

Polynomials

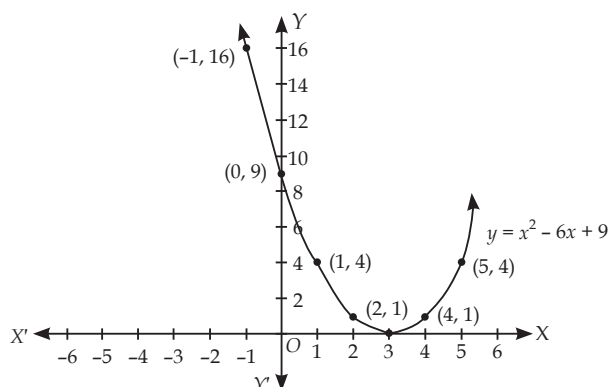


TRY YOURSELF

SOLUTIONS

1. Let $y = f(x) = x^2 - 6x + 9 = (x - 3)^2$
Now, we find values of y for different values of x .

x	-1	0	1	2	3	4	5
$y = f(x)$	16	9	4	1	0	1	4



The given curve represents a parabola opening upwards. Also, it intersects x axis at one point only. Hence, $y = f(x)$ has coincident zeroes.

\therefore Number of zeroes will be 1.

2. (i) Since, curve represents a parabola opening upwards.

$\therefore a > 0$.

(ii) Since, curve represents a parabola opening downwards.

$\therefore a < 0$.

3. (i) Let $p(y) = y^3 - 2y^2 - \sqrt{3}y + 1/2$

Yes, it is a polynomial. $p(y)$ is of the form $a_0 + a_1y + a_2y^2 + a_3y^3$, where, $a_0 = \frac{1}{2}$, $a_1 = -\sqrt{3}$, $a_2 = -2$, $a_3 = 1$ (all being real numbers).

Clearly, $p(y)$ is of degree 3.

- (ii) Let $p(x) = \sqrt{7}x^4 - \sqrt{x} + 2x - 1/3$

$p(x)$ is not of the form $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$

Hence, $p(x)$ is not a polynomial in x .

4. We know, $x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$

\therefore Zeroes of $x^2 - 3$ are $\sqrt{3}$ and $-\sqrt{3}$.

Now, sum of zeroes = $(\sqrt{3}) + (-\sqrt{3}) = 0$

$$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{and product of zeroes} = (\sqrt{3})(-\sqrt{3}) = -3 = \frac{(-3)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

5. Let $f(x) = x^3 - 27x + 54$

Now, $f(-6) = (-6)^3 - 27(-6) + 54$

$$= -216 + 162 + 54 = -216 + 216 = 0$$

$$f(3) = (3)^3 - 27(3) + 54 = 27 - 81 + 54 = 81 - 81 = 0$$

Hence, $-6, 3, 3$ are the zeroes of $f(x)$, where 3 is a repeated zero of $f(x)$.

Let $\alpha = -6$, $\beta = 3$, $\gamma = 3$ be the roots of $f(x)$.

$$\text{Now, } \alpha + \beta + \gamma = -6 + 3 + 3 = 0 = \frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = (-6)(3) + (3)(3) + 3(-6)$$

$$= -18 + 9 - 18 = -27 = \frac{(-27)}{1} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\beta\gamma = (-6)(3)(3) = -54 = \frac{-(54)}{1} = \frac{-(\text{Constant term})}{\text{Coefficient of } x^3}$$

6. Let $f(x) = x^2 + 3mx + 8m$

$\therefore 2$ is a root of $f(x) \therefore f(2) = 0$

$$\text{Now, } f(2) = (2)^2 + 3m(2) + 8m = 0$$

$$\Rightarrow 4 + 14m = 0 \Rightarrow 14m = -4 \Rightarrow m = \frac{-2}{7}$$

Let the other zero of $f(x)$ be α

$$\text{Sum of the roots} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\Rightarrow 2 + \alpha = \frac{-3m}{1} = -3 \times \frac{-2}{7} = \frac{6}{7} \Rightarrow \alpha = \frac{6}{7} - 2 = \frac{6-14}{7} = -\frac{8}{7}$$

7. Since, α and β are the zeroes of $f(x) = x^2 - x - 4$

$$\therefore \alpha\beta = \frac{(-4)}{1} = -4 \quad \dots(i) \text{ and } \alpha + \beta = \frac{-(-1)}{1} = 1 \quad \dots(ii)$$

$$\text{Now, } \frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{(\beta + \alpha) - (\alpha\beta)^2}{\alpha\beta} = \frac{(\alpha + \beta) - (\alpha\beta)^2}{\alpha\beta}$$

$$= \frac{1 - (-4)^2}{-4} \quad [\text{Using (i) and (ii)}]$$

$$= \frac{1 - 16}{-4} = \frac{-15}{-4} = \frac{15}{4}$$

8. Since, α and β are the zeroes of $f(t) = t^2 - 4t + 3$

$$\therefore \alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } t^2} = \frac{3}{1} = 3 \quad \dots(i)$$

$$\text{and } \alpha + \beta = \frac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2} = \frac{-(-4)}{1} = 4 \quad \dots(ii)$$

$$\text{Now, } \alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta) = (\alpha\beta)^3(\alpha + \beta)$$

$$= (3)^3(4) \quad [\text{Using (i) and (ii)}]$$

$$= 27 \times 4 = 108$$

9. Since, α and β are the zeroes of $p(z) = 5z^2 - 9z + 4$

$$\therefore \alpha + \beta = \frac{-(\text{Coefficient of } z)}{\text{Coefficient of } z^2} = -\frac{(-9)}{5} = \frac{9}{5} \quad \dots(i)$$

$$\text{and } \alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } z^2} = \frac{4}{5} \quad \dots(ii)$$

$$\text{Now, } \alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = \frac{4}{5} \left(\frac{9}{5} \right) = \frac{36}{25}$$

10. The quadratic polynomial $f(x)$ whose sum of zeroes, (S) and product of zeroes, (P) is given by $k(x^2 - Sx + P)$

$$\text{Here, } S = \sqrt{2} \text{ and } P = \frac{1}{3}$$

$$\therefore f(x) = k \left(x^2 - \sqrt{2}x + \frac{1}{3} \right), k \text{ being any non zero real number}$$

$$= k \left(\frac{3x^2 - 3\sqrt{2}x + 1}{3} \right) = 3x^2 - 3\sqrt{2}x + 1 \quad [\text{Taking } k = 3]$$

Hence, required quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$

11. Here, sum of zeroes (S) = $\sqrt{2}$

Sum of the product of zeroes taken two at a time (S') = $\sqrt{3}$.

$$\text{and product of zeroes, } P = \frac{1}{\sqrt{6}}$$

Now, required cubic polynomial is given by $k(x^3 - Sx^2 + S'x - P)$, k being any non zero real number.

$$k \left(x^3 - \sqrt{2}x^2 + \sqrt{3}x - \frac{1}{\sqrt{6}} \right) \\ k \left(\frac{\sqrt{6}x^3 - 2\sqrt{3}x^2 + 3\sqrt{2}x - 1}{\sqrt{6}} \right) = \sqrt{6}x^3 - 2\sqrt{3}x^2 + 3\sqrt{2}x - 1 \\ [\text{Taking } k = \sqrt{6}]$$

12. Given, α and β are zeroes of the polynomial, $f(x) = x^2 - 2x + 3$

$$\therefore \alpha + \beta = \frac{-(-2)}{1} = 2 \text{ and } \alpha\beta = 3$$

Let S and P denotes the sum and product of polynomial,

$$\text{whose roots are } \frac{\alpha-1}{\alpha+1} \text{ and } \frac{\beta-1}{\beta+1}$$

$$\text{Now, } S = \frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1} = \frac{(\alpha-1)(\beta+1) + (\alpha+1)(\beta-1)}{(\alpha+1)(\beta+1)}$$

$$= \frac{(\alpha\beta + \alpha - \beta - 1) + (\alpha\beta - \alpha + \beta - 1)}{\alpha\beta + (\alpha + \beta) + 1}$$

$$= \frac{2\alpha\beta - 2}{\alpha\beta + \alpha + \beta + 1} = \frac{2(\alpha\beta - 1)}{\alpha\beta + (\alpha + \beta) + 1} = \frac{2(3 - 1)}{3 + 2 + 1} = \frac{4}{6} = \frac{2}{3}$$

$$P = \frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)} = \frac{\alpha\beta - (\alpha + \beta) + 1}{\alpha\beta + (\alpha + \beta) + 1} = \frac{3 - (2) + 1}{3 + 2 + 1} = \frac{2}{6} = \frac{1}{3}$$

\therefore Cubic polynomial is $k(x^2 - Sx + P)$, k being any real number.

$$= k \left(x^2 - \frac{2}{3}x + \frac{1}{3} \right) = \frac{k(3x^2 - 2x + 1)}{3}$$

$$= 3x^2 - 2x + 1 \quad [\text{Taking } k = 3]$$

Thus, one of the polynomial is given by $3x^2 - 2x + 1$.

13. We have, $p(x) = 3x^3 + x^2 + 2x + 5$

$$g(x) = 1 + 2x + x^2 = x^2 + 2x + 1$$

Now, on dividing $p(x)$ by $g(x)$, we get the following :

$$\begin{array}{r} 3x - 5 \\ x^2 + 2x + 1 \overline{) 3x^3 + x^2 + 2x + 5} \\ \underline{(-) 3x^3 + 6x^2 + 3x} \\ -5x^2 - x + 5 \\ \underline{(-) 5x^2 + 10x + 5} \\ 9x + 10 \end{array}$$

\therefore Quotient = $3x - 5$, Remainder = $9x + 10$

14. Here, dividend = $f(x) = 4x^4 + 2x^3 - 2x^2 + x - 1$

$$\text{and divisor} = g(x) = x^2 + 2x - 3$$

Now, on dividing $f(x)$ by $g(x)$, we get the following :

$$\begin{array}{r} 4x^2 - 6x + 22 \\ x^2 + 2x - 3 \overline{) 4x^4 + 2x^3 - 2x^2 + x - 1} \\ \underline{(-) 4x^4 + 8x^3 - 12x^2} \\ -6x^3 + 10x^2 + x - 1 \\ \underline{(-) 6x^3 - 12x^2 + 18x} \\ 22x^2 - 17x - 1 \\ \underline{(-) 22x^2 + 44x - 66} \\ -61x + 65 \end{array}$$

Thus, remainder obtained is $-61x + 65$

\therefore We should add $61x - 65$ to $f(x)$ so that resulting polynomial is divisible by $x^2 + 2x - 3$.

15. Let $f(x) = 2x^3 + 3x^2 - 5x - 17$

$$q(x) = x - 2, r(x) = -x + 3$$

Then, by division algorithm, we get

$$f(x) = g(x)q(x) + r(x) \Rightarrow \frac{f(x) - r(x)}{q(x)} = g(x)$$

$$\Rightarrow g(x) = \frac{(2x^3 + 3x^2 - 5x - 17) - (-x + 3)}{x - 2} \\ = \frac{2x^3 + 3x^2 - 4x - 20}{x - 2}$$

On dividing $2x^3 + 3x^2 - 4x - 20$ by $x - 2$, we get the following :

$$\begin{array}{r} 2x^2 + 7x + 10 \\ x - 2 \overline{) 2x^3 + 3x^2 - 4x - 20} \\ \underline{(-) 2x^3 + 4x^2} \\ 7x^2 - 4x - 20 \\ \underline{(-) 7x^2 - 14x} \\ 10x - 20 \\ \underline{(-) 10x - 20} \\ 0 \end{array}$$

Thus, $g(x) = 2x^2 + 7x + 10$

16. Let $f(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$

Since, two zeroes of $f(x)$ are $\sqrt{2}$ and $-\sqrt{2}$

$\therefore (x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ is a factor of $f(x)$.

On dividing $f(x)$ by $x^2 - 2$, we get the following :

$$\begin{array}{r}
 x^2 - 2 \overline{) 2x^4 - 3x^3 - 3x^2 + 6x - 2} \quad (2x^2 - 3x + 1) \\
 \underline{(-) 2x^4 \quad (+) -4x^2} \\
 -3x^3 + x^2 + 6x - 2 \\
 \underline{(-) 3x^3 \quad (+) 6x} \\
 x^2 - 2 \\
 \underline{(-) x^2 \quad (+) 2} \\
 0
 \end{array}$$

By division algorithm, we have

$$f(x) = (x^2 - 2)(2x^2 - 3x + 1)$$

$$= (x + \sqrt{2})(x - \sqrt{2})(2x^2 - 2x - x + 1)$$

$$= (x + \sqrt{2})(x - \sqrt{2})[2x(x - 1) - 1(x - 1)]$$

$$= (x + \sqrt{2})(x - \sqrt{2})(2x - 1)(x - 1)$$

Hence, other zeroes are $\frac{1}{2}$ and 1.

17. Let $f(x) = x^4 + 4x^3 - x^2 - 10x + 6$

Since two zeroes of $f(x)$ are $-(1 + \sqrt{3})$ and $-(1 - \sqrt{3})$

$\therefore (x + (1 + \sqrt{3}))(x + (1 - \sqrt{3})) = (x + 1)^2 - (\sqrt{3})^2$

$$= x^2 + 1 + 2x - 3 = x^2 + 2x - 2$$

Now, we divide $f(x)$ by $x^2 + 2x - 2$

$$\begin{array}{r}
 x^2 + 2x - 2 \overline{) x^4 + 4x^3 - x^2 - 10x + 6} \quad (x^2 + 2x - 3) \\
 \underline{(-) x^4 \quad (+) 2x^3 \quad (-) 2x^2} \\
 2x^3 + x^2 - 10x + 6 \\
 \underline{(-) 2x^3 \quad (+) 4x^2 \quad (-) 4x \quad (+) 6} \\
 -3x^2 - 6x + 6 \\
 \underline{(-) 3x^2 \quad (+) 6x \quad (-) 6} \\
 0
 \end{array}$$

By division algorithm, we have

$$f(x) = (x^2 + 2x - 2)(x^2 + 2x - 3)$$

$$= (x^2 + 2x - 2)(x^2 + 3x - x - 3)$$

$$= (x^2 + 2x - 2)[x(x + 3) - 1(x + 3)]$$

$$= (x^2 + 2x - 2)(x - 1)(x + 3)$$

$$= [x + (1 + \sqrt{3})][x + (1 - \sqrt{3})](x - 1)(x + 3)$$

Thus, other zeroes are 1, -3.

