CHAPTER 11

Work and Energy

EXAM DRILL

ANSWERS

1. Average power $= \dfrac{\text{Total energy consumed}}{\text{Total time taken}}$

2. *K.E.* is zero at the highest point of its motion.

3(i) For $\theta = 90°$, work done is zero.

3(ii) $W = F \times s = 5 \times 2 = 10$ J

3(iii) Work done will be negative, if the force retards the motion of an object.

3(iv) When a force of 1 N acts on an object and the object moves a distance of 1 metre in the direction the force, then the work done by the force is 1 Joule.

4(i) The principle of conservation of energy states that the energy in a system can neither be created nor be destroyed . It can only be transformed from one form to another, but total energy of the system remains constant.

4(ii) Gravitational force is path independent.

4(iii) (a) : In the given situation, mechanical energy is conserved. Mechanical energy at any height is constant.

$\therefore$ Mechancial energy of the body = 800 + 0 = 800 J.

4(iv) (c) : $h = 2$ m, $m = 10$ kg

$E_p = mgh = 10 \times 10 \times 2 = 200$ J.

5. (b) : Potential energy increases as acceleration due to gravity (g) is more at poles.

OR

(b) : Here, Mass of the person, $m = 50$ kg

Height of the staircase, $h = 40 \times 15$ cm = 600 cm = 6 m

Time taken to run up to the staircase, $t = 6$ s

The power of the person will be

$$P = \frac{\text{work done}}{\text{time taken}} = \frac{mgh}{t} = \frac{(50\text{ kg})(10\text{ m/s}^2)(6\text{ m})}{6\text{ s}} = 500\text{ W}$$

6. (a) : 1 horse power (hp) = 746 W

7. (d) : No work is done by a man pushing a wall because there is no displacement takes place.

8. (a) : $mgh = K + mgh'$

$\Rightarrow \quad h' = h - \dfrac{K}{mg}$

OR

(c)

9. (b) : Body gain energy when work is done on it.

10. (d) : Angle between force and displacement should be 180° to obtain negative work.

11. (c) : If mass and velocity of a body are doubled,

Initial K.E. $= \dfrac{1}{2}mv^2$

Final K.E. $= \dfrac{1}{2}(2m)(2v)^2$

$= (8)\left(\dfrac{1}{2}mv^2\right)$

Kinetic energy increases 8 times of initial kinetic energy.

12. (b) : As $W = F \times s$ or $W = 7500 \times 1.5 \times 1000$ = 11250000 J = 11.25 MJ

OR

(c) : When we rub our hands, mechanical energy is converted into heat energy.

13. (a) : Since power is inversely proportional to time, crane P supplies more power.

14. (a) : Initial momentum = p

Initial kinetic energy = $E = \dfrac{p^2}{2m}$

if p increased by 50% then final momentum, $p' = \dfrac{3}{2}p$.

Final K.E. $(E') = \dfrac{p'^2}{2\text{m}}$

$$= \left(\frac{3p}{2}\right)^2 \times \frac{1}{2\text{m}} = \frac{9}{4}\frac{p^2}{2\text{m}} = 2.25\left(\frac{p^2}{2\text{m}}\right) = 2.25\,E$$

Percentage change in K.E.= $(E' - E) \times 100\%$

$K.E = 1.25\,E \times 100\% = 125\%$ of E

Thus, final K.E. increased by 125%

15. Momentum $= \sqrt{2 \times \text{mass} \times \text{kinetic energy}}$

OR

(i) A bird flying in the sky.

(ii) A body (*e.g.*, a stone) rolling down a hill.

16. (a) The potential energy of the car remains same since PE (= mgh) is independent of velocity.

(b) The kinetic energy of the car becomes four times, since $K.E.\left(= \dfrac{1}{2}mv^2\right)$ is proportional to square of velocity.

(c) The momentum of the car will also get doubled, since momentum ($p = mv$) is proportional to velocity.

17. (a) Potential energy stored in the rain drops.

(b) When water falls from a height, its potential energy is converted into kinetic energy and its velocity is maximum near at the foot of the fall. The velocity is suddenly reduced to zero, and hence its kinetic energy gets converted into heat energy by raising its temperature.

18. Consider a body of mass m, when placed at a height h from the earth's surface.

We apply a upward force F on body, $F = mg$

Work done by the force F,

$W = F \times h$

$W = mgh$

This work done on the object which changes its position is called gravitational potential energy

$P.E. = W$

$\therefore \quad P.E. = mgh.$

OR

Using $K.E. = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K.E.}{m}} = \sqrt{\frac{2 \times 10}{200 \times 10^{-3}\text{ kg}}}$

$= 10$ m/s

19. Relation between kinetic energy and momentum is

$$K.E. = \frac{p^2}{2m}$$

Let mass of the lighter body be m and velocity be v.

So, $K.E._1 = \frac{1}{2}mv^2$

And, mass of the heavier body is M and velocity is be V,

$K.E._2 = \frac{1}{2}MV^2$

Given $K.E._1 = K.E._2$

$$\frac{(mv)^2}{2m} = \frac{(MV)^2}{2M}$$

$\frac{(mv)^2}{(MV)^2} = \frac{m}{M}$. As $m < M$, $mv < MV$

The heavier body will have larger momentum.

20. (a) Positive : Since porter lift the suitcase by applying force in upward direction. Thus both force and displacement are in the same direction.

(b) Positive : As the suitcase is falling downward and force of gravity is also acting in downward direction therefore work done is positive.

(c) As the person does no work on the book. It is stationary so, work done is zero.

21. Power = 100 W

Time = 5 h

$\therefore$ Energy consumed in one day = Power × Time

= 100 W × 5 h = 500 Wh

$= \frac{500}{1000}$ kWh = 0.5 kWh

Since, 1 kWh = 1 unit

$\therefore$ Energy consumed = 0.5 units

22. As the gravitational potential energy turns into kinetic energy, so

$(E_g)_{top} = (E_k)_{bottom}$

$mgh_{top} = \frac{1}{2}mv_{bottom}^2$

$v_{bottom} = \sqrt{2gh_{top}} = \sqrt{2(9.8\text{ m s}^{-2})(0.15\text{ m})} = 1.7\text{ m s}^{-1}$

OR

Given, force applied (F) = 5 N

Displacement (s) = 2 m

Direction of displacement is parallel to applied force, So, $\theta = 0°$

From formula, $W = Fs \cos\theta$

$= 5\text{ N} \times 2\text{ m} \times \cos 0° = 10$ J

23. Given, mass of the sprinter, $m = 60$ kg

Initial velocity, $u = 0$

Final velocity, $v = 10$ m s^{-1}, time, $t = 3$ s

From equation of motion, $v = u + at$

$\Rightarrow \quad 10 = 0 + a \times 3$

$\Rightarrow$ Acceleration, $a = \frac{10}{3}$ m s^{-1}

As, $s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad s = 0 + \frac{1}{2} \times \frac{10}{3} \times 3 \times 3 = 15\text{ m}$

$\therefore$ Work done, (W) = Force (F) × Displacement (s)

$= 60 \times \frac{10}{3} \times 15 \quad [\because F = ma]$

= 3000 J

So, power, $P = \frac{W}{t} = \frac{3000}{3} = 1000\text{ W}$

24. Given, $m = 5$ kg, $u = 50$ m s^{-1}, $t = 5$ s, $g = -10$ m s^{-2}

Height covered by the body in 5 s is $h = ut + \frac{1}{2}gt^2$

$= 50 \times 5 - \frac{1}{2} \times 10 \times (5)^2 = 250 - 125 = 125$ m

Therefore, P.E. of the body after 5 s = mgh

= 5 × 10 × 125 = 6250 J

25. (a) Given, $m = 50$ kg, $h = 0.5$ m, $g = 10$ m s^{-2}

At highest point, kinetic energy is converted into potential energy.

$\therefore$ P.E. $= mgh = 50 \times 10 \times 0.5 = 250$ J

(b) Given, $m = 20$ kg, $v = 0.1$ m s^{-1}

As we know that, a moving body has kinetic energy.

$\therefore$ K.E. $= \frac{1}{2}mv^2 = \frac{1}{2} \times 20 \times (0.1)^2 = 0.1$ J

OR

Let M be the mass of the man, $M/2$ that of the boy and V and v be their respective velocities. As the K.E. of the man is half the K.E. of the boy, so

$$\frac{1}{2}MV^2 = \frac{1}{2}\cdot\frac{1}{2}\left(\frac{M}{2}\right)v^2 \text{ or } v^2 = 4V^2 \text{ or } v = 2V$$

When the velocity of the man is increased to $(V + 1)$, their kinetic energies become equal.

$$\therefore \quad \frac{1}{2}M(V+1)^2 = \frac{1}{2}\left(\frac{M}{2}\right)v^2 = \frac{1}{4}M.(2V)^2$$

or $\quad V^2 + 2V + 1 = 2V^2$ or $V^2 - 2V - 1 = 0$

$\therefore \quad V = \sqrt{2} + 1 = 2.414$ m s^{-1}

$v = 2(\sqrt{2} + 1) = 4.828$ m s^{-1}

26. (a) The energy possessed by a body by virtue of its motion is called its kinetic energy.

No, the kinetic energy of an object cannot be negative because both m and v^2 are always positive and $K.E. = \frac{1}{2}mv^2$

(b) Given, $m = 1200$ kg, $s = 40$ m, $t = 5$ s, u = 0

From, $s = ut + \frac{1}{2}at^2$

$40 = 0 \times t + \frac{1}{2} \times a \times (5)^2$

$a = \frac{40 \times 2}{25} = 3.2$ m s^{-2}

Now, $W = Fs = mas$

$= 1200 \times 3.2 \times 40 = 153600$ J

OR

(a) Commercial unit of energy is kilowatt-hour (kWh).

The SI unit of energy is joule.

Relation between kWh and J

$\therefore$ 1 kWh $= 3.6 \times 10^6$ J

(b) Given, $P = 2500$ W, $t = 4$ h

$\therefore E = P \times t = \frac{2500}{1000} \times 4 = 10$ kWh $= 10$ units

27. Heat energy : It is the energy possessed by a body by virtue of random motion of the molecules of the body.

Heat is also associated with the force of friction. Thus, work done by friction is not lost, but it is transferred as heat energy of the system.

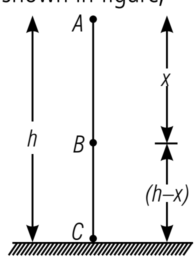
The energy possessed by a body due to its temperature is known as heat energy or thermal energy.

Electrical Energy : The flow of electric current causes bulbs to glow, fans to rotate and bells to ring. A definite amount of work has to be done in moving the free charge carriers in a particular direction through all the electrical appliances. This causes the expenditure of electrical energy. An urban Indian household consumes about 200 joule of energy per second on an average.

Chemical energy : Chemical energy arises from the fact that the molecules participating in the chemical reaction have different binding energies. A stable chemical compound has less energy than the separated parts. Chemical energy is associated with the forces that give rise to the stability of the substances. These forces bind atoms into molecules, molecules into polymeric chains etc. The chemical energy obtained from the combustion of coal, cooking gas, wood and petroleum have become indispensable for us.

28. To illustrate the law, let us calculate kinetic energy K.E., potential energy P.E. and total energy T.E. of a body falling freely under gravity.

Let m be the mass of the body held at position A, at a height h above the ground as shown in figure,



As the body is at rest at A, therefore,

At A : K.E. of the body = 0

P.E. of the body $= mgh$, where g is the acceleration due to gravity at A.

T.E. of the body = K.E. + P.E.

$= 0 + mgh$

$E_1 = mgh$ ...(i)

Let the body be allowed to fall freely under gravity, when it strikes the ground at C with a velocity v.

From, $v^2 - u^2 = 2as$

$v^2 - 0 = 2(g)h$

$v^2 = 2gh$ ...(ii)

$\therefore \quad$ At C : K.E. of the body, $\frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$

P.E. of the body = $mgh = mg\ (0) = 0$

Total energy of the body = K.E. + P.E.

$E_2 = mgh + 0 = mgh$ ...(iii)

In free fall, let the body cross any point B with a velocity v_1, where $AB = x$

From $v^2 - u^2 = 2as$

$v_1^2 - 0 = 2(g)\ x$

$v_1^2 = 2gx$

At B : K. E. of the body = $\frac{1}{2}mv_1^2 = \frac{1}{2}m(2gx) = mgx$

Height of the body at B above the ground = $CB = (h - x)$

∴ P.E. of the body at $B = mg\ (h - x)$

Total energy of the body at B = K.E. + P.E.

$E_3 = mgx + mg\ (h - x) = mgx + mgh - mgx$

$E_3 = mgh$ ...(iv)

from (i), (iii) and (iv) we find that

$E_1 = E_2 = E_3 = mgh$

29. **(a)** As the two bodies are dropped, they fall with the same acceleration of 9.8 m s^{-2}. When they are 10 m above the ground, they have already fallen through 14.9 m – 10 m = 4.9 m. The velocity at this point may be worked out from

$v^2 = u^2 + 2gh = 0 + 2 \times (9.8 \text{ m s}^{-2}) \times (4.9 \text{ m})$

$= 9.8 \times 9.8 \text{ m}^2 \text{ s}^{-2}$ or $v = 9.8 \text{ m s}^{-1}$.

The momentum of A is

$p_A = m_A v = (3.0 \text{ kg}) \times (9.8 \text{ m s}^{-1}) = 29.4 \text{ kg m s}^{-1}$

and that of B is

$p_B = m_B v = (10 \text{ kg}) \times (9.8 \text{ m s}^{-1}) = 98 \text{ kg m s}^{-1}$

(b) The bodies are at a height of 10 m above the ground. The potential energy of A is

$u_A = m_A gh = (3.0 \text{ kg}) \times (9.8 \text{ m s}^{-2}) \times (10 \text{ m}) = 294 \text{ J}$

and that of B is

$u_B = m_B gh = (10 \text{ kg}) \times (9.8 \text{ m s}^{-2}) \times (10 \text{ m}) = 980 \text{ J}$

(c) The kinetic energy of A is

$K_A = \frac{1}{2}m_A v^2 = \frac{1}{2} \times (3.0 \text{ kg}) \times (9.8 \text{ m s}^{-2})^2 \approx 144 \text{ J}$

and that of B is

$K_B = \frac{1}{2}m_B v^2 = \frac{1}{2} \times (10 \text{ kg}) \times (9.8 \text{ m s}^{-2})^2 \approx 480 \text{ J}$

OR

Mass of the car A, $m_A = 1000$ kg

Mass of the car B, $m_B = 1000$ kg

Initial velocity of car A, $u_A = 36 \text{ km/h} = 36\left(\frac{5}{18}\right) \text{m/s} = 10 \text{ m/s}$

$\left(\text{as } 1\text{km/h} = \frac{5}{18}\text{m/s}\right)$

Opposing force of friction, $F = 100$ N

Power of the engine of car $A = F\ u_A = (100 \text{ N})(10 \text{ m/s}) = 1000 \text{ W}$

(as power = force × velocity and N m/s = J/s = W)

Initial velocity of car B, $u_B = 0$

Final velocity of car A, $v_A = 0$, (as it comes to rest after colliding with car B.

If v_B is final velocity of car B when car A collides with it, applying the law conservation of momentum, initial momentum of cars A and B = final momentum of cars A and B

i.e., $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

or $1000 \times 10 + 1000 \times 0 = 1000 \times 0 + 1000 \times v_B$

or $v_B = 10$ m/s

Note : The cars A and B simply interchange their velocities, *i.e.*, car A comes to rest and car B moves with the initial velocity of car A, *i.e.*, 10 m/s.

30. Energy transformation taking place in different gadgets:

(a) Solar cell - light energy to electrical energy.

(b) Electric heater- electrical energy to heat energy.

(c) Magnetic compass- magnetic potential energy to kinetic energy

(d) Telephone- sound energy to electrical

(e) Electrical cell- chemical energy to electrical energy

(f) Heat engine- Heat or manual energy and chemical energy to mechanical energy.