x + 9 \quad \Rightarrow \quad 8x = 8 \Rightarrow x = 1$ ... (i)

Since, each angle of a square = 90°

$\therefore \quad ABC$ is a right angled triangle.

$\therefore \quad$ Using Pythagoras theorem, we have

$AB^2 + CB^2 = AC^2$

$\Rightarrow \quad [(x+1)^2 + (y-2)^2] + [(x-3)^2 + (y-2)^2] = [(3+1)^2 + (2-2)^2]$

$\Rightarrow \quad 2x^2 + 2y^2 + 2x - 4y - 6x - 4y + 1 + 4 + 9 + 4 = 16$

$\Rightarrow \quad 2x^2 + 2y^2 - 4x - 8y + 2 = 0$

$\Rightarrow \quad x^2 + y^2 - 2x - 4y + 1 = 0$ ... (ii)

Substituting the value of x from (i) into (ii), we have

$1 + y^2 - 2 - 4y + 1 = 0$

$\Rightarrow \quad y^2 - 4y = 0 \Rightarrow y(y-4) = 0$

$\Rightarrow \quad y = 0$ or $y = 4$

Hence, the two required other vertices are (1, 0) and (1, 4).

5. (i) By taking A as the origin and AD and AB as the coordinate axes. We have $P(4, 6)$, $Q(3, 2)$ and $R(6, 5)$ as the vertices of ΔPQR.

(ii) By taking C as the origin and CB and CD as the coordinate axes, then the vertices of ΔPQR are $P(-12, -2)$, $Q(-13, -6)$ and $R(-10, -3)$.

Now area of ΔPQR [when $P(4, 6)$, $Q(3, 2)$ and $R(6, 5)$ are the vertices]

$= \frac{1}{2}[4(2-5) + 3(5-6) + 6(6-2)]$

$= \frac{1}{2}[-12 - 3 + 24] = \frac{9}{2}$ sq. units

Area of ΔPQR [when $P(-12, -2)$, $Q(-13, -6)$ and $R(-10, -3)$ are the vertices]

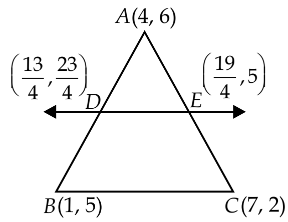
$= \frac{1}{2}[-12(-6+3) + (-13)(-3+2) + (-10)(-2+6)]$

$= \frac{1}{2}[-12(-3) + (-13)(-1) + (-10)4]$

$= \frac{1}{2}[36 + 13 - 40] = \frac{9}{2}$ sq.units

Thus, in both cases, the area of ΔPQR is the same.

6. We have, $\frac{AD}{AB} = \frac{1}{4}$



$\Rightarrow \quad \frac{AB}{AD} = \frac{4}{1} \Rightarrow \frac{AD + DB}{AD} = \frac{4}{1}$

$\Rightarrow \quad \frac{AD}{AD} + \frac{DB}{AD} = \frac{4}{1}$

$\Rightarrow \quad 1 + \frac{DB}{AD} = 1 + \frac{3}{1} \Rightarrow \frac{DB}{AD} = \frac{3}{1}$

$\Rightarrow \quad AD : DB = 1 : 3$

Thus, the point D divides AB in the ratio 1 : 3.

$\therefore \quad$ The coordinates of D are

$\left[\frac{(1\times 1) + (3\times 4)}{1+3}, \frac{(1\times 5) + (3\times 6)}{1+3}\right]$

$= \left[\frac{1+12}{4}, \frac{5+18}{4}\right] = \left[\frac{13}{4}, \frac{23}{4}\right]$

Similarly, $AE : EC = 1 : 3$ *i.e.*, E divides AC in the ratio 1 : 3.

$\therefore \quad$ Coordinates of E are

$\left[\frac{(1\times 7) + (3\times 4)}{1+3}, \frac{1\times 2 + 3\times 6}{1+3}\right]$

$= \left[\frac{7+12}{4}, \frac{2+18}{4}\right] = \left[\frac{19}{4}, 5\right]$

Now, area of ΔADE

$= \frac{1}{2}\left[4\left(\frac{23}{4} - 5\right) + \frac{13}{4}(5-6) + \frac{19}{4}\left(6 - \frac{23}{4}\right)\right]$

$= \frac{1}{2}\left[(23-20) + \frac{13}{4}(-1) + \frac{19}{4}\left(\frac{24-23}{4}\right)\right]$

$= \frac{1}{2}\left[3 - \frac{13}{4} + \frac{19}{16}\right] = \frac{1}{2}\left[\frac{48-52+19}{16}\right] = \frac{15}{32}$ sq.units

Area of $\Delta ABC = \frac{1}{2}[4(5-2) + 1(2-6) + 7(6-5)]$

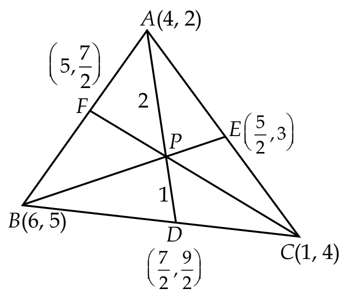
$= \frac{1}{2}[(4\times 3) + 1(-4) + 7\times 1]$

$= \frac{1}{2}[12 + (-4) + 7] = \frac{1}{2}(15) = \frac{15}{2}$ sq.units

Now, $\frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta ABC)} = \frac{\frac{15}{32}}{\frac{15}{2}} = \frac{15}{32} \times \frac{2}{15} = \frac{1}{16}$

$\Rightarrow \quad \text{ar}(\Delta ADE) : \text{ar}(\Delta ABC) = 1 : 16.$

7. We have the vertices of ΔABC as $A(4, 2)$, $B(6, 5)$ and $C(1, 4)$.



(i) Since AD is a median

$\therefore \quad D$ is the mid-point of BC.

$\therefore$ Coordinates of D are $\left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$

(ii) Since $AP : PD = 2 : 1$ *i.e.*, P divides AD in the ratio $2 : 1$.

$\therefore$ Coordinates of P are

$$\left(\frac{2\left(\frac{7}{2}\right)+(1\times 4)}{2+1}, \frac{2\left(\frac{9}{2}\right)+1\times 2}{2+1}\right) = \left(\frac{7+4}{3}, \frac{9+2}{3}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iii) Since, BE is the median.

$\therefore$ Coordinates of E are $\left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

$BQ : QE = 2 : 1 \Rightarrow$ The point Q divides BE in the ratio $2 : 1$.

$\therefore$ Coordinates of Q are

$$\left(\frac{2\left(\frac{5}{2}\right)+1\times 6}{2+1}, \frac{(2\times 3)+(1\times 5)}{2+1}\right)$$

$$= \left(\frac{5+6}{3}, \frac{6+5}{3}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

Since, CF is the median.

$\therefore$ Coordinates of F are $\left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$

Also, $CR : RF = 2 : 1$

$\Rightarrow$ The point R divides CF in the ratio $2 : 1$

So, Coordinates of R are

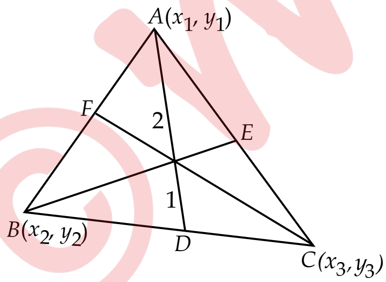
$$\left(\frac{2\times 5+1\times 1}{2+1}, \frac{2\times\frac{7}{2}+1\times 4}{2+1}\right)$$

$$= \left(\frac{10+1}{3}, \frac{7+4}{3}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iv) We observe that P, Q and R represent the same point.

(v) Here, we have $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of ΔABC. Let AD, BE and CF are its medians.

$\therefore$ D, E and F are the mid-points of BC, CA and AB respectively.



We know, the centroid is a point on a median, dividing it in the ratio $2 : 1$.

Considering the median AD, coordinates of D are

$$\left[\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right]$$

Let G be the centroid.

$\therefore$ Coordinates of the centroid are

$$\left[\frac{(1\times x_1)+2\left(\frac{x_2+x_3}{2}\right)}{1+2}, \frac{(1\times y_1)+2\left(\frac{y_2+y_3}{2}\right)}{1+2}\right]$$

$$= \left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right]$$

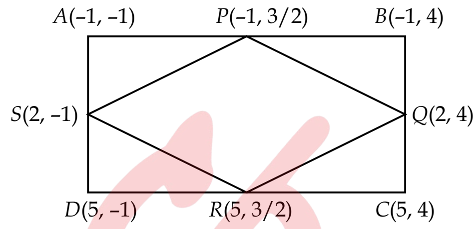
Similarly, considering the other medians we find that in each the coordinates of G are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$

i.e., The coordinates of the centroid are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

8. We have a rectangle whose vertices are $A(-1, -1)$, $B(-1, 4)$, $C(5, 4)$ and $D(5, -1)$.



$\because$ P is mid-point of AB

$\therefore$ Coordinates of P are $\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

Similarly, coordinates of Q are $\left(\frac{-1+5}{2}, \frac{4+4}{2}\right) = (2, 4)$

Coordinates of R are $\left(\frac{5+5}{2}, \frac{-1+4}{2}\right) = \left(5, \frac{3}{2}\right)$

Coordinates of S are $\left(\frac{-1+5}{2}, \frac{-1-1}{2}\right) = (2, -1)$

Now, $PQ = \sqrt{(2+1)^2+\left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$ units

$QR = \sqrt{(2-5)^2+\left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$ units

$RS = \sqrt{(2-5)^2+\left(-1+\left(-\frac{3}{2}\right)\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$ units

$SP = \sqrt{(2+1)^2+\left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$ units

$PR = \sqrt{(5+1)^2+\left(\frac{3}{2}-\frac{3}{2}\right)^2} = \sqrt{6^2+0} = 6$ units

$QS = \sqrt{(2-2)^2+(4+1)^2} = \sqrt{0+5^2} = 5$ units

We see that $PQ = QR = RS = SP$ *i.e.*, all sides of quadrilateral $PQRS$ are equal.

$\therefore$ It can be a square or a rhombus.

But its diagonals are not equal.

i.e., $PR \neq QS$

$\therefore$ $PQRS$ is a rhombus.