

Some Applications of Trigonometry



1. Here, AB is the pole and AC is the rope tied to the point C on the ground.

In right $\triangle ABC$,

$$\frac{AB}{AC} = \sin 30^\circ \Rightarrow \frac{AB}{AC} = \frac{1}{2} \Rightarrow \frac{AB}{20} = \frac{1}{2}$$

$$\Rightarrow AB = 20 \times \frac{1}{2} = 10 \text{ m}$$

Thus, the required height of the pole is 10 m.

2. Let the tree OP is broken at A and its top is touching the ground at B .

Now, in right $\triangle AOB$,

$$\frac{AO}{OB} = \tan 30^\circ$$

$$\Rightarrow \frac{AO}{8} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AO = \frac{8}{\sqrt{3}} \text{ m}$$

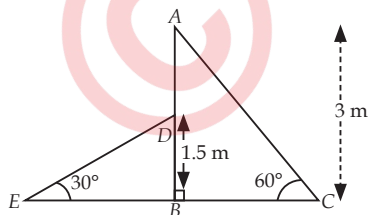
$$\text{Also, } \frac{AB}{OB} = \sec 30^\circ$$

$$\Rightarrow \frac{AB}{8} = \frac{2}{\sqrt{3}} \Rightarrow AB = \frac{2 \times 8}{\sqrt{3}} = \frac{16}{\sqrt{3}} \text{ m}$$

Now, height of the tree $OP = OA + AP = OA + AB$
[$\because AP = AB$]

$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 8\sqrt{3} \text{ m}$$

3. In the figure, DE is the slide for younger children, whereas AC is the slide for elder children.



In right $\triangle ABC$,

$$\therefore \frac{AB}{AC} = \sin 60^\circ$$

$$\Rightarrow \frac{3}{AC} = \frac{\sqrt{3}}{2} \Rightarrow AC = \frac{2 \times 3}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$

Again, in right $\triangle BDE$,

$$\frac{DE}{BD} = \csc 30^\circ = 2$$

$$\Rightarrow \frac{DE}{1.5} = 2 \Rightarrow DE = 2 \times 1.5 = 3 \text{ m}$$

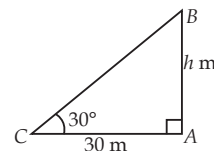
Thus, the lengths of slides are 3 m and $2\sqrt{3}$ m.

4. In right $\triangle ABC$, AB = height of the tower and point C is 30 m away from the foot of the tower.

$$\therefore AC = 30 \text{ m}$$

$$\text{Now, } \frac{AB}{AC} = \tan 30^\circ \Rightarrow \frac{h}{30} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$$



Thus, the required height of the tower is $10\sqrt{3}$ m.

5. Let OB = Length of the string
 $AB = 60$ m = Height of the kite.

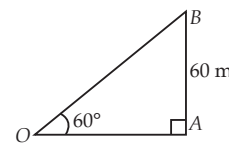
In right $\triangle AOB$,

$$\frac{OB}{AB} = \csc 60^\circ$$

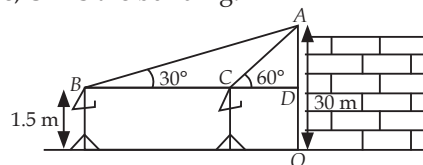
$$\Rightarrow \frac{OB}{60} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow OB = \frac{2 \times 60}{\sqrt{3}} = \frac{120 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 40\sqrt{3} \text{ m}$$

Thus, length of the string is $40\sqrt{3}$ m.



6. Here, OA is the building.



In right $\triangle ABD$,

$$\frac{AD}{BD} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BD = AD\sqrt{3} = 28.5\sqrt{3} \text{ m}$$

$$[\because AD = 30 \text{ m} - 1.5 \text{ m} = 28.5 \text{ m}]$$

Also, in right $\triangle ACD$,

$$\frac{AD}{CD} = \tan 60^\circ = \sqrt{3} \Rightarrow CD = \frac{AD}{\sqrt{3}} = \frac{28.5}{\sqrt{3}} \text{ m}$$

$$\text{Now, } BC = BD - CD = 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$$

$$= 28.5 \left[\sqrt{3} - \frac{1}{\sqrt{3}} \right] = 28.5 \left[\frac{3-1}{\sqrt{3}} \right]$$

$$= 28.5 \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{28.5 \times 2 \times \sqrt{3}}{3}$$

$$= 9.5 \times 2 \times \sqrt{3} = 19\sqrt{3} \text{ m}$$

Thus, the distance walked by the boy towards the building is $19\sqrt{3}$ m.

7. Let BC be the building of height 20 m and CD be the tower of height x m.

Let the point A be at a distance of y m from the foot of the building.

Now, in right $\triangle ABC$,

$$\frac{BC}{AB} = \tan 45^\circ = 1$$

$$\Rightarrow \frac{20}{y} = 1 \Rightarrow y = 20 \text{ i.e., } AB = 20 \text{ m.}$$

Now, in right $\triangle ABD$,

$$\frac{BD}{AB} = \tan 60^\circ$$

$$\Rightarrow \frac{20+x}{20} = \sqrt{3} \Rightarrow 20+x = 20\sqrt{3}$$

$$\Rightarrow x = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1) \text{ m.}$$

Thus, the height of the tower is $20(\sqrt{3} - 1)$ m.

8. In the figure, DC represents the statue of height 1.6 m and BC represents the pedestal of height h m.

Now, in right $\triangle ABC$,

$$\frac{AB}{BC} = \cot 45^\circ = 1$$

$$\Rightarrow \frac{AB}{h} = 1 \Rightarrow AB = h \text{ m}$$

Now, in right $\triangle ABD$,

$$\frac{BD}{AB} = \tan 60^\circ$$

$$\Rightarrow \frac{h+1.6}{h} = \sqrt{3} \Rightarrow h+1.6 = \sqrt{3}h$$

$$\Rightarrow h(\sqrt{3} - 1) = 1.6 \Rightarrow h = \frac{1.6}{\sqrt{3} - 1} = \frac{1.6}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$\Rightarrow h = \frac{1.6(\sqrt{3} + 1)}{2} = 0.8(\sqrt{3} + 1)$$

Thus, the height of the pedestal is $0.8(\sqrt{3} + 1)$ m.

9. In the figure, let AB be the building of height h m and CD be the tower of height 50 m.

Now, in right $\triangle ABC$,

$$\frac{AC}{AB} = \cot 30^\circ = \sqrt{3}$$

$$\Rightarrow \frac{AC}{h} = \sqrt{3} \Rightarrow AC = h\sqrt{3} \quad \dots(1)$$

In right $\triangle DCA$,

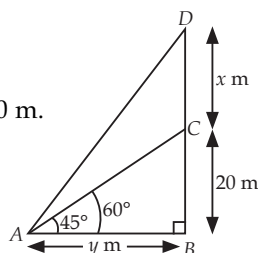
$$\frac{DC}{AC} = \tan 60^\circ$$

$$\Rightarrow \frac{50}{AC} = \sqrt{3} \Rightarrow AC = \frac{50}{\sqrt{3}} \quad \dots(2)$$

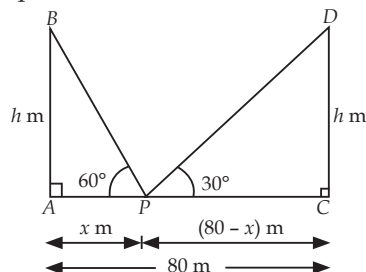
From (1) and (2), we get

$$\sqrt{3}h = \frac{50}{\sqrt{3}} \Rightarrow h = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3}$$

Thus, the height of the building is $16\frac{2}{3}$ m.



10. In the figure, let AB and CD are the poles of equal height h m and P be the point on the road at a distance of x m from the pole AB .



$$\therefore CP = (80 - x)$$

Now, in right $\triangle APB$,

$$\frac{AB}{AP} = \tan 60^\circ$$

$$\Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow h = x\sqrt{3} \quad \dots(1)$$

Again in right $\triangle CPD$,

$$\frac{CD}{CP} = \tan 30^\circ$$

$$\Rightarrow \frac{h}{(80 - x)} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{80 - x}{\sqrt{3}} \quad \dots(2)$$

From (1) and (2), we get

$$\sqrt{3}x = \frac{80 - x}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3} \times \sqrt{3} \times x = 80 - x \Rightarrow 3x = 80 - x$$

$$\Rightarrow 3x + x = 80 \Rightarrow 4x = 80 \Rightarrow x = \frac{80}{4} = 20$$

$$\therefore CP = 80 - x = 80 - 20 = 60$$

Now, from (1), we have $h = 20\sqrt{3}$

Thus, the required point is 20 m away from the first pole and 60 m away from the second pole and height of each pole is $20\sqrt{3}$ m.

11. In the figure, let AB be the TV tower of height h m and C be the point on the other bank of the canal at a distance of x m from B . D be another point 20 m away from point C .

$$\therefore BC = x \text{ m and } CD = 20 \text{ m}$$

Now, in right $\triangle ABC$,

$$\frac{AB}{BC} = \tan 60^\circ \Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow h = \sqrt{3}x \text{ m} \quad \dots(1)$$

In right $\triangle ABD$,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\Rightarrow \frac{h}{x+20} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{x+20}{\sqrt{3}} \text{ m} \quad \dots(2)$$

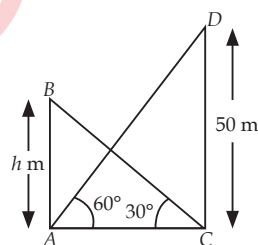
From (1) and (2), we get

$$\sqrt{3}x = \frac{x+20}{\sqrt{3}} \Rightarrow 3x = x+20$$

$$\Rightarrow 3x - x = 20 \Rightarrow 2x = 20 \Rightarrow x = \frac{20}{2} = 10 \text{ m}$$

Now, from (1), we get $h = 10\sqrt{3}$ m

Thus, the height of the tower is $10\sqrt{3}$ m and width of the canal is 10 m.



12. In the figure, let AB be the building of height 7 m.

Let $BC = AE = x$ m

Let CD be the height of the cable tower and $DE = h$ m.

$\therefore$ In right $\triangle DAE$,

$$\frac{DE}{EA} = \tan 60^\circ \Rightarrow \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow h = \sqrt{3}x \quad \dots(1)$$

Again, in right $\triangle ABC$,

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\Rightarrow \frac{7}{x} = 1 \Rightarrow x = 7 \quad \dots(2)$$

From (1) and (2), we get

$$h = 7\sqrt{3} \Rightarrow DE = 7\sqrt{3}$$

$$\therefore CD = CE + ED = (7 + 7\sqrt{3}) = 7(1 + \sqrt{3})$$

Thus, the height of the cable tower is $7(1 + \sqrt{3})$ m.

13. In the figure, let AB be the light house.

$\therefore AB = 75$ m

Let the positions of two ships be C and D such that angle of depression from A are 45° and 30° respectively.

Now, in right $\triangle ABC$,

$$\frac{AB}{BC} = \tan 45^\circ \Rightarrow \frac{75}{BC} = 1 \Rightarrow BC = 75 \text{ m}$$

Again, in right $\triangle ABD$,

$$\frac{AB}{BD} = \tan 30^\circ \Rightarrow \frac{75}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 75\sqrt{3} \text{ m}$$

Now, the distance between the two ships = CD

$$= BD - BC = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1) \text{ m}$$

Thus, the required distance between the ships is $75(\sqrt{3} - 1)$ m.

14. In the figure, let C be the position of the girl. A and P are two positions of the balloon. CD is the horizontal line from the eyes of the girl.

Here, $PD = AB = 88.2 \text{ m} - 1.2 \text{ m} = 87 \text{ m}$

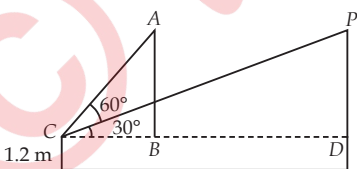
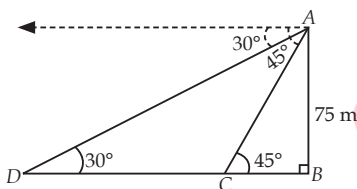
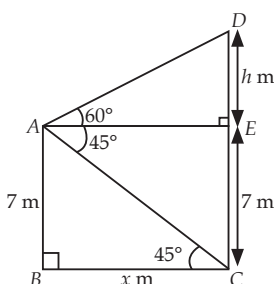
In right $\triangle ABC$,

$$\frac{AB}{BC} = \tan 60^\circ \Rightarrow \frac{87}{BC} = \sqrt{3} \Rightarrow BC = \frac{87}{\sqrt{3}} \text{ m}$$

In right $\triangle PDC$,

$$\frac{PD}{CD} = \tan 30^\circ \Rightarrow \frac{87}{CD} = \frac{1}{\sqrt{3}} \Rightarrow CD = 87\sqrt{3} \text{ m}$$

Now, $BD = CD - BC$



$$= 87\sqrt{3} - \frac{87}{\sqrt{3}} = 87\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 87 \times \left(\frac{3-1}{\sqrt{3}}\right) = \frac{2 \times 87}{\sqrt{3}}$$

$$= \frac{2 \times 87 \times \sqrt{3}}{3} = 2 \times 29 \times \sqrt{3} = 58\sqrt{3} \text{ m}$$

Thus, the required distance between the two positions of the balloon is $58\sqrt{3}$ m.

15. In the figure, let AB be the tower and C, D be the two positions of the car.

In right $\triangle ABD$,

$$\frac{AB}{AD} = \tan 60^\circ$$

$$\Rightarrow \frac{AB}{AD} = \sqrt{3} \Rightarrow AB = \sqrt{3}AD \quad \dots(1)$$

In right $\triangle ABC$,

$$\frac{AB}{AC} = \tan 30^\circ$$

$$\Rightarrow \frac{AB}{AC} = \frac{1}{\sqrt{3}} \Rightarrow AB = \frac{AC}{\sqrt{3}} \quad \dots(2)$$

From (1) and (2), we get

$$\sqrt{3}AD = \frac{AC}{\sqrt{3}}$$

$$\Rightarrow AC = \sqrt{3} \times \sqrt{3} \times AD = 3AD$$

Now, $CD = AC - AD = 3AD - AD = 2AD$

Since the distance $2AD$ is covered in 6 seconds,

$\therefore$ The distance AD will be covered in $\frac{6}{2}$ i.e., 3 seconds.

Thus, the time taken by the car to reach the tower from D is 3 seconds.

16. In the figure, let AB be the tower of height h m. C and D are the two points at a distance of 9 m and 4 m respectively from A . Let $\angle ACB = \theta$ $\therefore \angle ADB = 90^\circ - \theta$

In right $\triangle ABC$,

$$\frac{AB}{AC} = \tan \theta$$

$$\Rightarrow \frac{h}{9} = \tan \theta \quad \dots(1)$$

In right $\triangle ABD$,

$$\frac{AB}{AD} = \tan(90^\circ - \theta) = \cot \theta$$

$$\Rightarrow \frac{h}{4} = \cot \theta \quad \dots(2)$$

Multiplying (1) and (2), we get

$$\frac{h}{9} \times \frac{h}{4} = \tan \theta \times \cot \theta$$

$$\Rightarrow \frac{h^2}{36} = 1 \Rightarrow h^2 = 36 \Rightarrow h = \pm 6 \quad \therefore h = 6$$

Thus, the height of the tower is 6 m.

