Introduction to Trigonometry

CHAPTER 8

TRY YOURSELF SOLUTIONS

1. In ΔPQR, $RQ^2 = PR^2 + PQ^2 = 3^2 + 1^2 = 9 + 1 = 10$

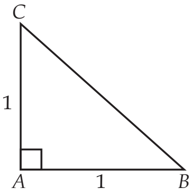
$\Rightarrow \quad RQ = \sqrt{10}$ cm

Now, $\cos R = \dfrac{PR}{RQ} = \dfrac{3}{\sqrt{10}}, \cos Q = \dfrac{PQ}{RQ} = \dfrac{1}{\sqrt{10}}$

$\cot Q = \dfrac{PQ}{PR} = \dfrac{1}{3}, \sec R = \dfrac{RQ}{PR} = \dfrac{\sqrt{10}}{3}$ and

$\tan R = \dfrac{PQ}{PR} = \dfrac{1}{3}$

2. In ΔABC, $BC^2 = AB^2 + AC^2$



$\Rightarrow \quad BC^2 = 1^2 + 1^2 = 2$

$\Rightarrow \quad BC = \sqrt{2}$ units

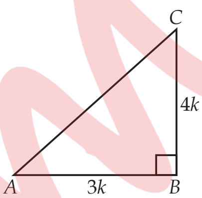
Now, $\sin B = \dfrac{AC}{BC} = \dfrac{1}{\sqrt{2}}$

$\cos B = \dfrac{AB}{BC} = \dfrac{1}{\sqrt{2}}, \tan B = \dfrac{AC}{AB} = \dfrac{1}{1} = 1,$

$\sec B = \dfrac{BC}{AB} = \dfrac{\sqrt{2}}{1} = \sqrt{2}, \operatorname{cosec} B = \dfrac{BC}{AC} = \dfrac{\sqrt{2}}{1} = \sqrt{2}$

and $\cot B = \dfrac{AB}{AC} = \dfrac{1}{1} = 1$

3. In ΔABC, $\tan A = \dfrac{BC}{AB} = \dfrac{4}{3}$.



Therefore, we have, $BC = 4k$ units and $AB = 3k$ units where k is a positive number.

By Pythagoras Theorem, we have

$AC^2 = AB^2 + BC^2 = (3k)^2 + (4k)^2 = 25k^2$

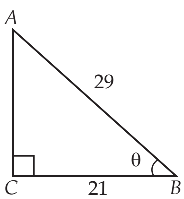
$\Rightarrow \quad AC = 5k$ units

Now, $\sin A = \dfrac{BC}{AC} = \dfrac{4k}{5k} = \dfrac{4}{5}$, $\cos A = \dfrac{AB}{AC} = \dfrac{3k}{5k} = \dfrac{3}{5}$,

$\cot A = \dfrac{AB}{BC} = \dfrac{3}{4}, \operatorname{cosec} A = \dfrac{AC}{BC} = \dfrac{5}{4}$

and $\sec A = \dfrac{AC}{AB} = \dfrac{5}{3}$.

4. In ΔACB, we have



$AC = \sqrt{AB^2 - BC^2} = \sqrt{(29)^2 - (21)^2}$

$= \sqrt{(29-21)(29+21)}$

$= \sqrt{(8)(50)} = \sqrt{400} = 20$ units

So, $\sin\theta = \dfrac{AC}{AB} = \dfrac{20}{29}, \cos\theta = \dfrac{BC}{AB} = \dfrac{21}{29}$

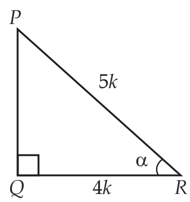
(i) $\cos^2\theta + \sin^2\theta = \left(\dfrac{21}{29}\right)^2 + \left(\dfrac{20}{29}\right)^2$

$= \dfrac{21^2 + 20^2}{29^2} = \dfrac{441 + 400}{841} = 1$

(ii) $\cos^2\theta - \sin^2\theta$

$= \left(\dfrac{21}{29}\right)^2 - \left(\dfrac{20}{29}\right)^2 = \dfrac{(21+20)(21-20)}{29^2} = \dfrac{41}{841}$

5. We have, $\sec\alpha = \dfrac{5}{4} = \dfrac{\text{Hypotenuse}}{\text{Base}}$



Let us draw a triangle PQR, right angled at Q such that $\angle PRQ = \alpha$, base $= QR = 4k$ units and hypotenuse $= PR = 5k$ units, where k is a positive number.

By Pythagoras theorem, we have

$PR^2 = PQ^2 + QR^2$

$\Rightarrow \quad 25k^2 = PQ^2 + 16k^2$

$\Rightarrow \quad PQ^2 = 25k^2 - 16k^2 = 9k^2 \Rightarrow PQ = 3k$ units

Now, $\tan\alpha = \dfrac{PQ}{QR} = \dfrac{3k}{4k} = \dfrac{3}{4}$

$\therefore \quad \dfrac{1 - \tan\alpha}{1 + \tan\alpha} = \dfrac{1 - \frac{3}{4}}{1 + \frac{3}{4}} = \dfrac{1}{7}$

6. Given, $m \cot A = n$

$\Rightarrow \quad m \cdot \dfrac{1}{\tan A} = n \qquad \left[\because \cot A = \dfrac{1}{\tan A}\right]$

$\Rightarrow \quad \tan A = \dfrac{m}{n} \qquad \ldots$(i)

Now, $\dfrac{m \sin A - n \cos A}{n \cos A + m \sin A} = \dfrac{m \cdot \frac{\sin A}{\cos A} - n \frac{\cos A}{\cos A}}{n \frac{\cos A}{\cos A} + m \cdot \frac{\sin A}{\cos A}}$

[Dividing both numerator and denominator by $\cos A$]

$= \dfrac{m \tan A - n}{n + m \tan A} = \dfrac{m \cdot \frac{m}{n} - n}{n + m \cdot \frac{m}{n}} \qquad$ [From (i)]

$= \dfrac{\frac{m^2 - n^2}{n}}{\frac{n^2 + m^2}{n}} = \dfrac{m^2 - n^2}{m^2 + n^2}$

7. We have, $3 \tan A = 4 \Rightarrow \tan A = 4/3$

Now, $\sqrt{\dfrac{\sec A - \operatorname{cosec} A}{\sec A + \operatorname{cosec} A}}$

$= \sqrt{\dfrac{\frac{1}{\cos A} - \frac{1}{\sin A}}{\frac{1}{\cos A} + \frac{1}{\sin A}}} = \sqrt{\dfrac{\sin A - \cos A}{\sin A + \cos A}} = \sqrt{\dfrac{\tan A - 1}{\tan A + 1}}$

[Dividing both numerator and denominator by $\cos A$]

$$= \sqrt{\frac{\frac{4}{3}-1}{\frac{4}{3}+1}} = \sqrt{\frac{\frac{1}{3}}{\frac{7}{3}}} = \sqrt{\frac{1}{7}} = \frac{1}{\sqrt{7}} \qquad \left[\because \tan A = \frac{4}{3}\right]$$

Hence proved.

8. L.H.S $= \dfrac{\operatorname{cosec}^2\theta - \cos^2\theta}{\cot^2\theta} = \dfrac{\frac{1}{\sin^2\theta} - \cos^2\theta}{\frac{\cos^2\theta}{\sin^2\theta}}$

$$= \frac{\frac{1-\cos^2\theta\sin^2\theta}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = \frac{1-\cos^2\theta\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} - \frac{\cos^2\theta\sin^2\theta}{\cos^2\theta} = \sec^2\theta - \sin^2\theta = \text{R.H.S}$$

Hence proved

9. We have, $5\cos\theta = 7\sin\theta$

$\Rightarrow \dfrac{\cos\theta}{\sin\theta} = \dfrac{7}{5}$ ...(i)

Now, $\dfrac{7\sin\theta + 5\cos\theta}{5\sin\theta + 7\cos\theta} = \dfrac{7 + 5\frac{\cos\theta}{\sin\theta}}{5 + 7\frac{\cos\theta}{\sin\theta}}$

[Dividing both numerator and denominator by sin θ]

$$= \frac{7 + 5\left(\frac{7}{5}\right)}{5 + 7\left(\frac{7}{5}\right)} \qquad \text{[Using (i)]}$$

$$= \frac{7+7}{5+\frac{49}{5}} = \frac{14}{\frac{74}{5}} = \frac{14\times 5}{74} = \frac{70}{74} = \frac{35}{37}$$

10. (i) We have, $\dfrac{4\cos^2 30° - 5\sin 90°}{6\sec^2 30° + 2\cos 90°}$

$$= \frac{4\left(\frac{\sqrt{3}}{2}\right)^2 - 5\times 1}{6\left(\frac{2}{\sqrt{3}}\right)^2 + 2\times 0} = \frac{4\times\frac{3}{4} - 5}{6\times\frac{4}{3}} = \frac{3-5}{8} = \frac{-2}{8} = -\frac{1}{4}$$

(ii) We have, $\dfrac{\cos 30°}{\sin^2 45°} - \tan 60° + 5\sin 0°$

$$= \frac{\sqrt{3}/2}{\left(\frac{1}{\sqrt{2}}\right)^2} - \sqrt{3} + 5\times 0 = \frac{\sqrt{3}}{2}\times\frac{2}{1} - \sqrt{3} = 0$$

11. Here, L.H.S. $= \dfrac{1+\sin A}{\cos A}$

$$= \frac{1+\sin 60°}{\cos 60°} \qquad [\because A = 60°]$$

$$= \frac{1+\sqrt{3}/2}{1/2} = 2+\sqrt{3}$$

Now, R.H.S. $= \dfrac{\cos A}{1-\sin A} = \dfrac{\cos 60°}{1-\sin 60°}$ $[\because A = 60°]$

$$= \frac{\frac{1}{2}}{1-\frac{\sqrt{3}}{2}} = \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

∴ L.H.S. = R.H.S.

12. We have, $\sin(A-B) = \frac{1}{2}$ and $\cos(A+B) = \frac{1}{2}$

We know that, $\sin 30° = \frac{1}{2}$ and $\cos 60° = \frac{1}{2}$

∴ $A - B = 30°$ ...(i)

$A + B = 60°$ ...(ii)

On adding (i) and (ii), we get

$2A = 90° \Rightarrow A = 45°$

On substituting the value of A in (i), we get

$45° - B = 30° \Rightarrow B = 15°$

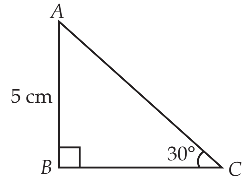
13. When $A = 10°$, we have,

$$\frac{3\sin 3A + 2\cos(5A+10)°}{\sqrt{3}\tan 3A + \operatorname{cosec}(5A-20)°} = \frac{3\sin 30° + 2\cos 60°}{\sqrt{3}\tan 30° + \operatorname{cosec} 30°}$$

$$= \frac{3\left(\frac{1}{2}\right)+2\left(\frac{1}{2}\right)}{\sqrt{3}\left(\frac{1}{\sqrt{3}}\right)+2} = \frac{\frac{3}{2}+1}{1+2} = \frac{\left(\frac{5}{2}\right)}{3} = \frac{5}{6}$$

14. We have, $\dfrac{\sin 90°}{\tan 45°} + \dfrac{1}{\sec 30°} = \dfrac{1}{1} + \dfrac{1}{2/\sqrt{3}} = 1 + \dfrac{\sqrt{3}}{2}$

15. In ΔABC,



$\tan C = \dfrac{AB}{BC}$

$\Rightarrow \tan 30° = \dfrac{5}{BC} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{5}{BC}$

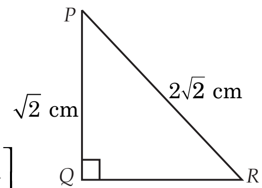
$\Rightarrow BC = 5\sqrt{3}$ cm

Now, $\sin 30° = \dfrac{AB}{AC}$

$\Rightarrow \dfrac{1}{2} = \dfrac{5}{AC} \Rightarrow AC = 10$ cm

16. Given, $PQ = \sqrt{2}$ cm

and $PR = 2\sqrt{2}$ cm.



Clearly, $\sin R = \dfrac{PQ}{PR} = \dfrac{\sqrt{2}}{2\sqrt{2}} = \dfrac{1}{2}$

$\Rightarrow \sin R = \sin 30° \quad \left[\because \sin 30° = \frac{1}{2}\right]$

∴ $\angle R = 30°$

Now, in ΔPQR, $\angle P + \angle Q + \angle R = 180°$

[By angle sum property of triangle]

$\Rightarrow \angle P + 90° + 30° = 180° \Rightarrow \angle P = 180° - 120°$

∴ $\angle P = 60°$

17. $\cot 85° + \cos 75° = \cot(90° - 5°) + \cos(90° - 15°)$

$= \tan 5° + \sin 15°$

18. We have, $\tan 46° - \cot 44° = \tan(90° - 44°) - \cot 44°$

$= \cot 44° - \cot 44°$ $[\because \tan(90° - \theta) = \cot\theta]$

$= 0$

19. We have, $\cot 10° \cdot \cot 30° \cdot \cot 80°$

$= \cot(90° - 80°) \cdot \cot 30° \cdot \cot 80°$

$= \tan 80° \cdot \cot 80° \cdot \sqrt{3}$ $[\because \cot 30° = \sqrt{3}]$

$= 1 \times \sqrt{3}$ $[\because \tan\theta\cdot\cot\theta = 1]$

$= \sqrt{3}$

20. We are given that $\sin 3A = \cos(A - 26°)$ ... (i)

Since $\sin 3A = \cos(90° - 3A)$,

∴ $\cos(90° - 3A) = \cos(A - 26°)$

Since 90° – 3A and A – 26° are both acute angles, therefore,

$90° - 3A = A - 26°$

$\Rightarrow 4A = 116° \Rightarrow A = 29°$

21. We have, sec 35° sin 55° = sec 35° sin (90° – 35°)

= sec 35° cos 35° $[\because \sin(90° - \theta) = \cos\theta]$

$= \frac{1}{\cos 35°}\cos 35°$ $\left[\because \sec\theta = \frac{1}{\cos\theta}\right]$

= 1 ...(1)

cos 55° cosec 35° = cos (90° – 35°) cosec 35°

= sin 35° cosec 35° $[\because \cos(90° - \theta) = \sin\theta]$

$= \sin 35° \frac{1}{\sin 35°}$ $\left[\because \text{cosec}\theta = \frac{1}{\sin\theta}\right]$

= 1 ...(2)

$\therefore$ sec 35° sin 55° + cos 55° cosec 35°

= 1 + 1 = 2.

22. We have, sin 43° sin 47° – cos 43° cos 47°

= sin 43° sin (90° – 43°) – cos 43° cos (90° – 43°)

= sin 43° cos 43° – cos 43° sin 43° = 0

23. Since, A and B are complementary angles.

$\therefore A + B = 90° \Rightarrow B = 90° - A$

$\Rightarrow \cot B = \cot(90° - A) = \tan A$ $[\because \cot(90° - \theta) = \tan\theta]$

$\therefore \cot B = \frac{5}{7}$

24. We have, $\sec^2\theta - \frac{1}{\text{cosec}^2\theta - 1}$

$= \sec^2\theta - \frac{1}{\cot^2\theta}$ $[\because \text{cosec}^2\theta = 1 + \cot^2\theta]$

$= \sec^2\theta - \tan^2\theta$ $\left[\because \tan\theta = \frac{1}{\cot\theta}\right]$

= 1

25. L.H.S. = sec A (1 – sin A)(sec A + tan A)

$= \left(\frac{1}{\cos A}\right)(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$

$= \frac{(1 - \sin A)(1 + \sin A)}{\cos^2 A} = \frac{1 - \sin^2 A}{\cos^2 A}$

$= \frac{\cos^2 A}{\cos^2 A} = 1$ = R.H.S.

26. L.H.S. = $\cot A + \tan A = \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$

$= \frac{\cos^2 A + \sin^2 A}{\sin A \cos A} = \frac{1}{\sin A \cos A}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{1}{\sin A} \cdot \frac{1}{\cos A} = \text{cosec} A \sec A$

= R.H.S. $\left[\because \text{cosec} A = \frac{1}{\sin A} \text{ and } \sec A = \frac{1}{\cos A}\right]$

27. L.H.S = $\frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + (1 + \cos\theta)^2}{\sin\theta(1 + \cos\theta)} = \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta(1 + \cos\theta)}$

$= \frac{1 + 1 + 2\cos\theta}{\sin\theta(1 + \cos\theta)}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{2 + 2\cos\theta}{\sin\theta(1 + \cos\theta)}$

$= \frac{2(1 + \cos\theta)}{\sin\theta(1 + \cos\theta)} = \frac{2}{\sin\theta}$ = 2 cosec θ = R.H.S.

28. We have, $\sin^2\theta(1 + \cot^2\theta) = \sin^2\theta \,\text{cosec}^2\theta$

$= \sin^2\theta \cdot \frac{1}{\sin^2\theta} = 1$

29. We know that, $\text{cosec} A = \frac{1}{\sin A} = \frac{1}{\sqrt{1 - \cos^2 A}}$

Also, $\tan A = \frac{\sin A}{\cos A} = \frac{\sqrt{1 - \cos^2 A}}{\cos A}$

And $\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sqrt{1 - \cos^2 A}}$

30. Given, $\tan\theta = \frac{12}{5}$

$\because \sin\theta = \frac{\tan\theta}{\sqrt{1 + \tan^2\theta}} = \frac{\frac{12}{5}}{\sqrt{1 + \left(\frac{12}{5}\right)^2}} = \frac{\frac{12}{5}}{\sqrt{\frac{25 + 144}{25}}}$

$= \frac{\frac{12}{5}}{\sqrt{\frac{169}{25}}} = \frac{\frac{12}{5}}{\frac{13}{5}} = \frac{12}{13}$

Now, $\frac{1 + \sin\theta}{1 - \sin\theta} = \frac{1 + \frac{12}{13}}{1 - \frac{12}{13}} = \frac{\frac{13 + 12}{13}}{\frac{13 - 12}{13}} = 25$

31. $\sec\theta = \sqrt{1 + \tan^2\theta} = \sqrt{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$= \sqrt{1 + \frac{1}{3}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$

$\therefore \cos\theta = \frac{1}{\sec\theta} = \frac{1}{2/\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{1 - \frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

Now, $3\sin^2\theta + 7\cos^2\theta$

$= 3\left(\frac{1}{2}\right)^2 + 7\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} + \frac{21}{4} = \frac{24}{4} = 6$