

Areas Related to Circles

CHAPTER 12



TRY YOURSELF

SOLUTIONS

1. Let d be the diameter of the circle whose area is equal to sum of areas of two circles of diameters $d_1 = 20$ cm and $d_2 = 48$ cm.

$$\therefore \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{d_1}{2}\right)^2 + \pi \left(\frac{d_2}{2}\right)^2$$

$$\Rightarrow \left(\frac{d}{2}\right)^2 = \left(\frac{20}{2}\right)^2 + \left(\frac{48}{2}\right)^2$$

$$\Rightarrow \frac{d^2}{4} = 100 + 576 \Rightarrow \frac{d^2}{4} = 676 \Rightarrow d^2 = 676 \times 4$$

$$\Rightarrow d = 26 \times 2 = 52 \text{ cm}$$

2. Let r cm be the radius of the protractor.

Given, perimeter of semi-circular protractor = 180 cm

$$\Rightarrow (\pi + 2)r = 180$$

$$\Rightarrow \left(\frac{22}{7} + 2\right)r = 180 \Rightarrow \frac{36}{7}r = 180 \Rightarrow r = \frac{180 \times 7}{36} = 35$$

Hence, diameter of protractor = $2r = 2 \times 35 = 70$ cm

3. Let r and R be the inner and outer radius of circular track.

Given, circumference of inner track = 308 m

$$\Rightarrow 2\pi r = 308$$

$$\Rightarrow r = \frac{308}{2\pi} = \frac{308}{2 \times 22} \times 7 = 7 \times 7 = 49 \text{ m}$$

$\therefore$ Outer radius, $R = r + \text{width of track} = 49 + 7 = 56$ m

$\therefore$ Circumference of outer track = $2\pi R$

$$= 2 \times \frac{22}{7} \times 56 = 352 \text{ m}$$

Now, cost of putting fence of 1 m = ₹ 30

$\therefore$ Cost of putting fence of 352 m = ₹ $(30 \times 352) = ₹ 10560$

4. Let R and r be the radius of outer and inner track respectively.

Given, outer circumference = 352 m

$$\Rightarrow 2\pi R = 352$$

$$\Rightarrow R = \frac{352}{2\pi} = \frac{352}{2 \times 22} \times 7 = 8 \times 7 = 56 \text{ m}$$

Also, inner circumference = 264 m

$$\Rightarrow 2\pi r = 264$$

$$\Rightarrow r = \frac{264}{2\pi} = \frac{264}{2 \times 22} \times 7 = 6 \times 7 = 42 \text{ m}$$

$\therefore$ Width of track = $R - r = 56 - 42 = 14$ m

Hence, area of track = $\pi(R^2 - r^2) = \pi(R + r)(R - r)$

$$= \frac{22}{7} (56 + 42) \times 14 = 22 \times 98 \times 2 = 4312 \text{ m}^2$$

5. Let r cm be the radius of circle.

Given, circumference of circle - diameter of circle = 75 cm

$$\Rightarrow 2\pi r - 2r = 75$$

$$\Rightarrow 2r \left(\frac{22}{7} - 1\right) = 75 \Rightarrow 2r \times \frac{15}{7} = 75 \Rightarrow r = \frac{75 \times 7}{15 \times 2} = \frac{35}{2}$$

$\therefore$ Area of circle = πr^2

$$= \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} = \frac{11 \times 5 \times 35}{2} = 962.5 \text{ cm}^2$$

6. Radius of wheel = 50 cm

$$\therefore \text{Circumference of wheel} = 2\pi r = \left(2 \times \frac{22}{7} \times 50\right) \text{ cm}$$

Distance travelled = 44 km = $(44 \times 1000 \times 100) \text{ cm}$

$$\therefore \text{Number of revolutions} = \frac{\text{Distance travelled}}{\text{Circumference of wheel}}$$

$$= \frac{44 \times 1000 \times 100 \times 7}{2 \times 22 \times 50} = 14000$$

7. Given that, radius of a circle (r) = 28 cm and measure of central angle (θ) = 45°

$$\therefore \text{Area of a sector of a circle} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times (28)^2 = 22 \times 4 \times 28 \times \frac{1}{8}$$

$$= 22 \times 14 = 308 \text{ cm}^2$$

Area of major sector = $\pi r^2 - \text{Area of minor sector}$

$$= \frac{22}{7} \times 28 \times 28 - 308 = 2464 - 308 = 2156 \text{ cm}^2$$

8. Angle described by hour hand in 1 hour = 30°

Time covered from 8 a.m. to 11 a.m. = 3 hours

$\therefore$ Angle described by hour hand in 3 hours = $3 \times 30^\circ = 90^\circ$

Area swept by hour hand in 3 hours = Area of sector of angle 90° in a circle of radius 5.6 cm

$$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (5.6)^2 = \frac{1}{4} \times \frac{22}{7} \times 5.6 \times 5.6 = 24.64 \text{ cm}^2$$

9. Given, length of arc = 26.4 cm

$$\Rightarrow \frac{\theta}{360^\circ} \times 2\pi r = 26.4 \Rightarrow \frac{80^\circ}{360^\circ} \times 2\pi r = 26.4 \quad [\because \theta = 80^\circ]$$

$$\Rightarrow r = \frac{26.4 \times 360^\circ}{2\pi \times 80^\circ} \text{ cm}$$

$$\therefore \text{Area of minor sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

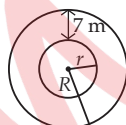
$$= \frac{80^\circ}{360^\circ} \times \pi \times \left(\frac{26.4 \times 360^\circ}{2\pi \times 80^\circ}\right)^2$$

$$= \frac{360^\circ}{80^\circ} \times \frac{7}{4 \times 22} \times 26.4 \times 26.4 = 249.48 \text{ cm}^2$$

10. Length of arc = 22 m

[Given]

$$\Rightarrow \frac{\theta}{360^\circ} \times 2\pi r = 22 \Rightarrow \frac{\theta}{360^\circ} \times 2 \times \frac{22}{7} \times 8.4 = 22$$



$$\Rightarrow \theta = \frac{22 \times 7 \times 360^\circ}{2 \times 22 \times 8.4} = 150^\circ$$

$$\therefore \text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{150^\circ}{360^\circ} \times \frac{22}{7} \times (8.4)^2$$

$$= \frac{5}{12} \times 22 \times 1.2 \times 8.4 = 92.4 \text{ cm}^2$$

11. Let $r = 7.5$ cm and $R = 11$ cm.

$$\therefore \text{Area of shaded region} = \text{Area of sector of radius 11 cm}$$

$$- \text{Area of sector of radius 7.5 cm}$$

$$= \frac{\theta}{360^\circ} \times \pi R^2 - \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{\theta}{360^\circ} \pi (11^2 - 7.5^2) = \frac{120^\circ}{360^\circ} \times \frac{22}{7} (11 + 7.5)(11 - 7.5)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 18.5 \times 3.5 = 67.83 \text{ cm}^2$$

12. Let O be the centre and AB is the chord of the circle having radius 10 cm.

Draw $BL \perp OA$

In right angled $\triangle OLB$,

$$\frac{BL}{OB} = \sin 30^\circ = \frac{1}{2}$$

$$\Rightarrow \frac{BL}{10} = \frac{1}{2} \quad (\because OB = 10 \text{ cm})$$

$$\Rightarrow BL = \frac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$$

Area of the segment APB = Area of the sector $OAPB$
- Area of the $\triangle AOB$

$$= \frac{30^\circ}{360^\circ} \times \pi \times (r)^2 - \frac{1}{2} \times OA \times BL$$

$$= \left\{ \frac{1}{12} \pi \times (10)^2 - \frac{1}{2} \times 10 \times 5 \right\} \text{cm}^2$$

$$= \left(\frac{25\pi}{3} - 25 \right) \text{cm}^2 = 25 \times \left(\frac{\pi}{3} - 1 \right) \text{cm}^2$$

$$= 25 \times \left(\frac{3.14}{3} - 1 \right) \text{cm}^2 = \left(\frac{25 \times 0.14}{3} \right) \text{cm}^2 = 1 \frac{1}{6} \text{ cm}^2$$

13. Draw $BM \perp OA$

In right $\triangle OMB$, $\frac{BM}{OB} = \sin 45^\circ$

$$\Rightarrow \frac{BM}{4\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow BM = 4 \text{ m}$$

Area of shaded region

= Area of sector OAB - Area of $\triangle OAB$

$$= \frac{\theta}{360^\circ} \pi r^2 - \frac{1}{2} \times OA \times BM$$

$$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 4\sqrt{2} \times 4\sqrt{2} - \frac{1}{2} \times 4\sqrt{2} \times 4$$

$$= \frac{1}{8} \times \frac{22}{7} \times 16 \times 2 - 2\sqrt{2} \times 4 = 12.57 - 11.28 = 1.29 \text{ m}^2$$

14. Let O be the centre of circle.

We have, radius (r) = 12 cm and $\theta = 90^\circ$

So, area of sector $OAPB = \frac{\theta}{360^\circ} \pi r^2$

$$= \frac{90^\circ}{360^\circ} \times 3.14 \times 12^2 = 113.04 \text{ cm}^2$$

Now, area of $\triangle AOB$

$$= \frac{1}{2} \times 12 \times 12 = 72 \text{ cm}^2$$

$\therefore$ Area of the minor segment APB = Area of sector

$$OAPB - \text{Area of } \triangle AOB = (113.04 - 72) \text{ cm}^2 = 41.04 \text{ cm}^2$$

$$\text{Area of circle} = \pi r^2 = 3.14 \times 12^2 = 452.16 \text{ cm}^2$$

$\therefore$ Area of major segment ALB = Area of circle

$$- \text{Area of minor segment } APB$$

$$= (452.16 - 41.04) \text{ cm}^2 = 411.12 \text{ cm}^2$$

15. We have, radius (r) = 14 cm and $\theta = 60^\circ$

Area of minor segment = Area of sector $OAPB$

- Area of $\triangle AOB$

$$= \frac{\theta \pi r^2}{360^\circ} - \frac{1}{2} r^2 \sin \theta$$

$$= \frac{60^\circ \times 22 \times 14 \times 14}{7 \times 360^\circ} - \frac{1}{2} \times 14 \times 14 \times \sin 60^\circ$$

$$= \frac{22 \times 14}{3} - 7 \times 14 \times \frac{\sqrt{3}}{2} = 102.67 - 84.87 = 17.8 \text{ cm}^2$$

16. Here, AB is tangent to the circle.

$\therefore \angle OAB = 90^\circ$

In right $\triangle OAB$,

$$\frac{AB}{OA} = \tan 60^\circ \Rightarrow \frac{AB}{7} = \sqrt{3}$$

$$\Rightarrow AB = 7\sqrt{3} \text{ cm}$$

By Pythagoras theorem,

$$OB^2 = OA^2 + AB^2 = 7^2 + (7\sqrt{3})^2 = 49 + 147$$

$$\Rightarrow OB^2 = 196 \Rightarrow OB = 14 \text{ cm}$$

$$\therefore BC = OB - OC = 14 - 7 = 7 \text{ cm}$$

$$\text{Length of arc, } \widehat{AC} = \frac{\theta}{360^\circ} \times 2\pi r = \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 7$$

$$= \frac{1}{6} \times 2 \times 22 = 7.33 \text{ cm}$$

$\therefore$ Perimeter of shaded region = Length of arc

$$\widehat{AC} + AB + BC$$

$$= 7.33 + 7\sqrt{3} + 7 = (14.33 + 7\sqrt{3}) \text{ cm}$$

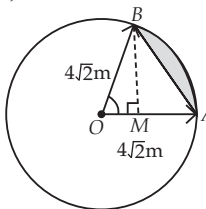
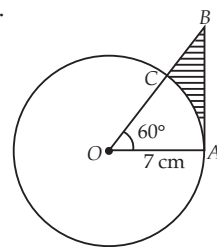
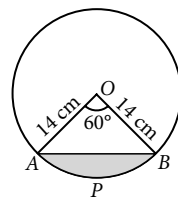
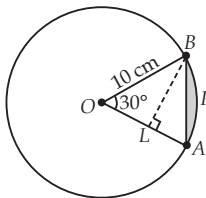
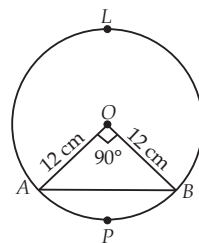
Now, area of shaded region = Area of $\triangle AOB$

- Area of sector OAC

$$= \frac{1}{2} \times OA \times AB - \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{1}{2} \times 7 \times 7\sqrt{3} - \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{49\sqrt{3}}{2} - \frac{1}{6} \times 22 \times 7 = \frac{49\sqrt{3}}{2} - \frac{154}{6} = \frac{7}{2} \left(7\sqrt{3} - \frac{22}{3} \right) \text{ cm}^2$$



17. We know that angle in a semi-circle is 90° .

$$\therefore \angle ACB = 90^\circ$$

Since, $\triangle ACB$ is right angle triangle.

$\therefore$ By Pythagoras theorem,

$$AB^2 = AC^2 + BC^2 = 12^2 + 5^2$$

$$\Rightarrow AB^2 = 144 + 25 = 169$$

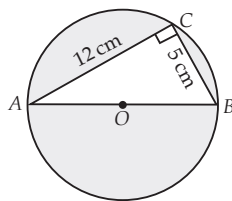
$$\Rightarrow AB = 13 \text{ cm}$$

$$\therefore \text{Radius of circle, } r = \frac{13}{2} \text{ cm}$$

Area of shaded region = Area of circle - Area of $\triangle ABC$

$$= \pi r^2 - \frac{1}{2} \times BC \times AC = \frac{22}{7} \times \frac{13}{2} \times \frac{13}{2} - \frac{1}{2} \times 5 \times 12$$

$$= 132.78 - 30 = 102.78 \text{ cm}^2$$



18. Radius of quadrant, $r = 8.4$ cm.

$$\therefore \text{Area of a quadrant} = \frac{1}{4} \pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 8.4 \times 8.4$$

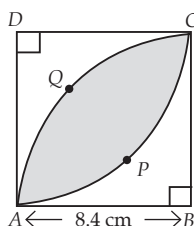
$$= 22 \times 0.3 \times 8.4 = 55.44 \text{ cm}^2$$

$$\therefore \text{Area of 2 quadrants} = 2 \times 55.44 = 110.88 \text{ cm}^2$$

Side of square $ABCD$ = Radius of quadrant = 8.4 cm

$$\therefore \text{Area of square } ABCD = (8.4)^2 = 70.56 \text{ cm}^2$$

$$\therefore \text{Area of shaded region } APCQA = \text{Area of 2 quadrants} - \text{Area of square} = 110.88 - 70.56 = 40.32 \text{ cm}^2$$



19. Area of equilateral triangle = $196\sqrt{3} \text{ cm}^2$ (Given)

$$\Rightarrow \frac{\sqrt{3}}{4} (\text{side})^2 = 196\sqrt{3}$$

$$\Rightarrow (\text{side})^2 = 196 \times 4 \Rightarrow \text{side} = 14 \times 2 = 28 \text{ cm}$$

$\therefore$ Side of equilateral triangle is 28 cm.

Now, circle and equilateral triangle formed by the same wire

$\therefore$ Perimeter of equilateral triangle = Perimeter of circle

$$\Rightarrow 3 \times 28 = 2\pi r \Rightarrow r = \left(\frac{3 \times 28}{2\pi} \right) \text{ cm}$$

$$\text{Area of circle} = \pi r^2 = \pi \times \left(\frac{3 \times 28}{2\pi} \right)^2$$

$$= \pi \times \frac{3 \times 28}{2\pi} \times \frac{3 \times 28}{2\pi} = \frac{3 \times 28 \times 3 \times 28 \times 7}{2 \times 2 \times 22} = 561.27 \text{ cm}^2$$

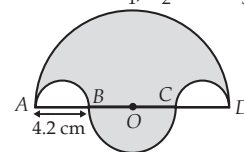
20. Let the radii of largest semi-circle, smallest semi-circle and semi-circle with diameter BC be r_1 , r_2 and r_3 respectively.

Given, $AD = 16.8$ cm

$$\therefore r_1 = \frac{16.8}{2} = 8.4 \text{ cm}$$

Also, $AB = 4.2$ cm

$$\therefore r_2 = \frac{4.2}{2} = 2.1 \text{ cm}$$



So, $r_3 = OA - AB = 8.4 - 4.2 = 4.2$ cm

(i) Length of boundary = Circumference of largest semi-circle $ABCD$ + Circumference of semi-circle with diameter BC .

$$= \pi r_1 + 4.2 + 4.2 + \pi r_3 = \pi(8.4 + 4.2) + 8.4$$

$$= \frac{22}{7} \times 12.6 + 8.4 = 39.6 + 8.4 = 48 \text{ cm}$$

$$(ii) \text{Area of shaded region} = \frac{\pi r_1^2}{2} + \frac{\pi r_3^2}{2} - 2 \left(\frac{1}{2} \pi r_2^2 \right)$$

$$= \pi \left(\frac{r_1^2}{2} + \frac{r_3^2}{2} - r_2^2 \right) = \frac{22}{7} \left[\frac{(8.4)^2}{2} + \frac{(4.2)^2}{2} - (2.1)^2 \right]$$

$$= \frac{22}{7} (35.28 + 8.82 - 4.41) = 124.74 \text{ cm}^2$$

21. We have, side of square $ABCD = 16.1$ cm

$$\therefore \text{Area of square } ABCD = (16.1)^2 = 259.21 \text{ cm}^2$$

Radius of quadrant, r = side of square = 16.1 cm

$$\therefore \text{Area of quadrant } OBCD = \frac{1}{4} \times \frac{22}{7} \times (16.1)^2 = 203.665 \text{ cm}^2$$

Area of shaded region = Area of square $ABCD$

$$- \text{Area of quadrant } OBCD = 259.21 - 203.665 = 55.545 \text{ cm}^2$$

