CHAPTER 11

Constructions

TRY YOURSELF — SOLUTIONS

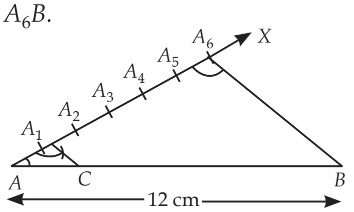
1. **Steps of Construction**

Step 1 : Draw a line segment $AB = 12$ cm.

Step 2 : Draw a ray AX making an acute angle with the line segment AB.

Step 3 : Locate $(1 + 5 =)$ 6 points $A_1, A_2, \ldots, A_6$ on AX such that $AA_1 = A_1A_2 = \ldots = A_5A_6$.

Step 4 : Join A_6B.



Step 5 : Through A_1, draw A_1C parallel to A_6B, meeting AB at C, such that $\angle AA_6B = \angle AA_1C$.

Hence, $AC : CB = 1 : 5$.

Justification : In ΔABA_6, we observe that A_1C is parallel to A_6B, therefore by basic proportionality theorem, we have

$$\frac{AA_1}{A_1A_6} = \frac{AC}{BC} \quad i.e., \quad \frac{AC}{BC} = \frac{1}{5} \qquad \left[\text{By construction, we get } \frac{AA_1}{A_1A_6} = \frac{1}{5}\right]$$

$\Rightarrow \quad AC : CB = 1 : 5.$

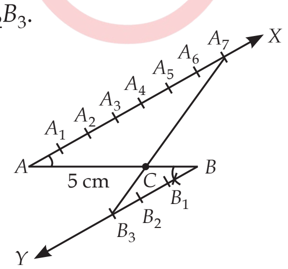
2. **Steps of Construction**

Step 1 : Draw a line segment $AB = 5$ cm.

Step 2 : Draw a ray AX making an acute angle with AB.

Step 3 : Draw a ray BY parallel to AX such that $\angle ABY = \angle BAX$.

Step 4 : Locate points $A_1, A_2, A_3, A_4, \ldots A_7$ on AX and B_1, B_2, B_3 on BY such that $AA_1 = A_1A_2 = A_2A_3 = \ldots = A_6A_7 = BB_1 = B_1B_2 = B_2B_3$.



Step 5 : Join A_7B_3 that intersects AB at C. Then, $AC : CB = 7 : 3$.

On measuring two parts, we get $AC = 3.5$ cm and $CB = 1.5$ cm.

Justification : In ΔACA_7 and ΔBCB_3, we have

$\angle A_7AC = \angle CBB_3$ $\quad [\because \angle XAB = \angle ABY]$

$\angle ACA_7 = \angle BCB_3$ $\quad$ [Vertically opposite angles]

$\therefore \quad \Delta ACA_7 \sim \Delta BCB_3$ $\quad$ [AA similarity]

$$\Rightarrow \quad \frac{AC}{BC} = \frac{AA_7}{BB_3}$$

$$\Rightarrow \quad \frac{AC}{BC} = \frac{7}{3} \qquad \left[\because \frac{AA_7}{BB_3} = \frac{7}{3}\right]$$

3. **Steps of Construction**

Step 1 : Draw a line segment $AB = 4.9$ cm.

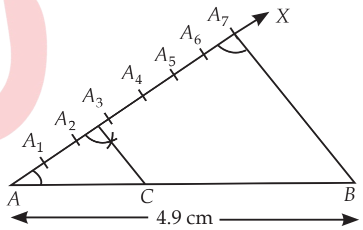
Step 2 : Draw a ray AX making an acute angle with AB.

Step 3 : Locate $(3 + 4 =)$ 7 points $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ on AX such that $AA_1 = A_1A_2 = \ldots = A_6A_7$.

Step 4 : Join A_7B.

Step 5 : Through A_3, draw A_3C parallel to A_7B, meeting AB at C, such that $\angle AA_7B = \angle AA_3C$.

Hence, $AC = CB = 3 : 4$.



On measuring two parts, we get $AC = 2.1$ cm and $CB = 2.8$ cm.

Justification : In ΔABA_7, we observe that A_3C is parallel to A_7B, therefore, by basic proportionality theorem, we have

$$\frac{AA_3}{A_3A_7} = \frac{AC}{BC} \quad i.e., \quad \frac{AC}{CB} = \frac{3}{4} \qquad \left[\text{By construction, we get } \frac{AA_3}{A_3A_7} = \frac{3}{4}\right]$$

$\Rightarrow \quad AC : CB = 3 : 4$

4. **Steps of Construction**

Step 1 : Construct a ΔABC in which $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 70°$.

Step 2 : Below BC, draw a ray BX such that $\angle CBX$ is an acute angle.

Step 3 : Along BX, mark off seven points $B_1, B_2, B_3, B_4, B_5, B_6$ and B_7 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.

Step 4 : Join B_7C.

Step 5 : From B_5, draw $B_5D \parallel B_7C$, meeting BC at D.

Step 6 : From D, draw $ED \parallel AC$, meeting BA at E.

Thus, ΔEBD is the required triangle.

Justification : Since $ED \parallel AC$

$\therefore \quad \Delta EBD \sim \Delta ABC$

$$\Rightarrow \quad \frac{EB}{AB} = \frac{BD}{BC} = \frac{ED}{AC} \qquad ...(i)$$

Now, in ΔBDB_5 and ΔBCB_7

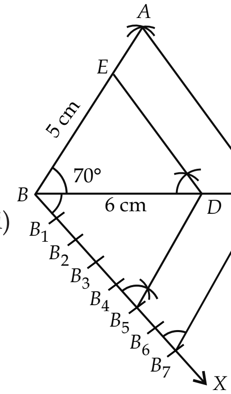
Since, $B_5D \parallel B_7C$

$\therefore \quad \Delta BDB_5 \sim \Delta BCB_7$

$$\Rightarrow \quad \frac{BD}{BC} = \frac{B_5D}{B_7C} = \frac{BB_5}{BB_7} = \frac{5}{7} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{EB}{AB} = \frac{BD}{BC} = \frac{ED}{AC} = \frac{5}{7}$$



5. Steps of Construction

Step 1 : Construct ΔABC in which BC = 6 cm, $\angle C$ = 90° and AC = 8 cm.

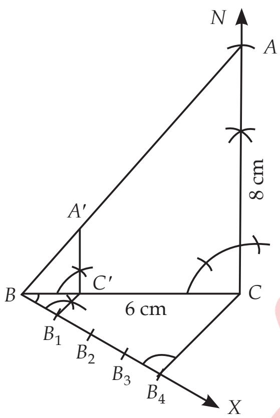
Step 2 : Below BC, draw any ray BX making an acute angle with it.

Step 3 : Locate 4 points B_1, B_2, B_3 and B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$ and join B_4C.

Step 4 : Draw a line through B_1 parallel to B_4C intersecting BC at C'.

Step 5 : Draw a line through C' parallel to the line CA intersecting BA at A'.

Hence, $\Delta A'BC'$ is the required similar triangle whose sides are $\frac{1}{4}$ times the corresponding sides of ΔABC.



6. Steps of Construction

Step 1 : Construct an equilateral triangle PQR with sides 4.2 cm.

Step 2 : Draw a ray QX such that $\angle RQX$ is an acute angle.

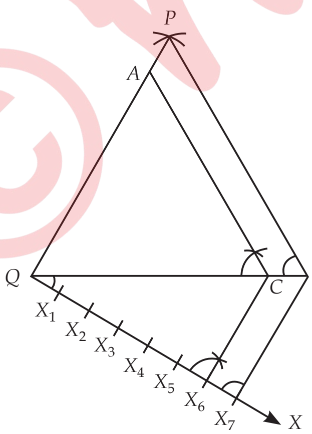
Step 3 : Along QX, mark off seven points X_1, X_2, X_3, X_4, X_5, X_6 and X_7 such that $QX_1 = X_1X_2 = X_2X_3 = X_3X_4 = X_4X_5 = X_5X_6 = X_6X_7$.

Step 4 : Join RX_7.

Step 5 : From X_6, draw $X_6C \parallel X_7R$, meeting QR at C.

Step 6 : From C, draw $AC \parallel PR$, meeting PQ at A.

Thus, ΔAQC is the required similar triangle.



7. Steps of Construction

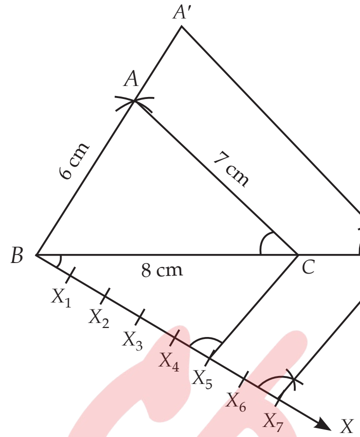
Step 1 : Construct a ΔABC such that AB = 6 cm, BC = 8 cm and AC = 7 cm.

Step 2 : Draw a ray BX such that $\angle CBX$ is an acute angle.

Step 3 : Mark 7 points X_1, X_2, X_3, X_4, X_5, X_6 and X_7 on BX such that $BX_1 = X_1X_2 = X_2X_3 = X_3X_4 = X_4X_5 = X_5X_6 = X_6X_7$.

Step 4 : Join X_5 to C.

Step 5 : Draw a line through X_7 intersecting BC produced at C' such that $X_5C \parallel X_7C'$.



Step 6 : Draw a line through C', parallel to CA to intersect BA produced at A'. Thus, $\Delta A'BC'$ is the required similar triangle.

8. Steps of Construction

Step 1 : Construct the given ΔABC.

Step 2 : Draw a ray BX such that $\angle CBX$ is an acute angle.

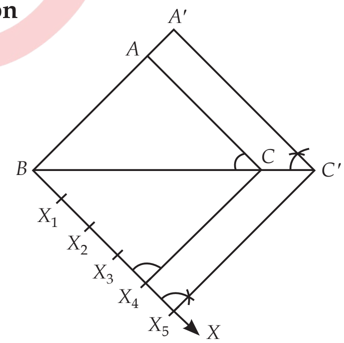
Step 3 : Mark 5 points X_1, X_2, X_3, X_4, X_5 on BX such that $BX_1 = X_1X_2 = X_2X_3 = X_3X_4 = X_4X_5$.

Step 4 : Join X_4 to C.

Step 5 : Draw a line through X_5 intersecting BC produced at C′ such that $X_4C \parallel X_5C'$.

Step 6 : Draw a line through C' parallel to AC intersecting, BA produced at A'.

Thus, $\Delta A'BC'$ is the required triangle.



9. Steps of Construction

Step 1 : Draw a circle of radius 5.2 cm and take a point T on the circle.

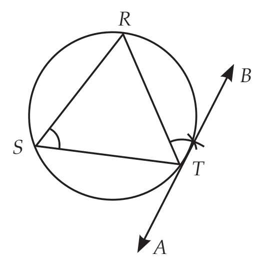
Step 2 : Draw a chord TR through the point T in the circle.

Step 3 : Take a point S on the major arc and join ST and SR.

Step 4 : On taking TR as base, construct $\angle RTB = \angle RST$.

Step 5 : Produce BT to A.

Thus, ATB is the required tangent at point T.



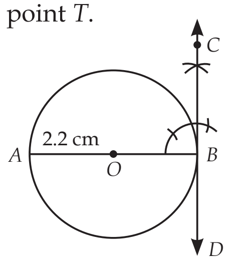
10. Steps of Construction

Step 1 : Draw a circle with O as centre and radius 2.2 cm.

Step 2 : Draw diameter AOB.

Step 3 : Take OB as base and construct $\angle OBC$ = 90° at B.

Step 4 : Produce CB to D to get the required tangent CBD.



11. Steps of Construction

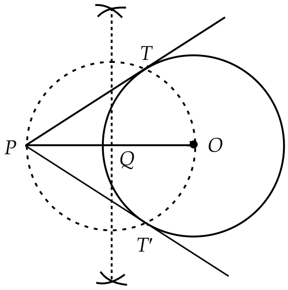
Step 1 : Draw a circle with O as centre and radius 5 cm.

Step 2 : Mark a point P outside the circle such that OP = 7 cm.

Step 3 : Join OP and draw its perpendicular bisector, which cuts OP at M.

Step 4 : Draw a circle with M as centre and radius equal to MP to intersect the given circle at the points T and T'.

Step 5 : Join PT and PT'.



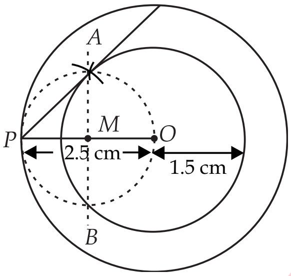
Hence, PT and PT' are the required tangents.

12. Steps of Construction

Step 1 : Draw two concentric circles with centre O and radii 1.5 cm and 2.5 cm.

Step 2 : Take any point P on the outer circle.

Step 3 : Join PO and bisect it and mark the mid-point of PO as M.



Step 4 : Taking M as centre and OM or MP as radius, draw a circle such that this circle intersects the inner circle (of radius 1.5 cm) at A and B.

Step 5 : Join AP.

Thus, PA is the required tangent.

By measurement, we have PA = 2 cm

Verification:

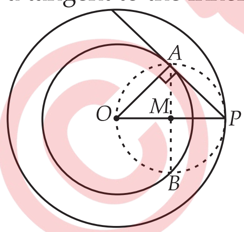
Join OA. As PO is diameter,

$\therefore \quad \angle PAO = 90°$ [Angle in a semi-circle]

$\Rightarrow \quad PA \perp OA$

$\because OA$ is a radius of the inner circle.

$\therefore \quad PA$ has to be a tangent to the inner circle.



In right ΔPAO,

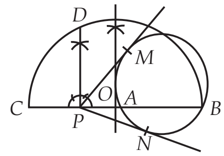
PO = 2.5 cm, OA = 1.5 cm.

$$\therefore \quad PA = \sqrt{(2.5)^2 - (1.5)^2} = \sqrt{6.25 - 2.25} = \sqrt{4} = 2\,\text{cm}$$

13. Steps of constructions :

Step 1 : Draw a circle of radius 4.5 cm and take a point P outside the circle.

Step 2 : Through P, draw a secant PAB which intersects the circle at A and B and extend it to C in opposite direction of AB such that $PC = PA$.



Step 3 : Now, bisect BC and take its mid-point as O. Draw a semi-circle with centre O and radius OB (or OC).

Step 4 : Draw $PD \perp BC$ which intersects the semi-circle at D.

Step 5 : With P as centre and radius PD, draw two arcs which intersects the given circle at points M and N.

Step 6 : Join PM and PN.

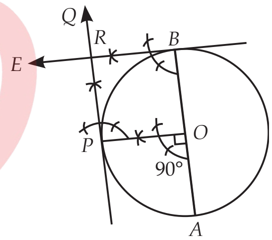
Thus, PM and PN are the required tangents to the given circle.

14. Steps of Construction

Step 1 : Draw a circle with centre O and radius 3 cm.

Step 2 : Draw any diameter AOB.

Step 3 : Take a point P on the circle such that $\angle AOP = 90°$.



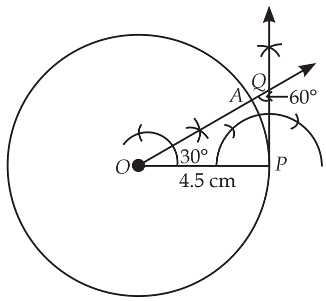
Step 4 : Draw $PQ \perp OP$ and $BE \perp OB$. Let PQ and BE intersect at R.

Hence, RB and RP are the required tangents inclined at an angle of 90°.

15. Steps of Construction

Step 1 : Draw a circle with centre O and radius 4.5 cm.

Step 2 : Construct an angle $\angle AOP$ equal to complement of 60° *i.e.*, $\angle AOP = 30°$.



Step 3 : Draw perpendicular to OP at P which meet OA produced at Q.

Clearly, PQ is the required tangent such that $\angle OQP$ = 60°.