

Pair of Linear Equations in Two Variables



TRY YOURSELF

SOLUTIONS

1. The given system of equations is

$$3x - 2y = 4 \quad \dots(i) \quad 2x + y = 5 \quad \dots(ii)$$

Putting $x = 2$ and $y = 1$ in (i), we get

$$\text{L.H.S.} = 3 \times 2 - 2 \times 1 = 4 = \text{R.H.S.}$$

Putting $x = 2$ and $y = 1$ in (ii), we get

$$\text{L.H.S.} = 2 \times 2 + 1 = 5 = \text{R.H.S.}$$

Thus, $x = 2$ and $y = 1$ satisfy both the equations of the given system.

Hence, $x = 2, y = 1$ is a solution of the given system of equations.

2. The given system of equations is

$$2x + 7y = 11 \quad \dots(i) \quad x - 3y = -3 \quad \dots(ii)$$

Putting $x = 3$ and $y = 2$ in (ii), we get

$$\text{L.H.S.} = 2 \times 3 + 7 \times 2 = 20 \neq \text{R.H.S.}$$

So, $x = 3$ and $y = 2$ does not satisfy (i)

Putting $x = 3$ and $y = 2$ in (i), we get

$$\text{L.H.S.} = 3 - 3 \times 2 = -3 = \text{R.H.S.}$$

So, $x = 3$ and $y = 2$ satisfy (ii), but not (i).

Hence, $x = 3, y = 2$ is not a solution of the given system of equations.

3. Given system of equations is

$$4x + 5y = 9 \Rightarrow y = \frac{9-4x}{5} \quad \dots(i)$$

$$8x + 10y = 18 \Rightarrow y = \frac{18-8x}{10} \quad \dots(ii)$$

Table of solutions for (i) is :

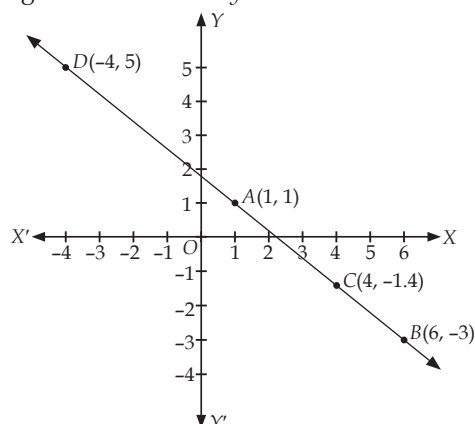
x	1	6
y	1	-3

Table of solution for (ii) is :

x	4	-4
y	-1.4	5

Now plot the points $A(1, 1)$ and $B(6, -3)$ and join them to get the line $4x + 5y = 9$.

Similarly, plot the points $C(4, -1.4)$ and $D(-4, 5)$ and join them to get the line $8x + 10y = 18$



Here, the lines represented by (i) and (ii) are coincident to each other.

4. Given system of equations is

$$2x + y - 7 = 0 \Rightarrow y = 7 - 2x \quad \dots(i)$$

$$-x + 2y - 4 = 0 \Rightarrow y = \frac{x+4}{2} \quad \dots(ii)$$

Table of solutions for (i) is :

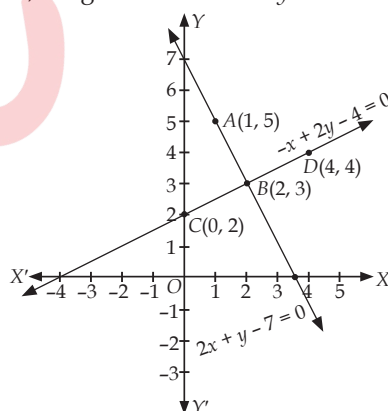
x	1	2
y	5	3

Table of solutions for (ii) is :

x	0	4
y	2	4

By plotting the points $A(1, 5)$ and $B(2, 3)$ and joining them, we get the line $2x + y - 7 = 0$.

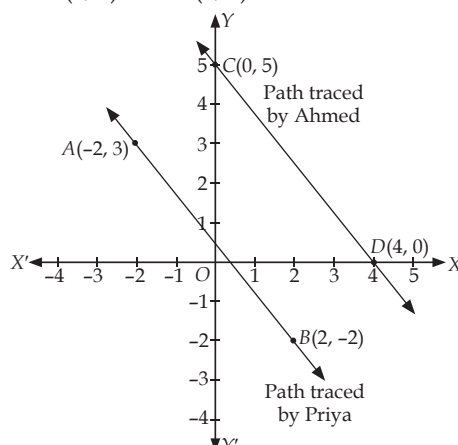
Similarly, plotting the points $C(0, 2)$ and $D(4, 4)$ and joining them, we get the line $-x + 2y - 4 = 0$.



Clearly, both lines intersect each other at $B(2, 3)$.

5. Let path of Priya is represented by the straight line AB , where $A(-2, 3)$ and $B(2, -2)$.

And path of Ahmed is represented by the straight line CD , where $C(0, 5)$ and $D(4, 0)$.



Plotting points $A(-2, 3)$, $B(2, -2)$ and joining them, we get path traced by priya.

Similarly, plotting points $C(0, 5)$, $D(4, 0)$ and joining them, we get path traced by Ahmed.

Here, the two lines do not intersect i.e., two lines (or path traced) are parallel to each other.

6. Let the cost of one book be ₹ x and that of one pen be ₹ y .

According to the condition-I, we have

$$5x + 7y = 79 \quad \dots(i)$$

According to the condition-II, we have

$$7x + 5y = 77 \quad \dots(ii)$$

∴ (i) and (ii) are the required algebraic representation of the given situation.

7. Let the digit in the units place be x and digit in the tens place be y . Then, $x = 2y$

$$\text{and number} = 10y + x$$

Number obtained by reversing the digits = $10x + y$

Also, Number + 27 = Number obtained by interchanging the digits.

$$\therefore 10y + x + 27 = 10x + y$$

$$\Rightarrow 9x - 9y = 27 \Rightarrow x - y = 3$$

Thus, the algebraic representation of given situation is $x - 2y = 0$ and $x - y = 3$.

8. Let the age of the father be x years and the sum of the ages of his 2 children be y years.

$$\text{Then, } x = 2y$$

After 18 years,

Age of the father = $(x + 18)$ years

Sum of the ages of his 2 children

= $(y + 18 + 18)$ years = $(y + 36)$ years

According to the question, $x + 18 = y + 36$

$$\Rightarrow x - y = 18$$

Thus, the algebraic representation of the given situation is $x - 2y = 0$ and $x - y = 18$.

9. Let the numerator of the fraction be x and denominator be y . Then, the fraction is $\frac{x}{y}$.

Now, according to the condition-I, we have $\frac{x+2}{y+2} = \frac{4}{5}$

$$\Rightarrow 5x + 10 = 4y + 8 \quad (\text{Cross-multiply both side})$$

$$\Rightarrow 5x - 4y + 2 = 0 \quad \dots(i)$$

Also, according to the condition-II, we have

$$\frac{x-4}{y-4} = \frac{1}{2} \quad (\text{Cross-multiply both side})$$

$$\Rightarrow 2x - 8 = y - 4 \Rightarrow 2x - y - 4 = 0 \quad \dots(ii)$$

Thus, the algebraic representation of the given problem is

$$5x - 4y + 2 = 0 \text{ and } 2x - y - 4 = 0.$$

To represent it graphically, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table of solutions for (i) is :

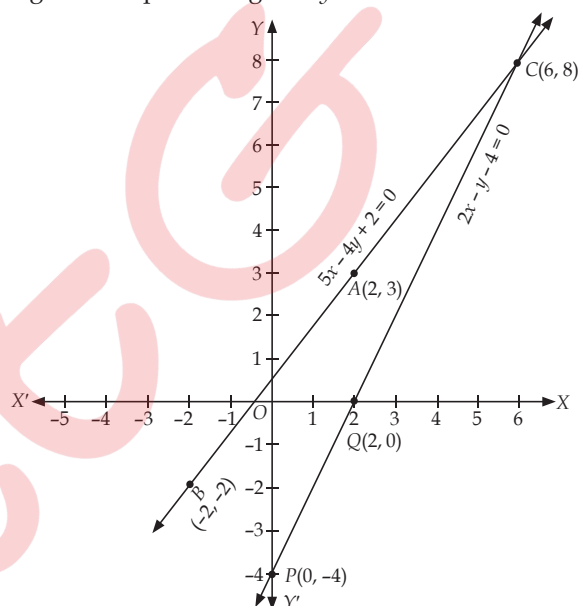
x	2	-2	6
$y = \frac{5x+2}{4}$	3	-2	8

Table of solutions for (ii) is :

x	0	2	6
$y = 2x - 4$	-4	0	8

So, we plot the points $A(2, 3)$, $B(-2, -2)$ and $C(6, 8)$ on graph paper and join them to get a straight line representing $5x - 4y + 2 = 0$.

Similarly, plot the points $P(0, -4)$, $Q(2, 0)$ and $C(6, 8)$ on same graph paper and join them to get a straight line representing $2x - y - 4 = 0$.



Clearly, the lines representing (i) and (ii) are intersecting each other at point $C(6, 8)$.

10. Let cost of one pencil be ₹ x and cost of one eraser be ₹ y .

According to the question, we have

$$3x + 2y = 8.5 \Rightarrow y = \frac{8.5 - 3x}{2} \quad \dots(i)$$

and $6x + 3y = 15$

$$\text{or } 2x + y = 5 \Rightarrow y = 5 - 2x \quad \dots(ii)$$

Thus, equations represented by (i) and (ii) is the algebraic representation of given situation.

Table of solutions for (i) is :

x	1.5	2.5
y	2	0.5

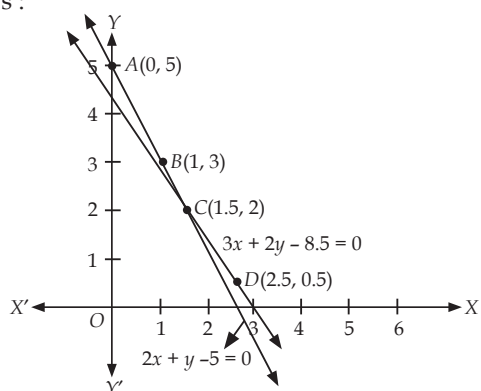
Table of solutions for (ii) is :

x	0	1
y	5	3

Plotting the points $C(1.5, 2)$ and $D(2.5, 0.5)$ on the graph paper and joining them, we get the line $3x + 2y = 8.5$.

Similarly, plotting the points $A(0, 5)$ and $B(1, 3)$ on graph paper and joining them, we get the line $2x + y = 5$.

∴ The graphical solution of given situations is as follows :



Graphically, we see that the lines are intersecting at point C(1.5, 2).

Hence, cost of 1 pencil and 1 eraser is ₹ 1.5 and ₹ 2 respectively.

11. The given pair of linear equation is

$$6x - 4y - 1 = 0 \Rightarrow y = \frac{6x-1}{4} \quad \dots(i)$$

$$\text{and } 2x - \frac{4}{3}y + 5 = 0 \Rightarrow y = \frac{3(2x+5)}{4} \quad \dots(ii)$$

Table of solutions for (i) is :

x	0.5	1
y	0.5	1.25

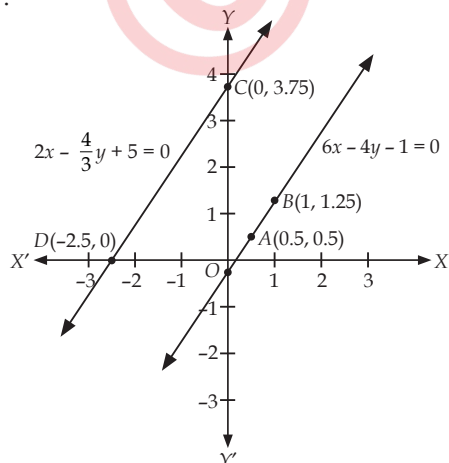
Table of solutions for (ii) is :

x	0	-2.5
y	3.75	0

Now, plotting the points A(0.5, 0.5) and B(1, 1.25) on the graph paper and joining them, we get the line $6x - 4y - 1 = 0$.

Similarly, plotting the points C(0, 3.75) and D(-2.5, 0) on the graph paper and joining them, we get the line $2x - \frac{4}{3}y + 5 = 0$.

The graphical representation of given equations is as follows :



Thus, given pair of equations has no solution as the two lines are parallel.

12. Given system of linear equations is

$$2x - y - 4 = 0 \Rightarrow y = 2x - 4 \quad \dots(i)$$

$$x + y + 1 = 0 \Rightarrow y = -1 - x \quad \dots(ii)$$

Table of solutions for (i) is :

x	1	2
y	-2	0

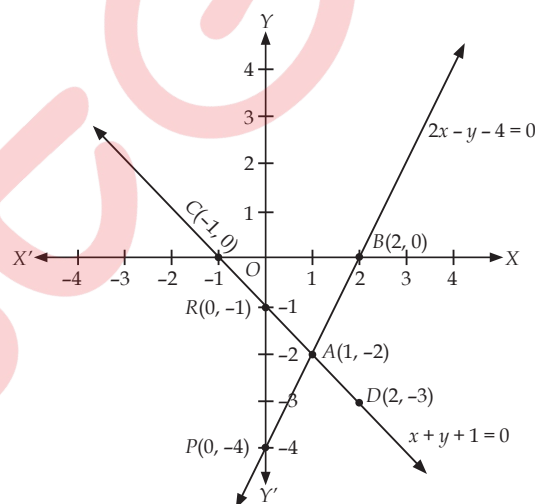
Table of solutions for (ii) is :

x	-1	2
y	0	-3

Now, plotting the points A(1, -2) and B(2, 0) on the graph paper and joining them, we get the line $2x - y - 4 = 0$.

Similarly, plotting the points C(-1, 0) and D(2, -3) on the graph paper and joining them, we get the line $x + y + 1 = 0$.

The graphical representation of given equations is as follows :



We see that both lines intersect at point A(1, -2) i.e., given system of equations has a unique solution given by $x = 1$ and $y = -2$.

From graph, line $2x - y - 4 = 0$ meets y -axis at P(0, -4) and line $x + y + 1 = 0$ meets y -axis at R(0, -1).

13. Compare the given equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

(i) For, $7x - 5y + 8 = 0$ and $7x + 8y - 9 = 0$, we have

$$a_1 = 7, b_1 = -5 \text{ and } c_1 = 8$$

$$\text{and } a_2 = 7, b_2 = 8 \text{ and } c_2 = -9$$

$$\text{Now, } \frac{a_1}{a_2} = \frac{7}{7} = 1; \frac{b_1}{b_2} = \frac{-5}{8}; \frac{c_1}{c_2} = \frac{8}{-9} = \frac{-8}{9}$$

$$\text{Since, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

∴ Lines are intersecting at a unique point.

(ii) For, $5x + 3y - 7 = 0$ and $15x + 9y - 21 = 0$, we have

$$a_1 = 5, b_1 = 3, c_1 = -7$$

$$\text{and } a_2 = 15, b_2 = 9, c_2 = -21$$

$$\text{Now, } \frac{a_1}{a_2} = \frac{5}{15} = \frac{1}{3}; \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}; \frac{c_1}{c_2} = \frac{-7}{-21} = \frac{1}{3}.$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ Lines are coincident.

14. Compare the given equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

(i) For, $6x - 3y - 18 = 0$, $2x - y - 4 = 0$

$a_1 = 6$, $b_1 = -3$, $c_1 = -18$

and $a_2 = 2$, $b_2 = -1$, $c_2 = -4$

Here, $\frac{a_1}{a_2} = \frac{6}{2} = 3$; $\frac{b_1}{b_2} = \frac{-3}{-1} = 3$ and $\frac{c_1}{c_2} = \frac{-18}{-4} = \frac{9}{2}$

∴ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, given pair of equations is inconsistent.

(ii) For, $x - 4y + 5 = 0$ and $16y - 2x + 20 = 0$, we have

$a_1 = 1$, $b_1 = -4$, $c_1 = 5$

and $a_2 = -2$, $b_2 = 16$, $c_2 = 20$

Now, $\frac{a_1}{a_2} = \frac{-1}{2}$, $\frac{b_1}{b_2} = \frac{-4}{16} = \frac{-1}{4}$, $\frac{c_1}{c_2} = \frac{5}{20} = \frac{1}{4}$

∴ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, given pair of equations is consistent.

15. We have, $7x - 15y = 2$

and $x + 2y = 3$

From (ii), $x = 3 - 2y$

Substituting the value of x from (iii) in (i), we get

$7(3 - 2y) - 15y = 2$

$\Rightarrow 21 - 14y - 15y = 2$

$\Rightarrow -29y = -19 \Rightarrow y = \frac{19}{29}$

Substituting the value of y in (iii), we get

$x = 3 - 2\left(\frac{19}{29}\right) = \frac{49}{29}$

∴ $x = \frac{49}{29}$, $y = \frac{19}{29}$ is the required solution.

16. We have, $\sqrt{2}x + \sqrt{5}y = 0$

and $\sqrt{6}x + \sqrt{15}y = 0$

From (i), $x = -\frac{\sqrt{5}}{\sqrt{2}}y$

Substituting the value of x from (iii) in (ii), we get

$\sqrt{6} \times \left(-\frac{\sqrt{5}}{\sqrt{2}}y\right) + \sqrt{15}y = 0$

$\Rightarrow -\sqrt{15}y + \sqrt{15}y = 0 \Rightarrow 0 = 0$, which is a true statement.

Hence, given pair of linear equations has infinitely many solutions.

Now, let us find these solutions.

Put $y = k$ (any real constant) in (iii), we get

$x = -\frac{\sqrt{5}}{\sqrt{2}}k$

Hence, $x = -\frac{\sqrt{5}}{\sqrt{2}}k$, $y = k$ is the required solution, where k is any real number.

17. Let the cost of 1 yellow candy is ₹ x and 1 orange candy is ₹ y ,

According to the question,

$x + 8y = 19$... (i)

and $2x + 11y = 28$... (ii)

From (i), $x = 19 - 8y$... (iii)

Substituting the value of x from (iii) in (ii), we get

$2(19 - 8y) + 11y = 28 \Rightarrow 38 - 16y + 11y = 28$

$\Rightarrow 38 - 5y = 28 \Rightarrow 5y = 10 \Rightarrow y = 2$

Substituting $y = 2$ in (iii), we get

$x = 19 - 8(2) = 3$

Hence, cost of 1 yellow candy is ₹ 3 and 1 orange candy is ₹ 2.

18. We have, $5ax + 6by = 28$... (i)

$3ax + 4by = 18$... (ii)

Multiplying (i) by 3 and (ii) by 5, we get

$15ax + 18by = 84$... (iii)

$15ax + 20by = 90$... (iv)

Subtracting (iii) from (iv), we get

$2by = 6 \Rightarrow y = \frac{3}{b}$

Putting $y = \frac{3}{b}$ in (i), we get

... (i) $5ax + 6b\left(\frac{3}{b}\right) = 28 \Rightarrow 5ax + 18 = 28$

... (ii) $\Rightarrow 5ax = 10 \Rightarrow x = \frac{2}{a}$

... (iii) $\therefore x = \frac{2}{a}$ and $y = \frac{3}{b}$ is the required solution.

19. We have, $99x + 101y = 499$... (i)

$101x + 99y = 501$... (ii)

Adding (i) and (ii), we get

$200x + 200y = 1000$... (iii)

$\Rightarrow x + y = 5$

Subtracting (ii) from (i), we get

$-2x + 2y = -2$

$\Rightarrow x - y = 1$... (iv)

Adding (iii) and (iv), we get

$2x = 6 \Rightarrow x = 3$

Put $x = 3$ in (iv), we get

$3 - y = 1 \Rightarrow y = 2$

Hence, $x = 3$ and $y = 2$ is the required solution.

20. Let digit at ten's place = x and digit at unit's place = y

Then, original number = $10x + y$

Number obtained by reversing the digit of given number = $10y + x$

According to the question, $x + y = 10$

and $(10x + y) + 36 = 10y + x$

$\Rightarrow 9x - 9y = -36$ or $x - y = -4$

∴ Given system of linear equations becomes

$x + y = 10$... (i)

and $x - y = -4$... (ii)

Adding (i) and (ii), we get

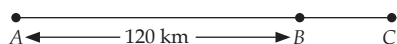
$2x = 6 \Rightarrow x = 3$

Using $x = 3$ in (i), we get

$3 + y = 10 \Rightarrow y = 7$

∴ Required number = $10x + y = 10(3) + 7 = 37$

21. Let the speed of two cars be x km/hr. and y km/hr.
Case I : When both cars travel in same direction and meet at point C (say)



Then, $AC - BC = AB$

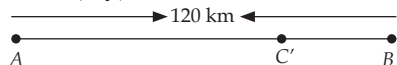
$$\Rightarrow \begin{aligned} & \text{(Distance travelled by car at A) -} \\ & \qquad \qquad \qquad \text{(Distance travelled by car at B)} \\ & = 120 \end{aligned}$$

$$\Rightarrow 6x - 6y = 120$$

(\because Distance = speed \times time and time = 6 hrs.)

$$\Rightarrow x - y = 20 \quad \dots(i)$$

Case II : When both cars travel towards each other and meet at point C' (say)



Then, $AC' + BC' = AB$

$$\Rightarrow \begin{aligned} & \text{(Distance travelled by car at A) +} \\ & \qquad \qquad \qquad \text{(Distance travelled by car at B)} = 120 \end{aligned}$$

$$\Rightarrow x \times 1 + y \times 1 = 120 \quad (\because \text{Time} = 1 \text{ hr., given})$$

$$\Rightarrow x + y = 120 \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2x = 140 \Rightarrow x = 70$$

Putting $x = 70$ in (i), we get $y = 50$.

Hence, speed of car starting from A is 70 km/hr and that starting from B is 50 km/hr.

22. Given system of equations is

$$2x + 3y - 7 = 0 \quad \dots(i)$$

$$5x - 4y - 6 = 0 \quad \dots(ii)$$

Solving (i) and (ii) by cross-multiplication method, we get

$$\frac{x}{\begin{vmatrix} 3 & -7 \\ -4 & -6 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -7 & 2 \\ -6 & 5 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & 3 \\ 5 & -4 \end{vmatrix}}$$

$$\Rightarrow \frac{x}{-18 - 28} = \frac{y}{-35 + 12} = \frac{1}{-8 - 15}$$

$$\Rightarrow \frac{x}{-46} = \frac{y}{-23} = \frac{1}{-23}$$

$$\Rightarrow \frac{x}{-46} = \frac{-1}{23} \text{ and } \frac{y}{-23} = \frac{-1}{23}$$

$$\Rightarrow x = \frac{46}{23} \text{ and } y = \frac{23}{23}$$

$\therefore x = 2$ and $y = 1$ is the required solution.

23. Let the fare from bus stand to Pitampura is ₹ x and that to Dilshad garden is ₹ y .

Then, according to question, we have

$$2x + 3y = 46 \text{ or } 2x + 3y - 46 = 0 \quad \dots(i)$$

$$\text{and } 3x + 5y = 74 \text{ or } 3x + 5y - 74 = 0 \quad \dots(ii)$$

Solving (i) and (ii) by cross-multiplication method, we get

$$\frac{x}{\begin{vmatrix} 3 & -46 \\ 5 & -74 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -46 & 2 \\ -74 & 3 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix}}$$

$$\Rightarrow \frac{x}{3(-74) - (-46)5} = \frac{y}{(-46)3 - 2(-74)} = \frac{1}{2 \times (5) - 3 \times (3)}$$

$$\Rightarrow \frac{x}{-222 + 230} = \frac{y}{-138 + 148} = \frac{1}{10 - 9} \Rightarrow \frac{x}{8} = \frac{y}{10} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{8} = 1 \text{ and } \frac{y}{10} = 1 \Rightarrow x = 8 \text{ and } y = 10$$

Hence, fare from the bus stand to Pitampura and Dilshad garden are ₹ 8 and ₹ 10 respectively.

24. Given equations are

$$4x - 5y - k = 0 \text{ and } 2x - 3y - 12 = 0$$

Here, $a_1 = 4$, $b_1 = -5$, $c_1 = -k$

and $a_2 = 2$, $b_2 = -3$, $c_2 = -12$

For unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{4}{2} \neq \frac{-5}{-3}, \text{ which is true.}$$

Hence, given system of equations has unique solution for any value of k i.e., for all real values.

25. Given equations are

$$kx + 2y - 5 = 0 \text{ and } 8x + ky - 20 = 0$$

Here, $a_1 = k$, $b_1 = 2$, $c_1 = -5$

and $a_2 = 8$, $b_2 = k$, $c_2 = -20$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{k}{8} = \frac{2}{k} = \frac{-5}{-20}$$

$$\text{From first two terms, } \frac{k}{8} = \frac{2}{k}$$

$$\Rightarrow k^2 = 16 \Rightarrow k = 4 \text{ and } -4$$

Thus, values of k are 4 and -4.

26. Given equations are

$$x + (k+1)y - 5 = 0$$

$$\text{and } (k+1)x + 9y - (8k-1) = 0$$

Here, $a_1 = 1$, $b_1 = k+1$, $c_1 = -5$

and $a_2 = k+1$, $b_2 = 9$, $c_2 = -(8k-1)$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{1}{k+1} = \frac{k+1}{9} = \frac{-5}{-(8k-1)}$$

Taking first two terms, we have

$$\frac{1}{k+1} = \frac{k+1}{9} \Rightarrow (k+1)^2 = 9 \Rightarrow k+1 = \pm 3$$

$$\Rightarrow k = 2 \text{ or } k = -4$$

Taking last two terms, we have

$$\frac{k+1}{9} = \frac{5}{8k-1}$$

...(iii)

Putting $k = 2$ in (iii), we have

$$\frac{3}{9} = \frac{5}{16-1} = \frac{1}{3}, \text{ which is true.}$$

Putting $k = -4$ in (iii), we have

$$\frac{-4+1}{9} = \frac{5}{-32-1}$$

$$\Rightarrow \frac{-3}{9} = \frac{-5}{-33}, \text{ which is not true.}$$

Hence, $k = 2$.

27. Given system of equations is

$$\frac{3}{x} + \frac{2}{y} - 13 = 0 \text{ and } \frac{4}{x} - \frac{5}{y} + 2 = 0$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in the given equations, we get

$$3u + 2v - 13 = 0 \quad \dots(i)$$

$$\text{and } 4u - 5v + 2 = 0 \quad \dots(ii)$$

$$\text{Form (i), } v = \frac{13 - 3u}{2} \quad \dots(iii)$$

Substituting the value of v in (ii), we get

$$4u - 5\left(\frac{13 - 3u}{2}\right) + 2 = 0$$

$$\Rightarrow 4u - \frac{65}{2} + \frac{15u}{2} + 2 = 0$$

$$\Rightarrow \frac{23u}{2} = \frac{61}{2} \Rightarrow u = \frac{61}{23}$$

Putting $u = \frac{61}{23}$ in (iii), we get

$$v = \frac{13 - 3\left(\frac{61}{23}\right)}{2} = \frac{299 - 183}{23 \times 2} = \frac{116}{23 \times 2} = \frac{58}{23}$$

$$\therefore u = \frac{61}{23} \text{ and } v = \frac{58}{23} \Rightarrow \frac{1}{x} = \frac{61}{23} \text{ and } \frac{1}{y} = \frac{58}{23}$$

$$\Rightarrow x = \frac{23}{61} \text{ and } y = \frac{23}{58}$$

28. We have, $\frac{xy}{x+y} = \frac{1}{9} \Rightarrow \frac{x+y}{xy} = \frac{9}{1}$

$$\Rightarrow \frac{1}{y} + \frac{1}{x} = 9 \quad \dots(i)$$

$$\text{and } \frac{xy}{x-y} = \frac{1}{11} \Rightarrow \frac{x-y}{xy} = 11 \Rightarrow \frac{1}{y} - \frac{1}{x} = 11 \quad \dots(ii)$$

Now, putting $\frac{1}{y} = u$ and $\frac{1}{x} = v$ in (i) and (ii), we get

$$u + v = 9 \quad \dots(iii)$$

$$u - v = 11 \quad \dots(iv)$$

Adding (iii) and (iv), we get

$$2u = 20 \Rightarrow u = 10$$

Putting the value of $u = 10$ in (iii), we get

$$10 + v = 9 \Rightarrow v = -1$$

$$\text{Now, } \frac{1}{y} = 10 \Rightarrow y = \frac{1}{10} \text{ and } \frac{1}{x} = -1 \Rightarrow x = -1$$

29. Let speed of sailor in still water be x km/hr and speed of current be y km/hr.

\therefore Speed of sailor in going downstream = $(x + y)$ km/hr and speed of sailor in going upstream = $(x - y)$ km/hr.

According to the question,

Time taken to go 8 km downstream = 40 minutes

$$\Rightarrow \frac{8}{x+y} = \frac{40}{60}$$

$$\Rightarrow \frac{1}{x+y} = \frac{1}{12} \Rightarrow x+y = 12 \quad \dots(i)$$

Also, time taken to go 8 km upstream = 1 hour

$$\Rightarrow \frac{8}{x-y} = 1 \Rightarrow x-y = 8 \quad \dots(ii)$$

On adding (i) and (ii) we get

$$2x = 20 \Rightarrow x = 10$$

Putting $x = 10$ in (i) we get

$$10 + y = 12 \Rightarrow y = 2$$

Hence, speed of sailor in still water is 10 km/hr and speed of current is 2 km/hr.

30. Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.

Then, speed of boat in upstream = $(x - y)$ km/hr

and speed of boat in downstream = $(x + y)$ km/hr

According to the question,

$$\frac{40}{x-y} + \frac{36}{x+y} = 8 \quad \dots(i)$$

$$\text{and } \frac{48}{x-y} + \frac{72}{x+y} = 12 \quad \dots(ii)$$

Putting $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$, in (i) and (ii), we get

$$40u + 36v = 8 \quad \dots(iii)$$

$$48u + 72v = 12 \quad \dots(iv)$$

Multiplying (iii) by 48 and (iv) by 40, we get

$$1920u + 1728v = 384 \quad \dots(v)$$

$$1920u + 2880v = 480 \quad \dots(vi)$$

Subtracting (v) from (vi), we get

$$1152v = 96$$

$$\Rightarrow v = \frac{1}{12}$$

Substituting the value of v in (iii), we get

$$40u + 36\left(\frac{1}{12}\right) = 8$$

$$\Rightarrow 40u + 3 = 8 \Rightarrow 40u = 5 \Rightarrow u = \frac{1}{8}$$

$$\therefore \frac{1}{x-y} = \frac{1}{8}$$

$$\Rightarrow x - y = 8 \quad \dots(vii)$$

$$\text{and } \frac{1}{x+y} = \frac{1}{12}$$

$$\Rightarrow x + y = 12 \quad \dots(viii)$$

Solving (vii) and (viii), we get

$$x = 10, y = 2$$

\therefore The speed of the boat = 10 km/hr and speed of the stream = 2 km/hr.

31. Suppose that one man and one boy alone can do the piece of work in x and y days respectively.

So, one day's work of a man = $\frac{1}{x}$ and

one day's work of a boy = $\frac{1}{y}$.

According to the question,

$$\left(\frac{4}{x} + \frac{6}{y}\right) = \frac{1}{5} \Rightarrow \frac{20}{x} + \frac{30}{y} = 1$$

$$\text{and } \left(\frac{3}{x} + \frac{4}{y}\right) = \frac{1}{7} \Rightarrow \frac{21}{x} + \frac{28}{y} = 1$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in both the equations, we have

$$20u + 30v - 1 = 0 \text{ and } 21u + 28v - 1 = 0$$

Solving by cross-multiplication method, we have

$$\frac{u}{30 \times -1 - 28 \times -1} = \frac{v}{-1 \times 20 - 21 \times -1} = \frac{1}{20 \times 20 - 30 \times 21}$$

$$\Rightarrow \frac{u}{-30 + 28} = \frac{v}{-21 + 20} = \frac{1}{28 \times 20 - 30 \times 21}$$

$$\Rightarrow \frac{u}{-2} = \frac{v}{-1} = \frac{1}{560 - 630} \Rightarrow \frac{u}{-2} = \frac{v}{-1} = \frac{1}{-70}$$

$$\Rightarrow u = \frac{-2}{-70} = \frac{1}{35} \text{ and } v = \frac{-1}{-70} = \frac{1}{70}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{35} \text{ and } \frac{1}{y} = \frac{1}{70}$$

$$\Rightarrow x = 35 \text{ and } y = 70$$

Hence, one man and one boy alone can finish the piece of work in 35 days and 70 days respectively.

32. Let 1 woman can finish the work in x days and 1 man can finish the work in y days.

$$\therefore \text{1 woman's one day work} = \frac{1}{x}$$

$$\text{and 1 man's one day work} = \frac{1}{y}$$

According to the question,

$$\frac{3}{x} + \frac{6}{y} = \frac{1}{5} \quad \dots(i)$$

$$\text{and } \frac{4}{x} + \frac{7}{y} = \frac{1}{4} \quad \dots(ii)$$

Multiplying (i) by 4 and (ii) by 3, we get

$$\frac{12}{x} + \frac{24}{y} = \frac{4}{5} \quad \dots(iii)$$

$$\frac{12}{x} + \frac{21}{y} = \frac{3}{4} \quad \dots(iv)$$

Subtracting (iv) from (iii), we get

$$\frac{3}{y} = \frac{4}{5} - \frac{3}{4} = \frac{16 - 15}{20} = \frac{1}{20} \Rightarrow y = 60$$

Substituting the value of y in (i), we get

$$\frac{3}{x} + \frac{6}{60} = \frac{1}{5} \Rightarrow \frac{3}{x} = \frac{1}{5} - \frac{1}{10} \Rightarrow \frac{3}{x} = \frac{1}{10}$$

$$\Rightarrow x = 30$$

Hence, 1 woman can do the work in 30 days and 1 man can do it in 60 days.

