Statistics

TRY YOURSELF

SOLUTIONS

1. Let us construct the following table for the given data.

Class-interval	Frequency (f_i)	Class mark (x_i)	f_ix_i
0 - 10	8	5	40
10 - 20	10	15	150
20 - 30	9	25	225
30 - 40	12	35	420
40 - 50	11	45	495
Total	$\Sigma f_i = 50$		$\Sigma f_ix_i = 1330$

$\therefore$ Mean $= \dfrac{\Sigma f_ix_i}{\Sigma f_i} = \dfrac{1330}{50} = 26.6$

2. Let us construct the following table for the given data.

Class-interval	Frequency (f_i)	Class mark (x_i)	f_ix_i
100 - 120	10	110	1100
120 - 140	20	130	2600
140 - 160	30	150	4500
160 - 180	15	170	2550
180 - 200	5	190	950
Total	$\Sigma f_i = 80$		$\Sigma f_ix_i = 11700$

$\therefore$ Mean $= \dfrac{\Sigma f_ix_i}{\Sigma f_i} = \dfrac{11700}{80} = 146.25$

3. The frequency distribution table from the given data can be drawn as:

Class interval	Class mark (x_i)	Frequency (f_i)	f_ix_i
0 - 2	1	1	1
2 - 4	3	2	6
4 - 6	5	3	15
6 - 8	7	p	$7p$
8 - 10	9	2	18
		$\Sigma f_i = 8 + p$	$\Sigma f_ix_i = 40 + 7p$

$\therefore$ Mean, $\bar{x} = \dfrac{\Sigma f_ix_i}{\Sigma f_i} \Rightarrow 5 = \dfrac{40+7p}{8+p}$

$\Rightarrow 40 + 5p = 40 + 7p \Rightarrow p = 0$

4. Let us construct the following table for the given data.

Class-interval	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - 175$	f_id_i
0 - 50	8	25	–150	–1200
50 - 100	15	75	–100	–1500
100 - 150	32	125	–50	–1600
150 - 200	26	175 = a(let)	0	0
200 - 250	12	225	50	600
250 - 300	7	275	100	700
Total	$\Sigma f_i = 100$			$\Sigma f_id_i = -3000$

$\therefore$ Mean $= a + \dfrac{\Sigma f_id_i}{\Sigma f_i} = 175 + \left(\dfrac{-3000}{100}\right) = 175 - 30 = 145$

5. Let us construct the following table for the given data.

Class-interval	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - 39$	f_id_i
18 - 24	12	21	–18	–216
24 - 30	16	27	–12	–192
30 - 36	24	33	–6	–144
36 - 42	16	39 = a(let)	0	0
42 - 48	8	45	6	48
48 - 54	4	51	12	48
Total	$\Sigma f_i = 80$			$\Sigma f_id_i = -456$

$\therefore$ Mean $= a + \dfrac{\Sigma f_id_i}{\Sigma f_i} = 39 + \left(\dfrac{-456}{80}\right) = 39 - 5.7 = 33.3$

6. Let us construct the following table for the given data.

Class-interval	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - 35$	f_id_i
0 - 10	4	5	–30	–120
10 - 20	4	15	–20	–80
20 - 30	7	25	–10	–70
30 - 40	20	35 = a(let)	0	0
40 - 50	12	45	10	120
50 - 60	8	55	20	160
60 - 70	5	65	30	150
Total	$\Sigma f_i = 60$			$\Sigma f_id_i = 160$

$\therefore$ Mean $= a + \dfrac{\Sigma f_id_i}{\Sigma f_i} = 35 + \dfrac{160}{60} = 35 + 2.67 = 37.67$

7. Here, $h = 10$

Now, let us construct the following table for the given data.

Class-interval	Frequency (f_i)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0 - 10	12	5	–2	–24
10 - 20	11	15	–1	–11
20 - 30	8	25 = a(let)	0	0
30 - 40	10	35	1	10
40 - 50	9	45	2	18
Total	$\Sigma f_i = 50$			$\Sigma f_i u_i = -7$

$\therefore$ Mean $= a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$

$= 25 + \left(\frac{-7}{50}\right) \times 10 = 25 - 1.4 = 23.6$

8. Here, $h = 10$

Now, let us construct the following table for the given data.

Class-interval	Frequency (f_i)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
25 - 35	6	30	–2	–12
35 - 45	10	40	–1	–10
45 - 55	8	50 = a(let)	0	0
55 - 65	12	60	1	12
65 - 75	4	70	2	8
Total	$\Sigma f_i = 40$			$\Sigma f_i u_i = -2$

$\therefore$ Mean $= a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h = 50 + \left(\frac{-2}{40}\right) \times 10$

$= 50 - 0.5 = 49.5$

9. Let the assumed mean be $a = 1150$.

Given, $h = 100$

Now, the frequency distribution table from the given data can be drawn as :

Class Interval	Class mark (x_i)	Frequency (f_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
800-900	850	10	–3	–30
900-1000	950	15	–2	–30
1000-1100	1050	8	–1	–8
1100-1200	1150	12	0	0
1200-1300	1250	x	1	x
1300-1400	1350	5	2	10
1400-1500	1450	3	3	9
Total		$\Sigma f_i = 53 + x$		$\Sigma f_i u_i = x - 49$

Now, mean $= a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$

$\Rightarrow 1080 = 1150 + \left(\frac{x - 49}{53 + x}\right) \times 100$

$\Rightarrow -70(53 + x) = 100x - 4900$

$\Rightarrow -3710 - 70x = 100x - 4900 \Rightarrow 1190 = 170x$

$\Rightarrow x = \frac{1190}{170} = 7$

10. Here, class intervals are not in inclusive form. So, we first convert them in inclusive form by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class. The given frequency distribution in inclusive form is as follows.

Age (in years)	Number of cases
4.5 - 14.5	6
14.5 - 24.5	11
24.5 - 34.5	21
34.5 - 44.5	23
44.5 - 54.5	14
54.5 - 64.5	5

We observe that the class 34.5 - 44.5 has the maximum frequency. So, it is the modal class.

$\therefore$ $l = 34.5, h = 10, f_1 = 23, f_0 = 21$ and $f_2 = 14$

Now, mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 34.5 + \left(\frac{23 - 21}{2 \times 23 - 21 - 14}\right) \times 10 = 34.5 + \frac{20}{11}$

$= 34.5 + 1.81 = 36.31$

11. From the given data, we observe that, highest frequency is 32, which lies in the class interval 10 - 15.

$\therefore$ Modal class is 10 - 15.

So, $l = 10, h = 5, f_0 = 24, f_1 = 32, f_2 = 28$

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 10 + \left(\frac{32 - 24}{2 \times 32 - 24 - 28}\right) \times 5$

$= 10 + \frac{8}{64 - 52} \times 5 = 10 + \frac{40}{12} = 10 + 3.33 = 13.33$

12. Here, mode = 340 which lies in the interval 300-400.

$\therefore$ Modal class = 300-400

Now, Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$\Rightarrow 340 = 300 + \left(\frac{20 - x}{2 \times 20 - x - 14}\right) \times 100$

$\Rightarrow 340 - 300 = \left(\frac{20 - x}{26 - x}\right) \times 100 \Rightarrow 6x = 96 \Rightarrow x = 16$

13. From the given data, we observe that the highest frequency is 60, which lies in the class interval 40 - 50.

$\therefore$ Model class is 40 - 50.

So, $l = 40, h = 10, f_0 = 50, f_1 = 60, f_2 = 40$.

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 40 + \left(\frac{60 - 50}{2 \times 60 - 50 - 40}\right) \times 10 = 40 + \frac{10}{120 - 90} \times 10$

$= 40 + \frac{100}{30} = 40 + 3.33 = 43.33$

14. The cumulative frequency table for the given data is as follows :

Age (in years)	Number of students (Cumulative frequency)
Less than 12	3
Less than 14	3 + 18 = 21
Less than 16	21 + 13 = 34
Less than 18	34 + 12 = 46
Less than 20	46 + 7 = 53
Less than 22	53 + 27 = 80

15. Given distribution is more than type distribution. Here, we observe that 82 students obtained marks more than or equal to 10. Further since 72 students have obtained marks more than or equal to 20. So, 82 – 72 = 10 students lie in the interval 10 - 20. Similarly, we can find the other classes and their corresponding frequencies. Now, we construct the continuous grouped frequency distribution as :

Marks	Number of students
10 - 20	82 – 72 = 10
20 - 30	72 – 58 = 14
30 - 40	58 – 43 = 15
40 - 50	43 – 23 = 20
50 - 60	23 – 11 = 12
More than or equal to 60	11

16. We make the class-intervals as below 240, 240 - 270, 270 - 300, 300 - 330, 330 - 360, 360 - 390, 390 - 420.
From given distribution, we observe that 1 factory consume electricity less than 240 kW. So, the frequency of class-interval below 240 is 1. Further, there are 4 factories which consume electricity less than 270 kW. Therefore, number of factories which consume electricity in the interval 240 - 270 is 4 – 1 = 3. Similarly, we can find other frequencies. Now we construct the frequency distribution table as follows :

Consumption (in kW)	Number of factories
Below 240	1
240 - 270	4 – 1 = 3
270 - 300	8 – 4 = 4
300 - 330	24 – 8 = 16
330 - 360	33 – 24 = 9
360 - 390	38 – 33 = 5
390 - 420	40 – 38 = 2

17. The cumulative frequency table for the given data can be drawn as:

Number of students	Number of days (f_i)	Cumulative frequency (*c.f.*)
5	1	1
6	5	1 + 5 = 6
7	11	6 + 11 = 17
8	14	17 + 14 = 31
9	16	31 + 16 = 47
10	13	47 + 13 = 60
11	10	60 + 10 = 70
12	70	70 + 70 = 140
13	4	140 + 4 = 144
15	1	144 + 1 = 145
18	1	145 + 1 = 146
20	1	146 + 1 = 147

Here, n = 147, which is odd.

$\therefore$ Median $= \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{147+1}{2}\right)^{\text{th}}$

$= 74^{\text{th}}$ observation = 12

($\because$ 74^{th} observation lie in the cumulative frequency 140)

18. The cumulative frequency table for the given data can be drawn as:

Variable	Frequency (f_i)	Cumulative frequency (*c.f.*)
15 - 25	8	8
25 - 35	10	18
35 - 45	x	$18 + x$
45 - 55	25	$43 + x$
55 - 65	40	$83 + x$
65 - 75	y	$83 + x + y$
75 - 85	15	$98 + x + y$
85 - 95	7	$105 + x + y$
Total	$\Sigma f_i = 105 + x + y$	

Since, median is 58, which lies in the interval 55 - 65.

$\therefore$ Median class is 55 - 65.

Also, sum of frequencies is 140.

$\therefore \frac{n}{2} = \frac{140}{2} = 70, f = 40$ and $c.f. = 43 + x$

$\therefore$ Median $= l + \left[\frac{\frac{n}{2} - c.f.}{f}\right] \times h$

$\Rightarrow 58 = 55 + \left[\frac{70 - (43 + x)}{40}\right] \times 10$

$\Rightarrow 3 = \frac{70 - 43 - x}{40} \times 10 \Rightarrow 12 = 27 - x$

$\Rightarrow x = 27 - 12 = 15$ ...(i)

Also, $105 + x + y = 140$

$\Rightarrow 105 + 15 + y = 140$ (From (i))

$\Rightarrow y = 140 - 120 \Rightarrow y = 20$

Hence, $x = 15, y = 20$

19. Here, the class-interval are in discontinuous form, we first convert them in continuous form by subtracting 0.5 from lower limit and adding 0.5 to the upper limit and then prepare cumulative frequency table as below:

Class- interval	Frequency (f_i)	Cumulative frequency (*c.f.*)	Class mark (x_i)	$u_i = \frac{x_i - a}{5}$	$f_i u_i$
10.5 - 15.5	2	2	13	–4	–8
15.5 - 20.5	3	2 + 3 = 5	18	–3	–9
20.5 - 25.5	6	5 + 6 = 11	23	–2	–12
25.5 - 30.5	7	11 + 7 = 18	28	–1	–7
30.5 - 35.5	14	18 + 14 = 32	33 = *a*(let)	0	0
35.5 - 40.5	12	32 + 12 = 44	38	1	12
40.5 - 45.5	4	44 + 4 = 48	43	2	8
45.5 - 50.5	2	48 + 2 = 50	48	3	6
Total	$\Sigma f_i = 50$				$\Sigma f_i u_i = -10$

Median : Here, $n = 50 \Rightarrow \frac{n}{2} = 25$

Cumulative frequency just greater than 25 is 32 and corresponding class-interval is 30.5 - 35.5.

$\therefore$ Median class is 30.5 - 35.5.

So, $l = 30.5, f = 14, h = 5, c.f. = 18$

$$\therefore \quad \text{Median} = l + \left[\frac{\frac{n}{2} - c.f.}{f}\right] \times h = 30.5 + \left[\frac{25-18}{14}\right] \times 5$$

$$= 30.5 + \frac{7}{14} \times 5 = 30.5 + 2.5 = 33$$

Mode : Here, maximum frequency is 14 and corresponding class - interval is 30.5 – 35.5

$\therefore$ Modal class is 30.5 – 35.5.

So, $l = 30.5, h = 5, f_0 = 7, f_1 = 14, f_2 = 12$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30.5 + \left(\frac{14-7}{2 \times 14 - 7 - 12}\right) \times 5 = 30.5 + \frac{7}{9} \times 5$$

$$= 30.5 + 3.88 = 34.38$$

Mean : We have, $\text{mean} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h = 33 + \left(\frac{-10}{50}\right) \times 5$

$= 33 - 1 = 32$

20. We have, mode = $12k$, mean = $15k$.

We know, 3 Median = Mode + 2 Mean

$\Rightarrow$ 3 Median = $12k + 2(15k)$

$\Rightarrow$ 3 Median = $12k + 30k = 42k$

$\Rightarrow$ Median = $\frac{42k}{3} = 14k$

21. We have, Mean = 9.5, Median = 10

We know, Mode = 3 Median – 2 Mean

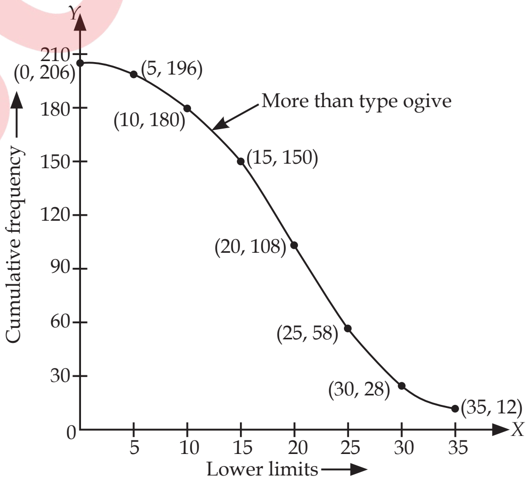
= 3 × 10 – 2 × 9.5 = 30 – 19 = 11

22. The "more than type" frequency distribution table for the given data is as follows :

Pocket Expenses	Cumulative frequency
More than or equal to 0	206
More than or equal to 5	206 – 10 = 196
More than or equal to 10	196 – 16 = 180
More than or equal to 15	180 – 30 = 150
More than or equal to 20	150 – 42 = 108
More than or equal to 25	108 – 50 = 58
More than or equal to 30	58 – 30 = 28
More than or equal to 35	28 – 16 = 12

Now, we plot the points (0, 206), (5, 196), (10, 180), (15, 150), (20, 108), (25, 58), (30, 28) and (35, 12).

The "more than type" ogive can be drawn on graph paper as follows :

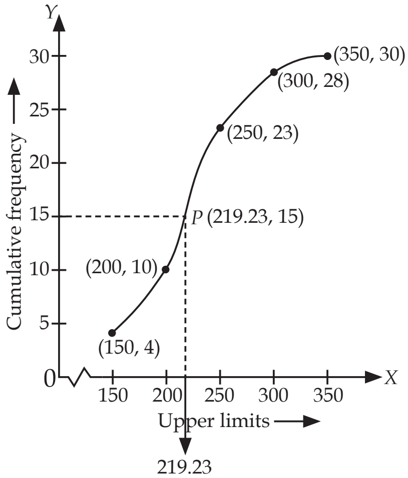


23. The "less than type" frequency distribution table for the given data is as follows:

Runs	Cumulative frequency
Less than 150	4
Less than 200	4 + 6 = 10
Less than 250	10 + 13 = 23
Less than 300	23 + 5 = 28
Less than 350	28 + 2 = 30

Now, we plot the points (150, 4), (200, 10), (250, 23), (300, 28), (350, 30).

The "less than type" ogive can be drawn on the graph paper as follows:



Now, locate $\frac{n}{2} = \frac{30}{2} = 15$ on the y-axis. From this point draw a line parallel to x-axis cutting the curve at P. From this point draw perpendicular to x-axis, the coordinate of point of intersection of perpendicular and x-axis is (219.23, 0).

Hence, median is 219.23.

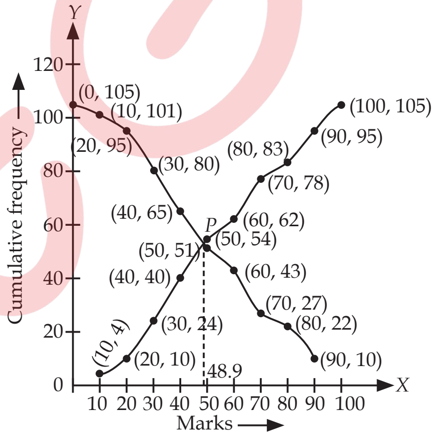
24. "Less than type" cumulative frequency distribution table is as follows:

Marks	Cumulative frequency
Less than 10	4
Less than 20	4 + 6 = 10
Less than 30	10 + 14 = 24
Less than 40	24 + 16 = 40
Less than 50	40 + 14 = 54
Less than 60	54 + 8 = 62
Less than 70	62 + 16 = 78
Less than 80	78 + 5 = 83
Less than 90	83 + 12 = 95
Less than 100	95 + 10 = 105

"More than type" cumulative frequency distribution table is as follows :

Marks	Cumulative frequency
More than or equal to 0	105
More than or equal to 10	105 – 4 = 101
More than or equal to 20	101 – 6 = 95
More than or equal to 30	95 – 14 = 81
More than or equal to 40	81 – 16 = 65
More than or equal to 50	65 – 14 = 51
More than or equal to 60	51 – 8 = 43
More than or equal to 70	43 – 16 = 27
More than or equal to 80	27 – 5 = 22
More than or equal to 90	22 – 12 = 10

The "less than type" and "more than type" ogives can be drawn on graph paper as follows:



The two ogives intersect at point P. Now we draw a perpendicular line from P to the x-axis, the intersection point on x-axis is (48.9, 0).

$\therefore$ Median = 48.9