Coordinate Geometry

CHAPTER 7

TRY YOURSELF — SOLUTIONS

1. We have, $P(-6, 3)$ and $Q(-1, -4)$
Using distance formula, we get

$$PQ=\sqrt{(-1+6)^2+(-4-3)^2}=\sqrt{25+49}=\sqrt{74}\text{ units}$$

2. We have, $P(a+b, a-b)$ and $Q(a-b, -a-b)$

$$\therefore\quad PQ=\sqrt{[(a-b)-(a+b)]^2+[(-a-b)-(a-b)]^2}$$
$$=\sqrt{(a-b-a-b)^2+(-a-b-a+b)^2}$$
$$=\sqrt{4b^2+4a^2}=2\sqrt{a^2+b^2}\text{ units}$$

3. Let $P(x, -2)$ and $Q(3, 2)$ be the given points.
Then, we have $PQ = 5$ [Given]

$\therefore\quad \sqrt{(3-x)^2+(2+2)^2}=5$

$\Rightarrow\quad (3-x)^2+16=25$ [Squaring both sides]

$\Rightarrow\quad x^2+9-6x+16=25$

$\Rightarrow\quad x^2-6x=0 \Rightarrow x(x-6)=0$

$\Rightarrow\quad x=0$ or $x=6$

4. Using distance formula, we have

$$AB=\sqrt{(6-3)^2+(0-5)^2}=\sqrt{9+25}=\sqrt{34}\text{ units}$$
$$BC=\sqrt{(1-6)^2+(-3-0)^2}=\sqrt{25+9}=\sqrt{34}\text{ units}$$
$$CD=\sqrt{(-2-1)^2+(2+3)^2}=\sqrt{9+25}=\sqrt{34}\text{ units}$$
$$DA=\sqrt{(3+2)^2+(5-2)^2}=\sqrt{25+9}=\sqrt{34}\text{ units}$$

Diagonal, $AC=\sqrt{(1-3)^2+(-3-5)^2}$
$$=\sqrt{4+64}=\sqrt{68}\text{ units}$$

and Diagonal, $BD=\sqrt{(-2-6)^2+(2-0)^2}$
$$=\sqrt{64+4}=\sqrt{68}\text{ units}$$

Since, $AB = BC = CD = DA$ *i.e.*, all the sides of given quadrilateral are equal in length and also, $AC = BD$ *i.e.*, diagonal are of same length.
Therefore, the given points are the vertices of a square $ABCD$.
Hence proved.

5. Using distance formula, we have

$$AB=\sqrt{(2+2)^2+(-1-3)^2}=\sqrt{16+16}=4\sqrt{2}\text{ units}$$
$$BC=\sqrt{(4-2)^2+(-3+1)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}\text{ units}$$
and $CA=\sqrt{(-2-4)^2+(3+3)^2}=\sqrt{36+36}=6\sqrt{2}$ units

Now, as $AB+BC=4\sqrt{2}+2\sqrt{2}=6\sqrt{2}$ units
i.e., $AB + BC = CA$
$\therefore$ A, B and C are collinear points.

6. Let $A(3, 2)$, $B\left(4, \frac{5}{2}\right)$ and $C(5, 3)$ are given points.
Using distance formula, we have

$$AB=\sqrt{(4-3)^2+\left(\frac{5}{2}-2\right)^2}=\sqrt{1+\frac{1}{4}}=\sqrt{\frac{5}{4}}=\frac{\sqrt{5}}{2}\text{ units}$$
$$BC=\sqrt{(5-4)^2+\left(3-\frac{5}{2}\right)^2}=\sqrt{1+\frac{1}{4}}=\sqrt{\frac{5}{4}}=\frac{\sqrt{5}}{2}\text{ units}$$
and $AC=\sqrt{(5-3)^2+(3-2)^2}=\sqrt{4+1}=\sqrt{5}$ units

Now, as $AB+BC=\frac{\sqrt{5}}{2}+\frac{\sqrt{5}}{2}=\sqrt{5}\text{ units}=AC$

$\therefore$ The given points are collinear.
Hence proved.

7. Given, point $P(x, y)$ is equidistant from the points $A(6, 1)$ and $B(1, 6)$

$\therefore\quad AP = PB \Rightarrow AP^2 = PB^2$

$\Rightarrow\quad (x-6)^2+(y-1)^2=(x-1)^2+(y-6)^2$

$\Rightarrow\quad x^2+36-12x+y^2+1-2y=x^2+1-2x+y^2+36-12y$

$\Rightarrow\quad -10x=-10y \Rightarrow x=y$

Hence proved.

8. Given, $A(a+b, b-a)$ and $B(a-b, a+b)$ are equidistant from $P(x, y)$.

$\therefore\quad AP = PB \Rightarrow (AP)^2 = (PB)^2$

$\Rightarrow\quad [x-(a+b)]^2+[y-(b-a)]^2=[x-(a-b)]^2+[y-(a+b)]^2$

$\Rightarrow\quad x^2+(a+b)^2-2(a+b)x+y^2+(b-a)^2-2(b-a)y$
$=x^2+(a-b)^2-2(a-b)x+y^2+(a+b)^2-2(a+b)y$

$\Rightarrow\quad -2[(a+b)x+(b-a)y]=-2[(a-b)x+(a+b)y]$

$\Rightarrow\quad (a+b)x+(b-a)y=(a-b)x+(a+b)y$

$\Rightarrow\quad (a+b-a+b)x=(a+b-b+a)y$

$\Rightarrow\quad 2bx=2ay$ or $bx=ay$

Hence proved.

9. Let P be $(0, y)$, which is equidistant from $A(4, 8)$ and $B(-6, 6)$.

$\therefore\quad AP = PB \Rightarrow AP^2 = PB^2$

$\Rightarrow\quad (4-0)^2+(8-y)^2=(0+6)^2+(y-6)^2$

$\Rightarrow\quad 16+64+y^2-16y=36+y^2+36-12y$

$\Rightarrow\quad 80-16y=72-12y$

$\Rightarrow \quad 4y = 8 \quad \Rightarrow \quad y = 2$

$\therefore$ Coordinates of P are (0, 2).

Now, $AP = \sqrt{(4-0)^2+(8-2)^2}$

$= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$ units

10. Let the point $P(0, y)$ divides the line segment joining the points $A(5, -6)$ and $B(-1, -4)$ in the ratio $k : 1$.

Using section formula, we have coordinates of P are $\left(\dfrac{-k+5}{k+1}, \dfrac{-4k-6}{k+1}\right)$.

Since, x-coordinate of P is zero

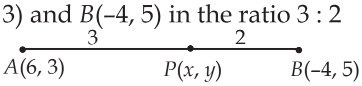
$\therefore \quad \dfrac{-k+5}{k+1} = 0 \quad \Rightarrow \quad -k+5 = 0 \quad \Rightarrow \quad k = 5$

Hence, the point P divides the line segment in the ratio 5 : 1.

Also, y-coordinate of $P = \dfrac{-4(5)-6}{5+1} = \dfrac{-20-6}{5+1} = \dfrac{-26}{6} = \dfrac{-13}{3}$

$\therefore$ Coordinates of P are $\left(0, \dfrac{-13}{3}\right)$.

11. Let $P(x, y)$ divides the line segment joining the points $A(6, 3)$ and $B(-4, 5)$ in the ratio 3 : 2



Using section formula, we have

$$(x, y) = \left(\frac{3\times(-4)+2\times 6}{3+2}, \frac{3\times 5+2\times 3}{3+2}\right)$$

$$= \left(\frac{-12+12}{5}, \frac{15+6}{5}\right) = \left(0, \frac{21}{5}\right)$$

12. Let $P(-3, p)$ divides the line segment joining the points $A(-5, -4)$ and $B(-2, 3)$ in the ratio $k : 1$.

Using section formula,

Coordinates of P are $\left(\dfrac{-2k-5}{k+1}, \dfrac{3k-4}{k+1}\right)$

But, coordinates of P are given as $(-3, p)$.

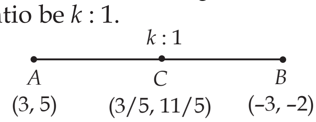
$\therefore \quad \dfrac{-2k-5}{k+1} = -3$ and $\dfrac{3k-4}{k+1} = p$

$\Rightarrow \quad -2k - 5 = -3k - 3 \quad \Rightarrow \quad k = 2$

Now, $p = \dfrac{3\times 2-4}{2+1} = \dfrac{6-4}{3} = \dfrac{2}{3}$

Hence, the ratio is 2 : 1 and $p = \dfrac{2}{3}$

13. Let the ratio be $k : 1$.



Using section formula, we have coordinates of C are $\left(\dfrac{-3k+3}{k+1}, \dfrac{-2k+5}{k+1}\right)$

But, coordinates of C are given as $\left(\dfrac{3}{5}, \dfrac{11}{5}\right)$.

$\therefore \quad \dfrac{-3k+3}{k+1} = \dfrac{3}{5}$ and $\dfrac{-2k+5}{k+1} = \dfrac{11}{5}$

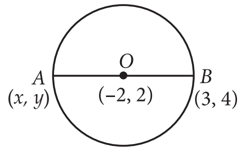
$\Rightarrow \quad -15k + 15 = 3k + 3$ and $-10k + 25 = 11k + 11$

$\Rightarrow \quad -18k = -12$ and $-21k = -14 \quad \Rightarrow \quad k = \dfrac{2}{3}$

Hence, the ratio is $\dfrac{2}{3} : 1$ *i.e.*, 2 : 3.

14. Let coordinates of the point A be (x, y) and O is the mid-point of AB.

Using mid-point formula, we have



$-2 = \dfrac{x+3}{2}$ and $2 = \dfrac{y+4}{2}$

$\Rightarrow \quad -4 = x + 3$ and $4 = y + 4$

$\Rightarrow \quad x = -7$ and $y = 0$

$\therefore$ Coordinates of A are $(-7, 0)$.

15. Let $A(-1, 0)$, $B(3, 1)$, $C(2, 2)$ and $D(x, y)$ be the vertices of a parallelogram, $ABCD$ taken in order.

$\because$ The diagonals of parallelogram bisect each other.

$\therefore$ Mid-point of AC = Mid-point of BD

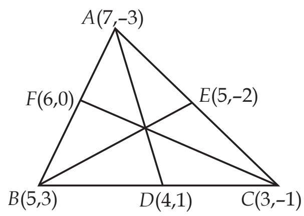
$\Rightarrow \quad \left(\dfrac{-1+2}{2}, \dfrac{0+2}{2}\right) = \left(\dfrac{3+x}{2}, \dfrac{1+y}{2}\right)$

$\Rightarrow \quad \left(\dfrac{1}{2}, 1\right) = \left(\dfrac{3+x}{2}, \dfrac{1+y}{2}\right) \quad \Rightarrow \quad \dfrac{3+x}{2} = \dfrac{1}{2}$ and $\dfrac{1+y}{2} = 1$

$\Rightarrow \quad 3 + x = 1$ and $1 + y = 2 \quad \Rightarrow \quad x = -2$ and $y = 1$

Hence, the coordinates of fourth vertex is (–2, 1)

16. Let D, E and F be the mid-point of the sides BC, CA and AB respectively. Then, the coordinates of D, E and F are



$D\left(\dfrac{5+3}{2}, \dfrac{3-1}{2}\right) = D\,(4, 1)$,

$E = \left(\dfrac{3+7}{2}, \dfrac{-1-3}{2}\right) = E(5, -2)$

and $F\left(\dfrac{7+5}{2}, \dfrac{-3+3}{2}\right) = F(6, 0)$

$\therefore \quad AD = \sqrt{(7-4)^2+(-3-1)^2} = \sqrt{9+16} = 5$ units;

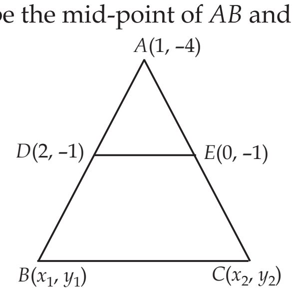
$BE = \sqrt{(5-5)^2+(-2-3)^2} = \sqrt{0+25} = 5$ units and

$CF = \sqrt{(6-3)^2+(0+1)^2} = \sqrt{9+1} = \sqrt{10}$ units

$\therefore$ Lengths of medians of ΔABC are 5 units, 5 units and $\sqrt{10}$ units.

17. Let $D(2, -1)$ and $E(0, -1)$ be the mid-point of AB and AC respectively.

Let coordinates of B and C are (x_1, y_1) and (x_2, y_2) respectively.



$\therefore$ Mid-point of AB

$= \left(\dfrac{x_1+1}{2}, \dfrac{y_1-4}{2}\right)$

$\Rightarrow \quad (2, -1) = \left(\dfrac{x_1+1}{2}, \dfrac{y_1-4}{2}\right)$ (Given)

$\Rightarrow \quad 2 = \dfrac{x_1+1}{2}$ and $-1 = \dfrac{y_1-4}{2}$

$\Rightarrow$ $4 = x_1 + 1$ and $-2 = y_1 - 4$ $\Rightarrow$ $x_1 = 3$ and $y_1 = 2$

$\therefore$ Coordinates of B are (3, 2).

Now, mid-point of $AC = \left(\frac{1+x_2}{2}, \frac{-4+y_2}{2}\right)$

$\Rightarrow$ $(0,-1) = \left(\frac{1+x_2}{2}, \frac{-4+y_2}{2}\right)$ (Given)

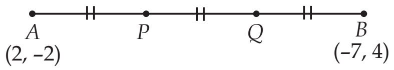
$\Rightarrow$ $0 = \frac{1+x_2}{2}$ and $-1 = \frac{-4+y_2}{2}$

$\Rightarrow$ $x_2 + 1 = 0$ and $-2 = -4 + y_2$ $\Rightarrow$ $x_2 = -1$ and $y_2 = 2$

$\therefore$ Coordinates of C are (–1, 2).

Now, mid-point of $BC = \left(\frac{3-1}{2}, \frac{2+2}{2}\right) = (1, 2).$

18. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ divides AB into three equal parts.



$\therefore$ P divides AB in the ratio 1 : 2

Using section formula, we have

$(x_1, y_1) = \left(\frac{-7+4}{1+2}, \frac{4-4}{1+2}\right) = \left(\frac{-3}{3}, 0\right) = (-1, 0)$

$\therefore$ Coordinates of P are (–1, 0)

Also, Q divides AB in the ratio 2 : 1.

Using section formula, we have

$(x_2, y_2) = \left(\frac{-14+2}{2+1}, \frac{8-2}{2+1}\right) = \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4, 2)$

$\therefore$ Coordinates of Q are (–4, 2).

19. Let $G(x, y)$ be the centroid of triangle.

$\therefore$ $(x, y) = \left(\frac{-2+4+4}{3}, \frac{3-3+5}{3}\right) = \left(\frac{6}{3}, \frac{5}{3}\right) = \left(2, \frac{5}{3}\right)$

20. Let G be the centroid of triangle.

$\therefore$ Centroid of ΔABC $(G) = \left(\frac{-1+0-5}{3}, \frac{3+4+2}{3}\right)$

$= \left(\frac{-6}{3}, \frac{9}{3}\right) = (-2, 3)$

Since, G lies on the median $x - 2y + k = 0$

So, coordinates of G must satisfy the equation of median.

$\therefore$ $-2 - 2 \times 3 + k = 0$ $\Rightarrow -2 - 6 + k = 0$ $\Rightarrow$ $k = 8$

21. Let $C(x, y)$ be the third vertex.

Given, centroid of $\Delta\, ABC$, $(G) = \left(\frac{5}{3}, -\frac{1}{3}\right)$

$\Rightarrow$ $\left(\frac{3-2+x}{3}, \frac{2+1+y}{3}\right) = \left(\frac{5}{3}, \frac{-1}{3}\right)$

$\Rightarrow$ $\left(\frac{1+x}{3}, \frac{3+y}{3}\right) = \left(\frac{5}{3}, \frac{-1}{3}\right)$

$\Rightarrow$ $\frac{1+x}{3} = \frac{5}{3}$ and $\frac{3+y}{3} = \frac{-1}{3}$

$\Rightarrow$ $1 + x = 5$ and $3 + y = -1$

$\Rightarrow$ $x = 4$ and $y = -4$

Hence, coordinates of C are (4, –4).

22. We have, $A(3, 0)$, $B(7,0)$ and $C(8,4)$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}[3(0-4) + 7(4-0) + 8(0-0)]$

$= \frac{1}{2}[-12 + 28 + 0] = \frac{1}{2}(16) = 8$ sq. units

23. The points $A(9, k)$, $B(4, -2)$ and $C(3, -3)$ are collinear.

$\therefore$ Area of $\Delta ABC = 0$

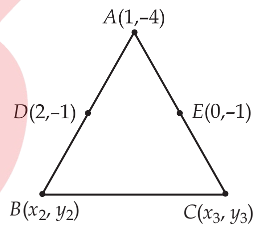
$\Rightarrow$ $\frac{1}{2}[9(-2+3) + 4(-3-k) + 3(k+2)] = 0$

$\Rightarrow$ $[9 - 12 - 4k + 3k + 6] = 0$

$\Rightarrow$ $[3 - k] = 0$ $\Rightarrow$ $k = 3$

24. Let $A(x_1, y_1) = (1, -4)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC.

Since, $D(2, -1)$ is the mid-point of AB.



$\therefore$ $2 = \frac{1+x_2}{2}$ and $-1 = \frac{-4+y_2}{2}$

$\Rightarrow$ $x_2 = 4 - 1 = 3$

and $y_2 = -2 + 4 = 2$

$\therefore$ $B(x_2, y_2) = (3, 2)$

Also, $E(0, -1)$ is the mid-point of AC.

$\therefore$ $0 = \frac{1+x_3}{2}$ and $-1 = \frac{-4+y_3}{2}$

$\Rightarrow$ $x_3 = -1$ and $y_3 = -2 + 4 = 2$

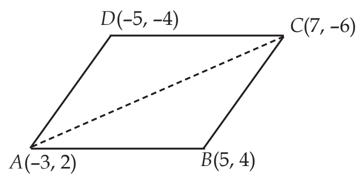
$\Rightarrow$ $C(x_3, y_3) = (-1, 2)$

Now, area of ΔABC

$= \frac{1}{2}[1(2-2) + 3(2+4) + (-1)(-4-2)]$

$= \frac{1}{2}[0 + 18 + 6] = \frac{1}{2} \times 24 = 12$ sq. units

25. We have, $A(-3, 2)$, $B(5, 4)$, $C(7, -6)$ and $D(-5, -4)$



Let us join diagonal AC.

Area of $\Delta ABC = \frac{1}{2}[-3(4+6) + 5(-6-2) + 7(2-4)]$

$= \frac{1}{2}[-3 \times 10 + 5 \times (-8) + 7(-2)]$

$= \frac{1}{2}[-30 - 40 - 14] = \frac{-84}{2} = -42$

Since area of triangle cannot be negative

$\therefore$ Area of ΔABC = 42 sq.units

Area of $\Delta ACD = \frac{1}{2}[-3(-6-(-4)) + 7(-4-2) - 5(2-(-6))]$

$= \frac{1}{2}[-3(-2) + 7(-6) - 5(8)] = \frac{1}{2}[6 - 42 - 40] = \frac{-76}{2} = -38$

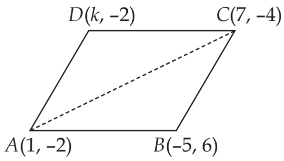
Since area of triangle cannot be negative

$\therefore$ Area of ΔACD = 38 sq.units

Now, area of quadrilateral $ABCD$

= Area of ΔABC + Area of ΔACD = 42 sq.units + 38 sq.units = 80 sq.units

26. Let $A(1, 2)$, $B(-5, 6)$, $C(7, -4)$ and $D(k, -2)$ be the vertices of quadrilateral $ABCD$.



Let us join diagonal AC.

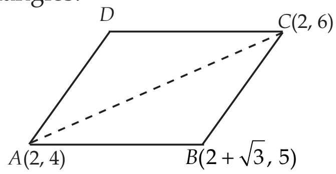
Area of quadrilateral $ABCD$ = Area of ΔABC + Area of ΔACD

$$\Rightarrow \frac{1}{2}[1(6-(-4))+(-5)(-4-2)+7(2-6)]$$
$$+\frac{1}{2}[(-4-(-2))+7(-2-2)+k(2-(-4))]=0$$

$$\Rightarrow \frac{1}{2}[10+30-28]+\frac{1}{2}[-2-28+6k]=0$$

$\Rightarrow$ $6 - 15 + 3k = 0 \Rightarrow 3k = 9 \Rightarrow k = 3$

27. Since, diagonals of parallelogram divides it into two congruent triangles.



$\therefore$ Area of ΔABC = Area of ΔACD.

$\therefore$ Area of parallelogram $ABCD$ = 2(Area of ΔABC)

$$=2\times\frac{1}{2}[2(5-6)+(2+\sqrt{3})(6-4)+2(4-5)]$$
$$=2(-1)+(2+\sqrt{3})(2)+2(-1)$$
$$=-2+4+2\sqrt{3}-2=2\sqrt{3}\text{ sq.units}$$