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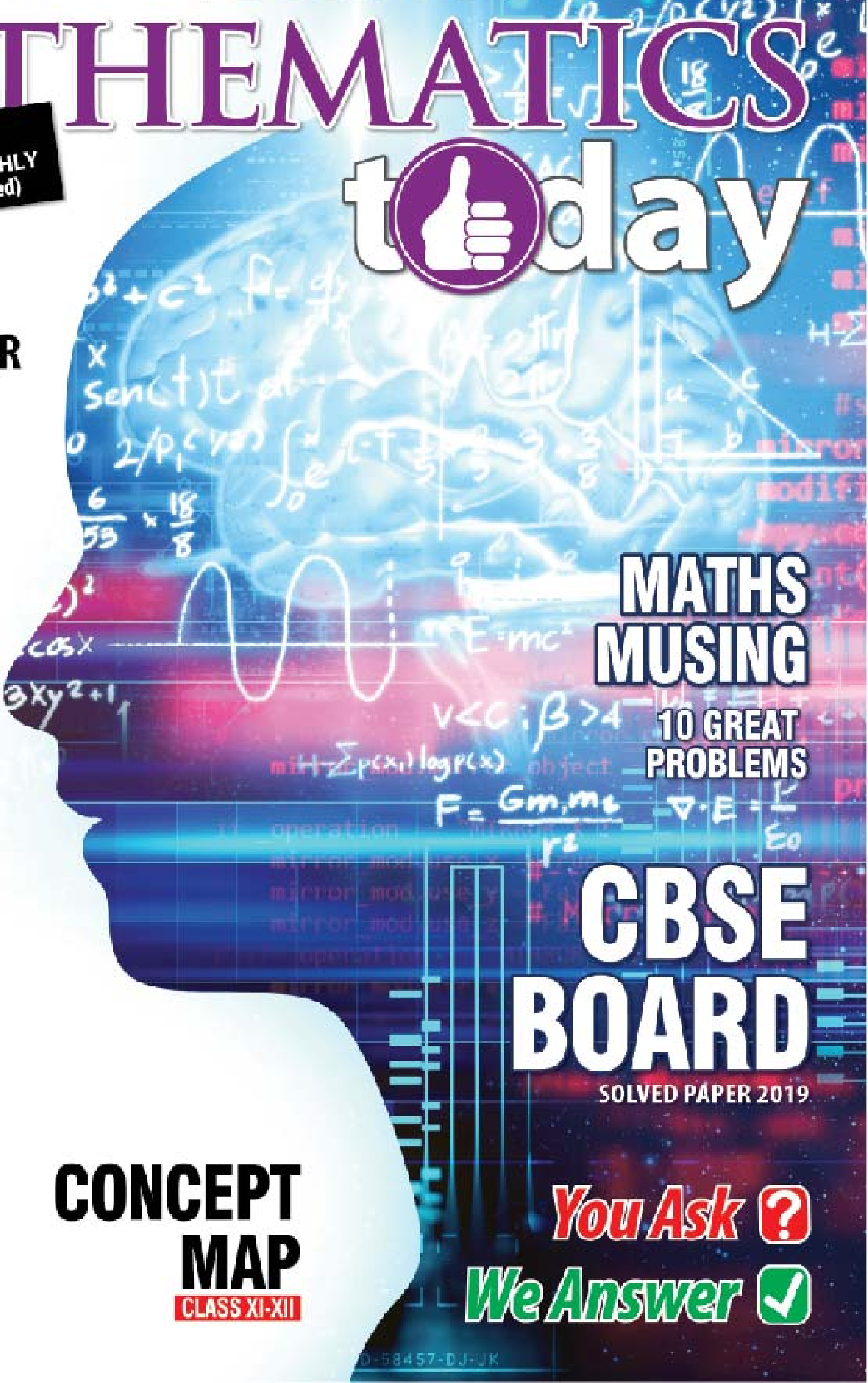

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10 GREAT PROBLEMS

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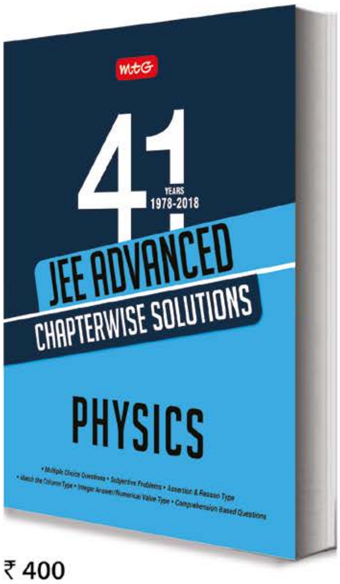

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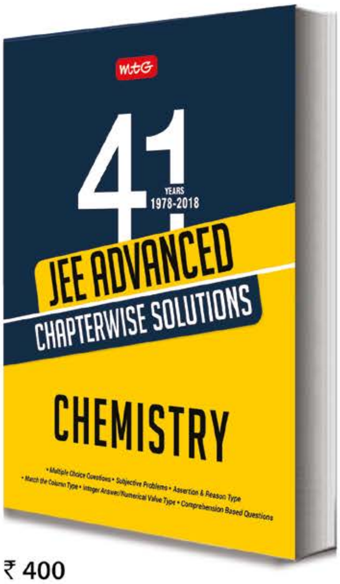

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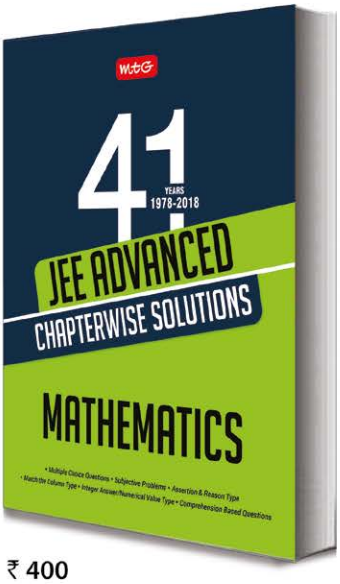

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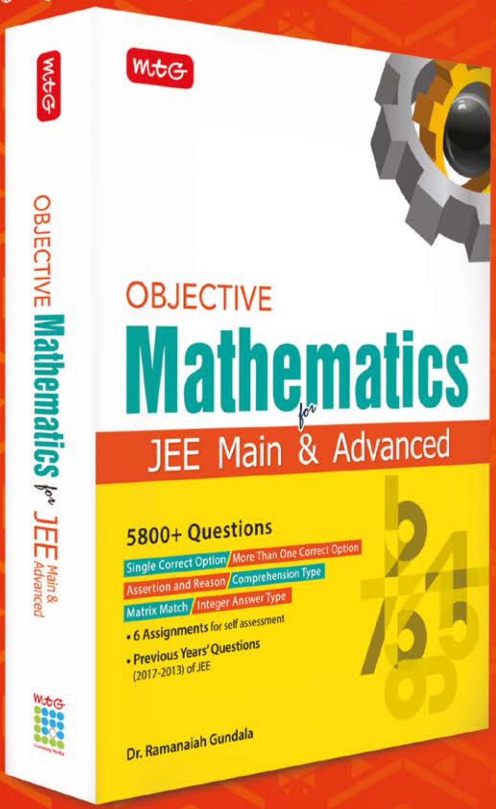
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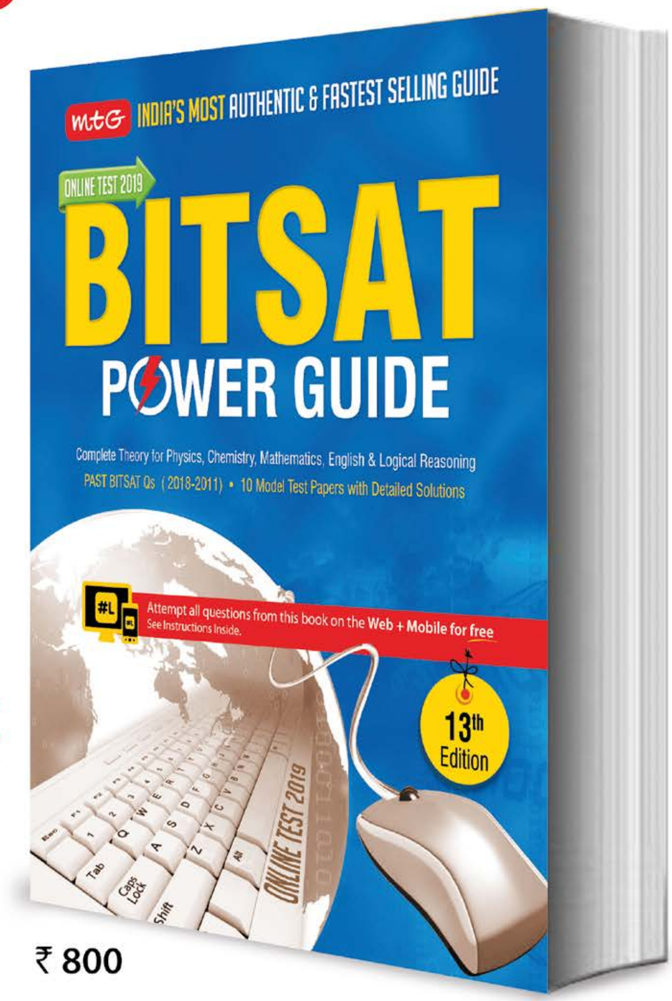

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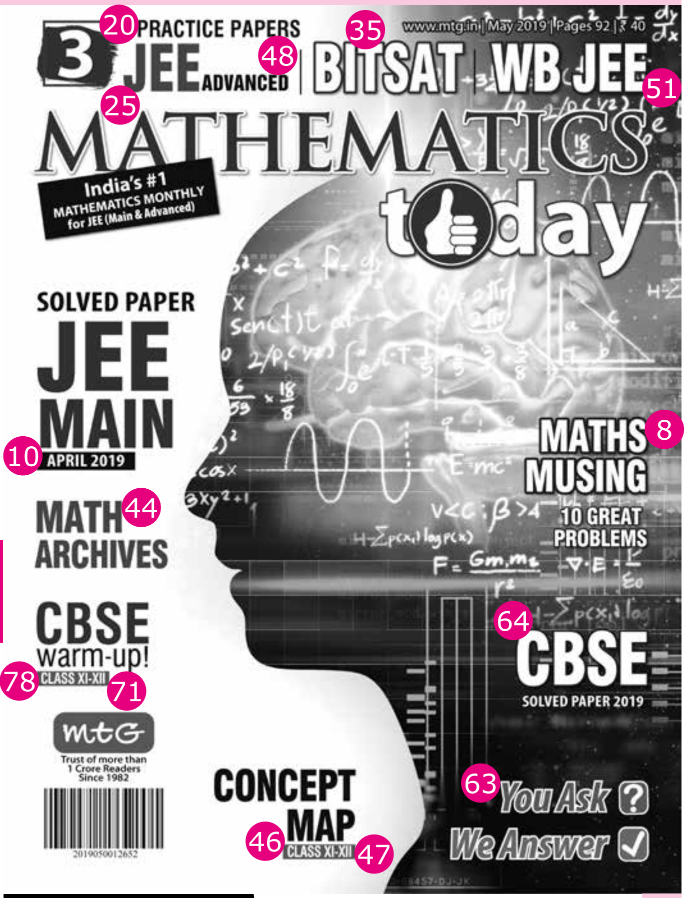

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM
Set 197

JEE MAIN

1. If $f(x)=\begin{cases}\dfrac{x\log\cos x}{\log(1+x^2)}, & x\neq 0\\ 0 \quad , & x=0\end{cases}$, then

(a) $f(x)$ is differentiable at $x = 0$
(b) $f(x)$ is continuous at $x = 0$
(c) $f(x)$ is not differentiable at $x = 0$
(d) Both (a) and (b)

2. If $f(x)=\begin{cases}e^{\cos x}\sin x, & \text{for } |x|\le 2\\ 2, & \text{otherwise}\end{cases}$, then $4\int_{-2}^{3} f(x)dx$ is

(a) 1 (b) 2 (c) 3 (d) 4

3. The general solution of the equation

$$\sum_{r=1}^{n}\cos(r^2x)\sin(rx)=\frac{1}{2} \text{ is}$$

(a) $2m\pi+\frac{\pi}{6}, m\in I$ (b) $\frac{(4m+1)}{n(n+1)}\cdot\frac{\pi}{2}, m\in I$

(c) $\frac{(4m-1)}{n(n+1)}\cdot\frac{\pi}{2}, m\in I$ (d) none of these

4. The line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the curve $xy = c^2$, $z = 0$, if c is equal to

(a) ±1 (b) $\pm 1/3$ (c) $\pm\sqrt{5}$ (d) none of these

5. $\theta=\tan^{-1}(2\tan^2\theta)-\tan^{-1}\left\{\left(\frac{1}{3}\right)\tan\theta\right\}$, if

(a) $\tan\theta = -2$ (b) $\tan\theta = -1$
(c) $\tan\theta = 3$ (d) $\tan\theta = 2$

JEE ADVANCED

6. Let $\vec{b}=4\hat{i}+3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy plane. All vectors in the same plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$ respectively, are given by

(a) $2\hat{i}+\hat{j}$ (b) $2\hat{i}-\hat{j}$

(c) $(-2\hat{i}+11\hat{j})/5$ (d) $(2\hat{i}+11\hat{j})/5$

COMPREHENSION

Consider the lines $L_1:\frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}$, $L_2:\frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$.

7. The unit vector perpendicular to both L_1 and L_2 is

(a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

8. The shortest distance between L_1 and L_2 is

(a) 0 (b) $17/\sqrt{3}$
(c) $41/5\sqrt{3}$ (d) $17/5\sqrt{3}$

NUMERICAL ANSWER TYPE

9. If $F(x)=\frac{1}{x^2}\int_4^x\{4t^2-2F'(t)\}\,dt$, then $9F'(4)$ equals

MATRIX MATCH

10. Match the columns :

	Column-I		Column-II
P.	$f(x) = 2x^2 - \log x$ increases on	1.	(0, 1/2)
Q.	$f(x) = x - 2\sin x$ decreases on	2.	(2, ∞)
R.	$f(x) = x^2 e^{-x}$ decreases on	3.	(– ∞, 0)

	P	Q	R
(a)	2	3	1, 3
(b)	2	1	2, 3
(c)	3	2	1
(d)	2	3	1, 2

See Solution Set of Maths Musing 196 on page no. 85

KNOWLEDGE SERIES

(for JEE / Olympiad Aspirants)

Myth

Before few days of JEE advanced only practice difficult and new questions.

Reality

Before any examination, revise all your concepts and practice same pattern questions and try to avoid new questions.

Suggestion - Just before seven days of exam, practice earlier JEE Advanced papers.

Please visit youtube to watch more myth and reality discussions on KCS EDUCATE channel.

Check your concepts & win prizes.

Knowledge Quiz - 18

If $x^2 + y^2 + z^2 = 1$, then what is the maximum value of $x + 2y + 3z$?

Please send your detailed solution before 15th May 2019 to **quiz@kcseducate.in** along with your name, father's name, class, school, address and contact number.

Winner Knowledge Quiz - 17

- **Vidisha Wahal (Class - XII), Mansukhbhai Kothari National School, Pune, Maharashtra.**
- **Pranay Kumar, (Class - XII), Kendriya Vidyalaya No.2, Kharagpur, West Bengal.**
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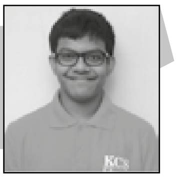

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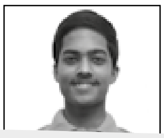

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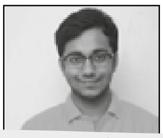

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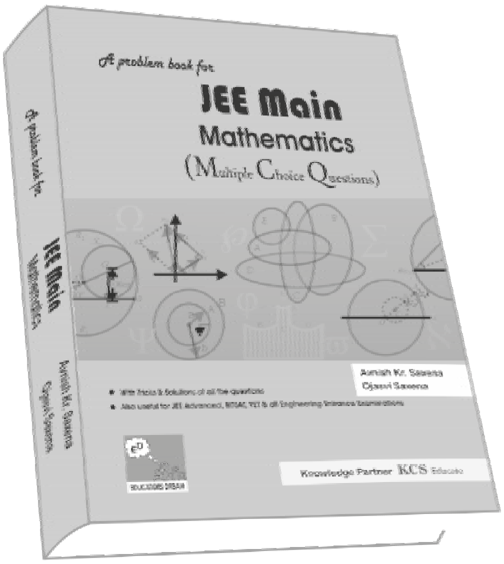

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JEE MAIN 2019

Held on 8th April (Morning Shift)

SOLVED PAPER

1. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is :
   (a) 3 (b) 5 (c) 4 (d) 2

2. The greatest value of $c \in R$ for which the system of linear equations
$$x - cy - cz = 0$$
$$cx - y + cz = 0$$
$$cx + cy - z = 0$$
has a non-trivial solution, is :
   (a) $\frac{1}{2}$ (b) -1 (c) 2 (d) 0

3. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is :
   (a) 45 (b) 48 (c) 40 (d) 49

4. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point (1, 1, 0) is :
   (a) $x - 3y - 2z = -2$ (b) $2x - z = 2$
   (c) $x - y - z = 0$ (d) $x + 3y + z = 4$

5. The sum of the series $2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 \cdot {}^{20}C_2 + 11 \cdot {}^{20}C_3 + \ldots + 62 \cdot {}^{20}C_{20}$ is equal to :
   (a) 2^{25} (b) 2^{24} (c) 2^{26} (d) 2^{23}

6. The length of the perpendicular from the point (2, –1, 4) on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is :
   (a) greater than 2 but less than 3
   (b) greater than 3 but less than 4
   (c) greater than 4
   (d) less than 2

7. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to :
   (a) $\frac{4}{17}$ (b) $\frac{128}{17}$ (c) $\frac{2}{17}$ (d) $\frac{64}{17}$

8. If $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$, $\beta = \tan^{-1}\left(\frac{1}{3}\right)$, where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to :
   (a) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (b) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$
   (c) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (d) $\tan^{-1}\left(\frac{9}{14}\right)$

9. Let $f : [0, 2] \to R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2 - x)$, then ϕ is :
   (a) decreasing on (0, 2)
   (b) increasing on (0, 2)
   (c) decreasing on (0, 1) and increasing on (1, 2)
   (d) increasing on (0, 1) and decreasing on (1, 2)

10. The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is :
   (a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{6}$ (c) $3\sqrt{6}$ (d) $\sqrt{\frac{3}{2}}$

11. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then :
   (a) $S_1 = \{-2, 1\}$; $S_2 = \{0\}$
   (b) $S_1 = \{-1\}$; $S_2 = \{0, 2\}$
   (c) $S_1 = \{-2, 0\}$; $S_2 = \{1\}$
   (d) $S_1 = \{-2\}$; $S_2 = \{0, 1\}$

12. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in :
(a) 1st, 2nd and 4th quadrants
(b) 1st and 2nd quadrants
(c) 4th quadrant (d) 1st quadrant

13. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is :
(a) 160 (b) 105 (c) 210 (d) 320

14. If $f(x)=\log_e\left(\frac{1-x}{1+x}\right)$, $|x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to :
(a) $2f(x^2)$ (b) $-2f(x)$ (c) $(f(x))^2$ (d) $2f(x)$

15. If $\cos(\alpha+\beta)=\frac{3}{5}$, $\sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha,\beta<\frac{\pi}{4}$ then $\tan(2\alpha)$ is equal to :
(a) $\frac{63}{52}$ (b) $\frac{21}{16}$ (c) $\frac{63}{16}$ (d) $\frac{33}{52}$

16. The sum of the co-efficients of all even degree terms in x in the expansion of

$$(x+\sqrt{x^3-1})^6+(x-\sqrt{x^3-1})^6, (x>1)$$

is equal to :
(a) 26 (b) 32 (c) 29 (d) 24

17. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is :
(a) If you are a citizen of India, then you are born in india.
(b) If you are not a citizen of India, then you are not born in India.
(c) If you are born in India, then you are not a citizen of India.
(d) If you are not born in India, then you are not a citizen of India.

18. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct?
(a) $P(A|B) = 1$ (b) $P(A|B) \geq P(A)$
(c) $P(A|B) \leq P(A)$ (d) $P(A|B) = P(B) - P(A)$

19. The sum of the solutions of the equation $|\sqrt{x}-2|+\sqrt{x}(\sqrt{x}-4)+2=0$, $(x > 0)$ is equal to :
(a) 4 (b) 12 (c) 10 (d) 9

20. If $f(x)=\frac{2-x\cos x}{2+x\cos x}$ and $g(x) = \log_e x$, $(x > 0)$ then the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x))\,dx$ is :
(a) $\log_e e$ (b) $\log_e 3$ (c) $\log_e 2$ (d) $\log_e 1$

21. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is :
(a) 3121 (b) 3203 (c) 3303 (d) 3221

22. If $2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^2$, $x\in\left(0,\frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to :
(a) $\frac{\pi}{6}-x$ (b) $2x-\frac{\pi}{3}$ (c) $x-\frac{\pi}{6}$ (d) $\frac{\pi}{3}-x$

23. $\int\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to :
(where c is a constant of integration.)
(a) $2x + \sin x + 2\sin 2x + c$
(b) $x + 2\sin x + \sin 2x + c$
(c) $x + 2\sin x + 2\sin 2x + c$
(d) $2x + \sin x + \sin 2x + c$

24. The area (in sq. units) of the region $A=\{(x,y)\in R\times R|0\le x\le 3, 0\le y\le 4, y\le x^2+3x\}$ is :
(a) $\frac{53}{6}$ (b) $\frac{26}{3}$ (c) 8 (d) $\frac{59}{6}$

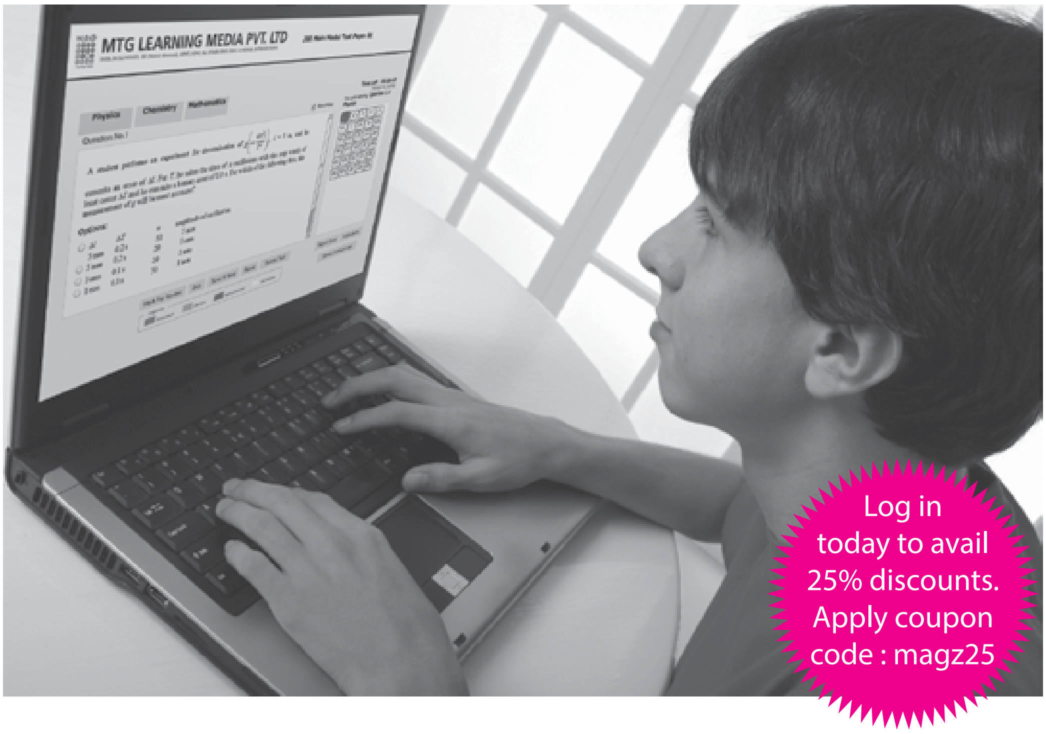
25. $\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals :
(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) 4

26. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is :
(a) 180 (b) 162 (c) 160 (d) 175

27. Let $A=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$, $(\alpha \in R)$ such that $A^{32}=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$. Then a value of α is :
(a) 0 (b) $\frac{\pi}{16}$ (c) $\frac{\pi}{64}$ (d) $\frac{\pi}{32}$

28. The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is :
(a) $\frac{7}{8}$ (b) 2 (c) $\frac{7}{4\sqrt{2}}$ (d) $\frac{11}{4\sqrt{2}}$

29. Let $O(0, 0)$ and $A(0, 1)$ be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is :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(a) $9x^2 + 8y^2 - 8y = 16$ (b) $8x^2 + 9y^2 - 9y = 18$
(c) $8x^2 - 9y^2 + 9y = 18$ (d) $9x^2 - 8y^2 + 8y = 16$

30. Let $y = y(x)$ be the solution of the differential equation, $(x^2+1)^2\dfrac{dy}{dx}+2x(x^2+1)y=1$ such that $y(0) = 0$. If $\sqrt{a}\ y(1)=\dfrac{\pi}{32}$, then the value of '$a$' is :

(a) $\dfrac{1}{16}$ (b) $\dfrac{1}{4}$ (c) 1 (d) $\dfrac{1}{2}$

SOLUTIONS

1. (c) : Given, α and β be the roots of the equation $x^2 - 2x + 2 = 0$.

Now, $x^2 - 2x + 2 = 0$

$\Rightarrow (x - 1)^2 + 1 = 0 \Rightarrow (x - 1)^2 = -1 = i^2$

$\Rightarrow x - 1 = \pm i \Rightarrow x = 1 + i, 1 - i$

Also, $\left(\dfrac{\alpha}{\beta}\right)^n = 1$ (Given)

If $\left(\dfrac{1+i}{1-i}\right)^n=1$, then $(i)^n = 1$

If $\left(\dfrac{1-i}{1+i}\right)^n=1$, then $(-i)^n = 1$

$\therefore (\pm i)^n = 1$

Thus, least value of n is 4.

2. (a) : For non-trivial solution, $\begin{vmatrix}1 & -c & -c\\ c & -1 & c\\ c & c & -1\end{vmatrix} = 0$

$\Rightarrow 1(1 - c^2) + c(-c - c^2) - c(c^2 + c) = 0$

$\Rightarrow 2c^3 + 3c^2 - 1 = 0 \Rightarrow (c + 1)^2 (2c - 1) = 0$

$\therefore$ Greatest value of $c = \dfrac{1}{2}$.

3. (b) : Let the remaining two observations are x_1 and x_2.

$\therefore$ Mean $(\bar{x})=\dfrac{2+4+10+12+14+x_1+x_2}{7}=8$

$\Rightarrow x_1 + x_2 = 14$ ...(i)

And variance $(\sigma^2) = \dfrac{\Sigma x_i^2}{n}-(\bar{x})^2$

$\Rightarrow 16=\dfrac{4+16+100+144+196+x_1^2+x_2^2}{7}-64$

$\Rightarrow 80=\dfrac{460+x_1^2+x_2^2}{7}$

$\Rightarrow x_1^2 + x_2^2 = 560 - 460 = 100$

$\Rightarrow x_1^2 + x_2^2 + 2x_1x_2 - 2x_1x_2 = 100$

$\Rightarrow (x_1 + x_2)^2 - 2x_1x_2 = 100$

$\Rightarrow x_1x_2=\dfrac{14^2-100}{2}$ [Using (i)]

$\Rightarrow x_1x_2 = 48$

4. (c) : The required plane is

$2x - y - 4 + \lambda (y + 2z - 4) = 0$ ...(i)

It passes through (1, 1, 0).

$\therefore 2 - 1 - 4 + \lambda(1 + 0 - 4) = 0$

$\Rightarrow -3 + \lambda(-3) = 0 \Rightarrow \lambda = -1$

Thus, the equation of plane is $2x - 2y - 2z = 0$

$\Rightarrow x - y - z = 0.$

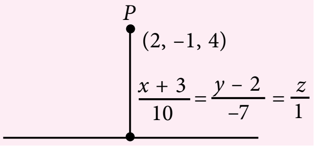
5. (a) : $2\cdot{}^{20}C_0 + 5\cdot{}^{20}C_1 + 8\cdot{}^{20}C_2 + 11\cdot{}^{20}C_3 + \ldots. + 62\cdot{}^{20}C_{20}$

$$=\sum_{r=0}^{20}(3r+2)^{20}C_r=3\sum_{r=0}^{20}r\,{}^{20}C_r+2\sum_{r=0}^{20}{}^{20}C_r$$

$$=3\sum_{r=1}^{20}r\,.\frac{20}{r}\,{}^{19}C_{r-1}+2\cdot2^{20}=3\times20\sum_{r=1}^{20}{}^{19}C_{r-1}+2\cdot2^{20}$$

$= 60\cdot2^{19} + 2\cdot2^{20} = 2^{20}(30 + 2) = 2^{20}\cdot 2^5 = 2^{25}$

6. (b) : Foot of perpendicular from a point on a line is $\vec{a}+\dfrac{\vec{b}\cdot(\vec{c}-\vec{a})\vec{b}}{\vec{b}\cdot\vec{b}}$, where



$\vec{a}=-3\hat{i}+2\hat{j},\vec{b}$

$=10\hat{i}-7\hat{j}+\hat{k},\ \vec{c}=2\hat{i}-\hat{j}+4\hat{k}$

$\therefore$ Foot of perpendicular

$=\vec{a}+\dfrac{(10\hat{i}-7\hat{j}+\hat{k})\cdot(5\hat{i}-3\hat{j}+4\hat{k})\vec{b}}{150}$

$=\vec{a}+\dfrac{(50+21+4)\vec{b}}{150}=\vec{a}+\dfrac{75}{150}\vec{b}=\vec{a}+\dfrac{\vec{b}}{2}$

$=\left(-3\hat{i}+2\hat{j}\right)+\dfrac{1}{2}\left(10\hat{i}-7\hat{j}+\hat{k}\right)=2\hat{i}-\dfrac{3}{2}\hat{j}+\dfrac{1}{2}\hat{k}$

$\therefore$ Length of perpendicular from (2, –1, 4) to $\left(2,-\dfrac{3}{2},\dfrac{1}{2}\right)=\sqrt{(2-2)^2+\left(-1+\dfrac{3}{2}\right)^2+\left(4-\dfrac{1}{2}\right)^2}$

$=\sqrt{\dfrac{1}{4}+\dfrac{49}{4}}=\sqrt{\dfrac{50}{4}}=\sqrt{\dfrac{25}{2}}=\dfrac{5}{\sqrt{2}}=3.54$

Thus, length of perpendicular is greater than 3 but less than 4.

7. (c) : The given equation of ellipse is $4x^2 + y^2 = 8$ ...(i)

Differentiating (i) w.r.t. x, we get

$$8x + 2y.\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-4x}{y}$$

Now, $\left(\frac{dy}{dx}\right)_{(1,\,2)} = \frac{-4}{2} = -2$ and $\left(\frac{dy}{dx}\right)_{(a,b)} = \frac{-4a}{b}$

Since, tangents are perpendicular to each other.

∴ $(-2)\left(-\frac{4a}{b}\right) = -1 \Rightarrow b = -8a$ ...(ii)

Also, (a, b) satisfy equation of ellipse.

∴ $4a^2 + b^2 = 8 \Rightarrow 4a^2 + (-8a)^2 = 8$ [Using (ii)]

⇒ $68a^2 = 8 \Rightarrow a^2 = \frac{2}{17}$.

8. (b) : Here $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$ and $\beta = \tan^{-1}\left(\frac{1}{3}\right)$

⇒ $\cos\alpha = \frac{3}{5}$ and $\tan\beta = \frac{1}{3}$

⇒ $\sin\alpha = \frac{4}{5}$, $\sin\beta = \frac{1}{\sqrt{10}}$, $\cos\beta = \frac{3}{\sqrt{10}}$

Now, $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$= \frac{4}{5} \times \frac{3}{\sqrt{10}} - \frac{3}{5} \times \frac{1}{\sqrt{10}} = \frac{9}{5\sqrt{10}}$$

$$\Rightarrow \alpha - \beta = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$$

9. (c) : We have, $\phi(x) = f(x) + f(2 - x)$

⇒ $\phi'(x) = f'(x) + f'(2 - x)(-1)$

$= f'(x) - f'(2 - x)$

Since $f''(x) > 0 \quad \forall\ x \in (0, 2)$ [Given]

∴ $f'(x)$ is increasing $\forall\ x \in (0, 2)$

Case I : If $x > 2 - x \Rightarrow 2x > 2 \Rightarrow x > 1$

⇒ $\phi'(x) > 0\ \forall\ x \in (1, 2)$

⇒ $\phi(x)$ is increasing on $(1, 2)$

Case II : If $x < 2 - x \Rightarrow 2x < 2 \Rightarrow x < 1$

⇒ $\phi'(x) < 0 \quad \forall\ x \in (0, 1)$

⇒ $\phi(x)$ is decreasing on $(0, 1)$

10. (d) : The vector perpendicular to the plane containing the vector $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$ is parallel to vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i}(3-2) - \hat{j}(3-1) + \hat{k}(2-1) = \hat{i} - 2\hat{j} + \hat{k}$$

∴ Magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on $\hat{i} - 2\hat{j} + \hat{k}$ is

$$\frac{|(2\hat{i} + 3\hat{j} + \hat{k}) \cdot (\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + \hat{k}|}$$

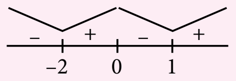
$$= \frac{|2 - 6 + 1|}{|\sqrt{1 + 4 + 1}|} = \frac{|-3|}{|\sqrt{6}|} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

11. (a) : Here $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

Now, $f'(x) = 36x^3 + 36x^2 - 72x$

$= 36x(x^2 + x - 2) = 36x(x - 1)(x + 2)$

Sign of $f'(x)$



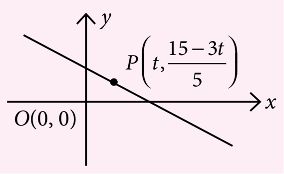
Since $f'(x)$ changes its sign from –ve to +ve about $x = -2, 1$. So, $x = -2, 1$ are the points of local minima.

∴ $S_1 = \{-2, 1\}$.

Since $f'(x)$ changes its sign from +ve to –ve about $x = 0$ only. So, $x = 0$ is a point of local maxima.

∴ $S_2 = \{0\}$

12. (b) : Let $P\left(t, \frac{15 - 3t}{5}\right)$ be any point on the straight line $3x + 5y = 15$.



∴ $\left|\frac{15 - 3t}{5}\right| = |t|$

⇒ $\frac{15 - 3t}{5} = t$ or $\frac{15 - 3t}{5} = -t$

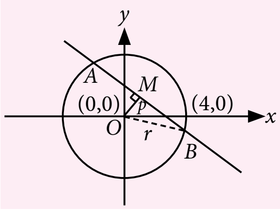
⇒ $15 - 3t = 5t$ or $15 - 3t = -5t$

$$\Rightarrow \quad 15 = 8t \text{ or } 15 = -2t \Rightarrow t = \frac{15}{8} \text{ or } t = \frac{-15}{2}$$

∴ Point P lies in 1st or 2nd quadrants.

13. (c) : Let p be the perpendicular distance from $O(0, 0)$ to line $x + y = n$.

$$\therefore \quad p = \frac{n}{\sqrt{2}} < 4 \text{ for } n = 1, 2, 3, 4, 5$$



Now, length of chord

$$= 2\sqrt{r^2 - p^2} = 2\sqrt{r^2 - \frac{n^2}{2}}$$

$$= 2\sqrt{16 - \frac{n^2}{2}} = \sqrt{64 - 2n^2}$$

Now, sum of the squares of the lengths of the chords $= 62 + 56 + 46 + 32 + 14 = 210$

14. (d) : Here, $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$

$$\text{Now, } f\left(\frac{2x}{1+x^2}\right) = \log_e\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$= \log_e\left(\frac{1+x^2-2x}{1+x^2+2x}\right) = \log_e\left(\frac{1-x}{1+x}\right)^2$$

$$= 2\log_e\left(\frac{1-x}{1+x}\right) = 2f(x)$$

15. (c) : Given, $\cos(\alpha + \beta) = \frac{3}{5} \Rightarrow \tan(\alpha + \beta) = \frac{4}{3}$

Also, $\sin(\alpha - \beta) = \frac{5}{13} \Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$

$\tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)]$

$$= \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \times \frac{5}{12}} = \frac{\frac{16+5}{12}}{1 - \frac{5}{9}} = \frac{21}{12} \times \frac{9}{4} = \frac{63}{16}$$

16. (d) : Even degree terms of given expression

$$= \left[{}^6C_0\, x^6 + {}^6C_2\, x^4\left(\sqrt{(x^3-1)}\right)^2 + {}^6C_4 x^2\left(\sqrt{x^3-1}\right)^4 + {}^6C_6 x^0\left(\sqrt{x^3-1}\right)^6\right] + \left[{}^6C_0 x^6 + {}^6C_2 x^4(-\sqrt{x^3-1})^2 + {}^6C_4 x^2\left(-\sqrt{x^3-1}\right)^4 + {}^6C_6 x^0\left(-\sqrt{x^3-1}\right)^6\right]$$

$$= 2[{}^6C_0 x^6 + {}^6C_2(x^7 - x^4) + {}^6C_4\,(x^8 + x^2 - 2x^5) + {}^6C_6\,(x^9 - 1 - 3x^6 + 3x^3)$$

∴ Required sum of coefficients

$= 2[{}^6C_0 - {}^6C_2 + 2 \times {}^6C_4 - {}^6C_6 - 3 \times {}^6C_6]$

$= 2[1 - 15 + 30 - 1 - 3] = 24$

17. (b) : The contrapositive of the statement "If you are born in India, then you are a citizen of India", is "If you are not a citizen of India, then you are not born in India".

18. (b) : Since, A and B are two non-null sets such that $A \subset B$, so $A, B \neq \phi$ and $A \cap B = A$

Using multiplication theorem, we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \geq P(A)$$

$$[\because \quad P(B) \leq 1 \Rightarrow \frac{1}{P(B)} \geq 1 \Rightarrow \frac{P(A)}{P(B)} \geq P(A)]$$

19. (c) : We have,

$$|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0, (x > 0)$$

$$\Rightarrow |\sqrt{x} - 2| + x - 4\sqrt{x} + 2 = 0$$

$$\Rightarrow |\sqrt{x} - 2| + x - 4\sqrt{x} + 2 + 2 - 2 = 0$$

$$\Rightarrow |\sqrt{x} - 2| + (\sqrt{x} - 2)^2 - 2 = 0 \qquad \text{...(i)}$$

Let $|\sqrt{x} - 2| = t$, then (i) becomes

$t^2 + t - 2 = 0 \Rightarrow t^2 + 2t - t - 2 = 0$

$\Rightarrow \quad t(t + 2) - 1(t + 2) = 0 \Rightarrow (t - 1)(t + 2) = 0$

$\Rightarrow \quad t = 1, t = -2$

Since $|\sqrt{x} - 2| = -2$ not possible

So, $\sqrt{x} - 2 = \pm 1 \Rightarrow \sqrt{x} = 3, \sqrt{x} = 1$

$\Rightarrow \quad x = 9, x = 1$

So, required sum $= 9 + 1 = 10$

20. (d) : $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ and $g(x) = \log_e x$

$$\Rightarrow \quad g(f(x)) = g\left(\frac{2 - x\cos x}{2 + x\cos x}\right) = \log_e\left(\frac{2 - x\cos x}{2 + x\cos x}\right)$$

$= h(x)$ (say)

$$\text{Now, } h(-x) = \log_e\left(\frac{2 + x\cos(-x)}{2 - x\cos(-x)}\right)$$

$$= \log_e\left(\frac{2 + x\cos x}{2 - x\cos x}\right) = \log_e\left(\frac{2 - x\cos x}{2 + x\cos x}\right)^{-1}$$

$$= -\log_e\left(\frac{2 - x\cos x}{2 + x\cos x}\right) = -h(x)$$

$$\text{Hence, } \int_{-\pi/4}^{\pi/4} g(f(x))dx = \int_{-\pi/4}^{\pi/4} \log_e\left(\frac{2 - x\cos x}{2 + x\cos x}\right)dx = 0$$

[∵ Function is odd]

$= \log_e 1$

21. (a) : We have, $100 < n < 200$ and H.C.F. $(91, n) > 1$

So, n can be multiple of 7 or 13 or both $[\because 91 = 7 \times 13]$

Let S_P = Sum of all natural numbers which are divisible by 7 and lie between 100 and 200

S_Q = Sum of all natural numbers which are divisible by 13 and lie between 100 and 200

S_R = Sum of all natural numbers which are divisible by both 7 and 13 and lie between 100 and 200

So, $S_P = 105 + 112 + \dots + 196$

$$= \frac{14}{2}[105 + 196] = 7(301) = 2107$$

$$S_Q = 104 + \dots + 195 = \frac{8}{2}[104 + 195] = 4(299) = 1196$$

$S_R = 182$

$\therefore$ Required Sum $= S_P + S_Q - S_R$

$= 2107 + 1196 - 182 = 3121$

22. (c) : We have, $2y = \left(\cot^{-1}\left(\dfrac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2$

$$= \left[\cot^{-1}\left(\frac{\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x}{\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x}\right)\right]^2 = \left[\cot^{-1}\left(\frac{\sin\left(x + \frac{\pi}{3}\right)}{\cos\left(x + \frac{\pi}{3}\right)}\right)\right]^2$$

$$= \left[\cot^{-1}\left(\tan\left(x + \frac{\pi}{3}\right)\right)\right]^2 = \left[\frac{\pi}{2} - \tan^{-1}\left(\tan\left(x + \frac{\pi}{3}\right)\right)\right]^2$$

$$= \left[\frac{\pi}{2} - \left(x + \frac{\pi}{3}\right)\right]^2 = \left[\frac{\pi}{6} - x\right]^2$$

$$\Rightarrow \quad 2\frac{dy}{dx} = 2\left(\frac{\pi}{6} - x\right)(-1) \quad \Rightarrow \quad \frac{dy}{dx} = x - \frac{\pi}{6}$$

23. (b) : Let $I = \displaystyle\int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\frac{x}{2}}dx = \int \frac{2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}dx$

$$= \int \frac{\sin\left(\frac{5x}{2} + \frac{x}{2}\right) + \sin\left(\frac{5x}{2} - \frac{x}{2}\right)}{\sin x}dx$$

$$= \int \frac{\sin(3x) + \sin 2x}{\sin x}dx$$

$$= \int \frac{3\sin x - 4\sin^3 x + 2\sin x\cos x}{\sin x}dx$$

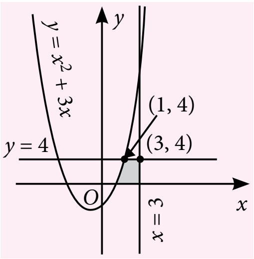
$$= \int (3 - 4\sin^2 x + 2\cos x)dx$$

$$= \int \left(3 - 4\left(\frac{1 - \cos 2x}{2}\right) + 2\cos x\right)dx$$

$$= \int (3 - 2 + 2\cos 2x + 2\cos x)dx$$

$$= x + 2\frac{\sin 2x}{2} + 2\sin x + C = x + \sin 2x + 2\sin x + C$$

24. (d) : Required area



$$= \int_0^1 (x^2 + 3x)dx + \int_1^3 4\,dx$$

$$= \left[\frac{x^3}{3} + \frac{3}{2}x^2\right]_0^1 + [4x]_1^3$$

$$= \left[\left(\frac{1}{3} + \frac{3}{2}\right) - (0)\right] + [12 - 4]$$

$$= \frac{11}{6} + 8 = \frac{59}{6} \text{ sq. units}$$

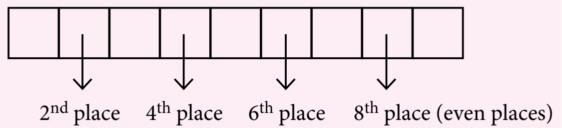
25. (c) : $\displaystyle\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}}$ $\quad \left(\dfrac{0}{0}\text{ form}\right)$

So, using L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{2\sin x\cos x}{\frac{-1}{2\sqrt{1 + \cos x}} \times (-\sin x)}$$

$$= \lim_{x\to 0} 2\cos x \times 2\sqrt{1 + \cos x} = 4\sqrt{2}$$

26. (a) : The given digits are 1, 1, 2, 2, 2, 2, 3, 4, 4.



Out of given digits, 3 are odd and 6 are even.

$\therefore$ Required number of such numbers in which odd digits occupy even places

$$= {}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{4!2!} = 4 \times 3 \times \frac{6 \times 5}{2} = 180$$

27. (c) : We have, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, $\alpha \in R$

$$\Rightarrow \quad A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \cdot \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & -\sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha + \cos\alpha\sin\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$=\begin{bmatrix}\cos 2\alpha\cos\alpha-\sin 2\alpha\sin\alpha & -\cos 2\alpha\sin\alpha-\sin 2\alpha\cos\alpha\\ \sin 2\alpha\cos\alpha+\cos 2\alpha\sin\alpha & -\sin 2\alpha\sin\alpha+\cos 2\alpha\cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos(2\alpha+\alpha) & -\sin(2\alpha+\alpha)\\ \sin(2\alpha+\alpha) & \cos(2\alpha+\alpha)\end{bmatrix}=\begin{bmatrix}\cos 3\alpha & -\sin 3\alpha\\ \sin 3\alpha & \cos 3\alpha\end{bmatrix}$$

Thus, we get

$$A^{32}=\begin{bmatrix}\cos 32\alpha & -\sin 32\alpha\\ \sin 32\alpha & \cos 32\alpha\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

$\Rightarrow$ $\cos 32\alpha = 0$ and $\sin 32\alpha = 1$

$$\Rightarrow \tan 32\alpha = \tan\frac{\pi}{2} \quad \Rightarrow \quad 32\alpha = n\pi+\frac{\pi}{2},\ n\in Z$$

For $n = 0$, $\alpha=\frac{\pi}{64}$

28. (c) : We are given the line

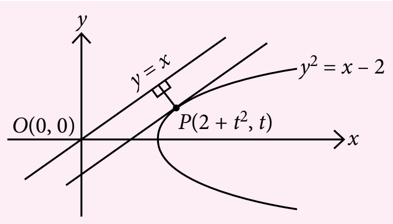
$$y = x \Rightarrow \frac{dy}{dx}=1$$

and the curve

$y^2 = x - 2$

$$\Rightarrow 2y\frac{dy}{dx}=1$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2y}$$



Let $P(t^2 + 2, t)$ be any point on the curve.

$$\text{Thus, } \left(\frac{dy}{dx}\right)_{(t^2+2,\,t)}=\frac{1}{2t}$$

$$\text{Now, } \frac{1}{2t}=1 \Rightarrow 2t=1 \Rightarrow t=\frac{1}{2}$$

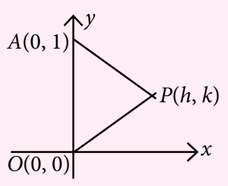
$$\text{So, } P\equiv\left(\frac{9}{4},\frac{1}{2}\right)$$

$$\text{Hence, shortest distance } = \frac{\left|\frac{9}{4}-\frac{1}{2}\right|}{\sqrt{1^2+1^2}}=\frac{7}{4\sqrt{2}} \text{ units}$$

29. (a) : Let coordinates of points P be (h, k).

Since, perimeter of $\Delta AOP = 4$

$\Rightarrow AP + OA + OP = 4$



$$\Rightarrow \sqrt{(h-0)^2+(k-1)^2}+1+\sqrt{h^2+k^2}=4$$

$$\Rightarrow \sqrt{h^2+k^2+1-2k}+\sqrt{h^2+k^2}=3$$

$$\Rightarrow \sqrt{h^2+k^2-2k+1}=3-\sqrt{h^2+k^2}$$

$$\Rightarrow h^2 + k^2 - 2k + 1 = 9 + h^2 + k^2 - 6\sqrt{h^2+k^2}$$

$$\Rightarrow -2k - 8 = -6\sqrt{h^2+k^2} \quad \Rightarrow \quad k + 4 = 3\sqrt{h^2+k^2}$$

$$\Rightarrow k^2 + 16 + 8k = 9(h^2 + k^2) \Rightarrow 9h^2 + 8k^2 - 8k = 16$$

Thus, the locus of point P is $9x^2 + 8y^2 - 8y = 16$

30. (a) : We have, $(x^2+1)^2\frac{dy}{dx}+2x(x^2+1)y=1$

$$\Rightarrow \frac{dy}{dx}+\left(\frac{2x}{x^2+1}\right)y=\frac{1}{(x^2+1)^2},$$

which is a linear differential equation.

$$\therefore \text{ I.F.} = e^{\int\frac{2x}{x^2+1}dx}=e^{\log(x^2+1)}=x^2+1$$

So, the required solution is

$$y \times \text{I.F.} = \int \text{I.F.}\times\frac{1}{(x^2+1)^2}dx+C$$

$$\Rightarrow y(x^2+1)=\int\frac{1}{x^2+1}dx+C=\tan^{-1}x+C$$

Using initial condition, $y(0) = 0$, we get

$0(0 + 1) = \tan^{-1} 0 + C \Rightarrow C = 0$

So, the solution is $y(x^2 + 1) = \tan^{-1} x$

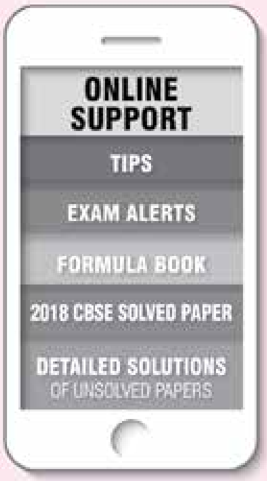
$$\Rightarrow y=\frac{\tan^{-1}x}{x^2+1}$$

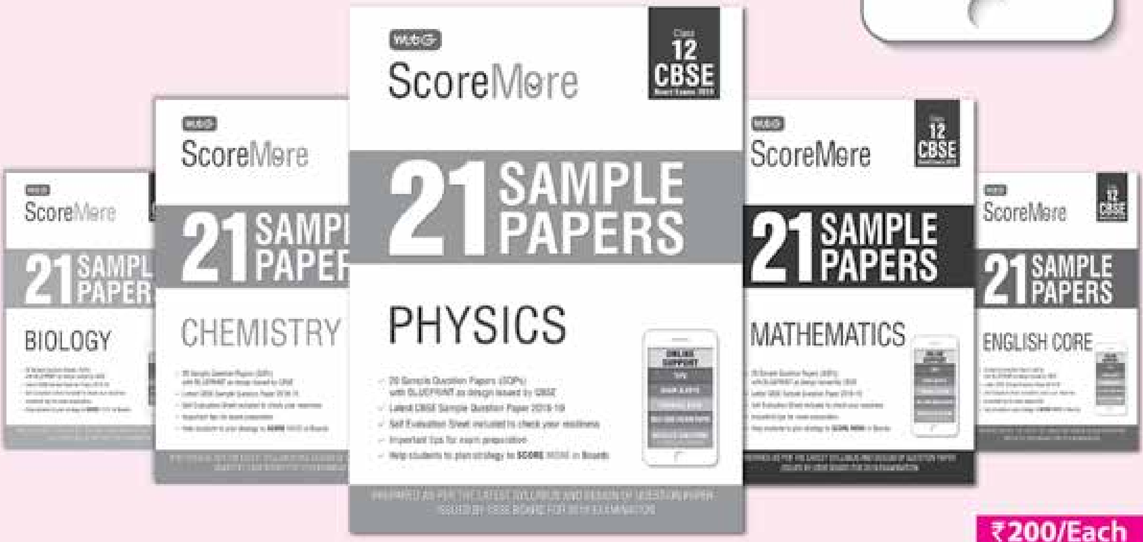
$$\text{Now, we have, } \sqrt{a}\ y(1)=\frac{\pi}{32}$$

$$\Rightarrow \sqrt{a}\left(\frac{\tan^{-1}(1)}{1+1}\right)=\frac{\pi}{32} \quad \Rightarrow \quad \sqrt{a}\left(\frac{\pi/4}{2}\right)=\frac{\pi}{32}$$

$$\Rightarrow \sqrt{a}=\frac{1}{4}\Rightarrow a=\frac{1}{16}$$

◆

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*ALOK KUMAR

Single Option Correct Type

1. Equation of angle bisector of the lines $3x - 4y + 1 = 0$ and $12x + 5y - 3 = 0$ containing the point (1, 2) is
(a) $3x + 11y - 4 = 0$ (b) $99x - 27y - 2 = 0$
(c) $3x + 11y + 4 = 0$ (d) $99x + 27y - 2 = 0$

2. Coefficient of x^{2009} in $(1 + x + x^2 + x^3 + x^4)^{1001}\ (1 - x)^{1002}$ is
(a) 0 (b) $4 \cdot {}^{1001}C_{501}$
(c) -2009 (d) none of these

3. If $P(z)$ and $A(z_1)$ be two variable points such that $zz_1 = |z|^2$ and $|z - \bar{z}\,| + |z_1 + \bar{z}_1| = 10$, then area enclosed by the curve formed by them is
(a) 25π (b) 20π (c) 50 (d) 100

4. Let $f(x) = x^2 + bx + c$, b is negative odd integer, $f(x) = 0$ has two distinct prime numbers as roots and $b + c = 15$. Then least value of $f(x)$ is
(a) $\frac{-233}{4}$ (b) $\frac{233}{4}$
(c) $-\frac{225}{4}$ (d) none of these

5. A square matrix A is said to be nilpotent of index m if $A^m = 0$, now, if for this A
$(I - A)^n = I + A + A^2 + \ldots + A^{m-1}$, then n is equal to
(a) 0 (b) m (c) $-m$ (d) -1

6. The sum of all the coefficients of those terms in the expansion of $(a + b + c + d)^8$ which contain b but not c is
(a) 6305 (b) 6561 (c) 256 (d) 4^8

7. Two cards are selected at random from a pack of ordinary playing cards. If they are found to be of different colours (Red & Black), then conditional probability that both are face cards is
(a) $\frac{36}{325}$ (b) $\frac{18}{169}$
(c) $\frac{9}{169}$ (d) none of these

8. The equation of normal to the curve $x + y = x^y$, where it cuts the x-axis is
(a) $y = x - 1$ (b) $x + y = 1$
(c) $12x + y + 2 = 0$ (d) $3x + y = 3$

One or More Than One Option(s) Correct Type

9. If 2 cos$x$ + sin$x$ = 1, then value of 7 cos$x$ + 6 sin$x$ can be
(a) 2 (b) 6 (c) 3 (d) 1

10. In a gambling between Mr. A and Mr. B, a machine continues tossing a fair coin until the two consecutive throws either HT or TT are obtained for the first time. If it is HT, Mr. A wins and if it is TT, Mr. B wins. Which of the following is (are) true?
(a) Probability of Mr. A winning is 3/4.
(b) Probability of Mr. B winning is 1/4.
(c) Given first toss is head, probability of Mr. A winning is 1.
(d) Given first toss is tail, probability of Mr. A winning is 1/2.

11. The projection of line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ on the plane $3x + 2y + z = 0$ is
(a) $\frac{x+1}{11} = \frac{y-1}{-9} = \frac{z-1}{-15}$
(b) $3x - 8y + 7z + 4 = 0 = 3x + 2y + z$
(c) $\frac{x+12}{11} = \frac{y+8}{-9} = \frac{z+14}{15}$
(d) $\frac{x+12}{11} = \frac{y+8}{-9} = \frac{z+14}{-15}$

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*Alok Kumar, a B Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

12. Let $f:\left[0,\frac{\pi}{2}\right]\to[0,1]$ be a differentiable function such that $f(0)=0,\ f\left(\frac{\pi}{2}\right)=1,$ then

(a) $f'(\alpha)=\sqrt{1-(f(\alpha))^2}$ for all $\alpha\in\left(0,\frac{\pi}{2}\right)$

(b) $f'(\alpha)=\frac{2}{\pi}$ for all $\alpha\in\left(0,\frac{\pi}{2}\right)$

(c) $f(\alpha)f'(\alpha)=\frac{1}{\pi}$ for at least one $\alpha\in\left(0,\frac{\pi}{2}\right)$

(d) $f'(\alpha)=\frac{8\alpha}{\pi^2}$ for at least one $\alpha\in\left(0,\frac{\pi}{2}\right)$

Comprehension Type

Paragraph for Q. No. 13 to 15

Let $(1+x)^m = C_0 + C_1x + C_2x^2 + \ldots + C_mx^m$, where $C_r = {}^mC_r = \frac{m!}{r!(m-r)!}$

$$\sum_{0\le i<j\le n}\sum f(i,j)=\sum_{0\le i<j\le n} f(n-i,n-j), \text{ if } f(i,j)=f(j,i)$$

13. $\sum_{0\le i<j\le m}\sum (i+j)\,C_i\,C_j$ is equal to

(a) $m(2^{m-1})$
(b) $m(2^{2m-1}-{}^{2m-1}C_{m-1})$
(c) $m(2^{m-1}-{}^{2m-1}C_{m-1})$
(d) none of these

14. $\sum_{0\le i<j\le m}\sum i\cdot C_j$ is equal to

(a) $m(m-1)\,2^{m-2}$
(b) $m(m-1)2^{m-3}$
(c) 2^{2m}
(d) $2^{2m}-{}^{2m}C_m$

15. $\sum_{0\le i<j\le m}\sum C_i^2$ is equal to

(a) $({}^{2m}C_m)^2$
(b) $(2^{2m}-{}^{2m}C_m)^2$
(c) $m\ {}^{2m-1}C_{m-1}$
(d) none of these

Matrix-Match Type

16. Let ABC be an isosceles triangle with $AB = AC$. If AB lies along $x + y = 10$ and AC lies along $7x - y = 30$ and area of triangle is 20 sq. units, then match the following :

Column-I		Column-II	
(A)	Coordinates of point B are	(p)	(10, 0)
(B)	Coordinates of point C are	(q)	(4, –2)
(C)	Centroid of ΔABC is	(r)	$\left(\frac{-5}{2},\frac{5}{2}\right)$
(D)	Circumcentre of ΔABC is	(s)	$\left(3,\frac{13}{3}\right)$

17. Match the following :

Column-I (Equation of pair of curves)		Column-II (Equation of a common tangent)	
(A)	$xy=-4$ and $x^2+16y=0$	(p)	$x+2y+1=0$
(B)	$x^2=8y$ and $y^2=x$	(q)	$x-y+4=0$
(C)	$7x^2+25y^2-175=0$ and $x^2+y^2-16=0$	(r)	$x+y+4\sqrt{2}=0$
(D)	$x^2+y^2-4=0$ and $x^2+y^2-8x+15=0$	(s)	$y=\frac{1}{\sqrt{15}}(x-8)$

SOLUTION SENDERS

Samurai Sudoku (April)

Kalur Nikhil	(Hyderabad)
Sriyaan Kodeboyena	(Hyderabad)
Sandeepa Dhara	(West Bengal)

Mathdoku (April)

Devjit Acharjee	(West Bengal)
Arkamitra Roy	(West Bengal)
Vishwa Banner	(Hyderabad)
Kalur Nikhil	(Hyderabad)
Sriyaan Kodeboyena	(Hyderabad)
Sai Kiran D	(Tamil Nadu)
Debesh Pattnaik	(Hyderabad)
Sulagna Nandimondal	

Samurai Sudoku (March)

Varun Chaturvedi	(Gujarat)
Shailaja Tekaday	(Maharashtra)

Mathdoku (March)

Rajashree Tekaday	(Maharashtra)
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Gomati Goswami	(Mumbai)
Satdhruti Paul	(Kerala)
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N. Jayanthi	(Hyderabad)

Maths Musing Set - 195

- Ravinder Gajula

SOLUTIONS

1. (b) : Since 3 × 1 – 4 × 2 + 1 and 12 × 1 + 5 × 2 – 3 are of the opposite sign, so required angle bisector is given by

$$\frac{3x-4y+1}{5}=-\left(\frac{12x+5y-3}{13}\right)$$

2. (a) : $(1 + x + x^2 + x^3 + x^4)^{1001}\ (1 - x)^{1002}$
$= (1 - x)\ (1 - x^5)^{1001}$, so all the powers of x will be of the form $5m$ or $5m + 1$ $(m \in I)$
So coefficient of x^{2009} will be 0.

3. (c) : $zz_1 = |z|^2 = z\bar{z} \Rightarrow z_1=\bar{z}$
Now let $z = x + iy \Rightarrow z_1 = x - iy$
Now, $|z - \bar{z}| + |z_1 + \bar{z}_1| = 10 \Rightarrow |x| + |y| = 5$
which represents a square of area 50 sq. units.

4. (c) : Let the roots are α, β
$\alpha + \beta = -b, \alpha\beta = c$
$\Rightarrow$ α is even, β is odd $\Rightarrow \alpha = 2.$
Now $b + c = 15$
$\Rightarrow -2 - \beta + 2\beta = 15 \Rightarrow \beta = 17$
So $f(x) = x^2 - 19x + 34$ and least value of $f(x)$ is $-\frac{225}{4}$.

5. (d) : Let $B = I + A + A^2 + ... + A^{m-1}$
$\Rightarrow B\ (I - A) = (I + A + A^2 + ... + A^{m-1})\ (I - A)$
$= I - A^m = I \qquad (\because A^m = 0)$
$\Rightarrow B = (I - A)^{-1} \Rightarrow n = -1$

6. (a) : Sum of the coefficients of the terms not containing c is 3^8 and of the terms not containing b and c both is 2^8, so required sum $= 3^8 - 2^8$.

7. (b) : Let A be the event that they are face cards, and B be the event that they are of different colours.

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{{}^{12}C_2-2\times{}^{6}C_2}{13\times 26}=\frac{18}{169}$$

8. (a) : At x-axis, $y = 0 \Rightarrow x = 1$
Now, $x + y = x^y \Rightarrow \ln (x + y) = y \ln x$
Differentiating both sides, we get

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)=\frac{y}{x}+\frac{dy}{dx}\ln x\ ;\ \left(\frac{dy}{dx}\right)_{(1,0)}=-1$$

So equation of normal is $y - 0 = x - 1$.

9. (a, b) : Given, $2 \cos x + \sin x = 1$,
$\Rightarrow 2 \cos x = 1 - \sin x$
On squaring, we have $4\cos^2 x = 1 - 2\sin x + \sin^2 x$

$$\Rightarrow 5\sin^2 x-2\sin x-3=0 \Rightarrow \sin x=1,-\frac{3}{5}$$

When $\sin x = 1$, then $\cos x = 0$ and so, $7 \cos x + 6 \sin x = 6$

When $\sin x=-\frac{3}{5}$, then $\cos x=\frac{4}{5}$

$\therefore$ $7 \cos x + 6 \sin x = 2$
Hence, $7 \cos x + 6 \sin x = 6$ or 2

10. (a, b, c, d) : If T comes in first toss then Mr. B can win in only one case that is TT.
$\Rightarrow$ Probability of Mr. B winning = 1/4
$\Rightarrow$ Probability of Mr.A winning = 3/4
Given first toss is head. Then the successive tosses in which Mr. A can win are T,HT, HHT, ...

$$\text{Probability}=\frac{1/2}{1-\frac{1}{2}}=1$$

Given first toss is tail. Then the successive tosses in which Mr. A can win are HT, HHT, HHHT,.....

$$\text{Probability}=\frac{1/4}{1-\frac{1}{2}}=\frac{1}{2}$$

11. (a, b) : Equation of a plane passing through the line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ is
$3x - y + 2z - 1 + \lambda(x + 2y - z - 2) = 0$
Since it is perpendicular to the given plane.

$$\therefore\ \lambda=-\frac{3}{2}$$

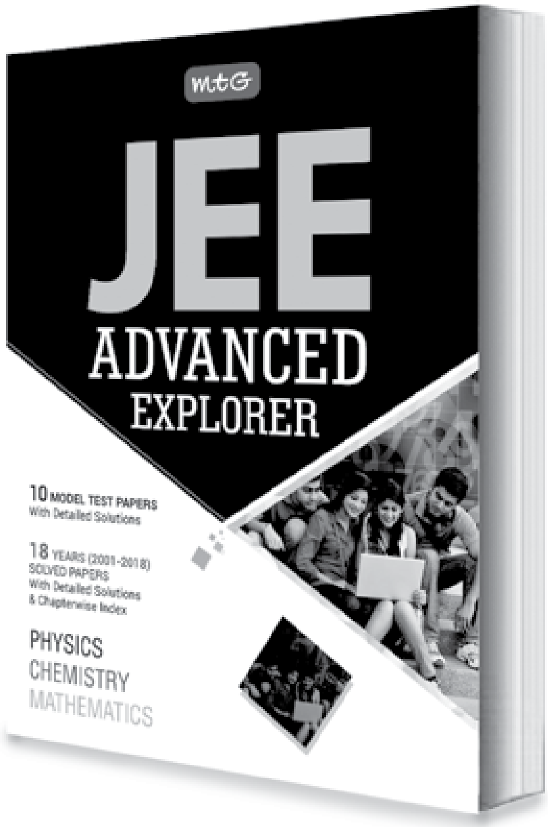
$\therefore$ Equation of the line of projection is
$3x - 8y + 7z + 4 = 0 = 3x + 2y + z$
Its direction ratios are $<11, -9, -15>$ and the point $(-1, 1, 1)$ lies on the line.

$\therefore$ $\frac{x+1}{11}=\frac{y-1}{-9}=\frac{z-1}{-15}$ is also the equation of the line of projection.

COMIC CAPSULE

Hindi : Rishtedar ke yaha shaadi hai.

Maths : Relation me function hai.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12. (c, d) : Let $f:\left[0,\frac{\pi}{2}\right]\to[0,1]$ be a differentiable function.

(a) Consider $g(x) = \sin^{-1}f(x) - x$

Since $g(0) = 0,\ g\left(\frac{\pi}{2}\right) = 0$

$\therefore$ There is atleast one value of $\alpha\in\left(0,\frac{\pi}{2}\right)$ such that

$$g'(\alpha)=\frac{f'(\alpha)}{\sqrt{1-(f(\alpha))^2}}-1=0$$

i.e., $f'(\alpha)=\sqrt{1-(f(\alpha))^2}$ for at least one value of α but may not be for all $\alpha\in\left(0,\frac{\pi}{2}\right)$

$\therefore$ False

(b) Consider $g(x)=f(x)-\frac{2x}{\pi}$

Since $g(0)=0, g\left(\frac{\pi}{2}\right)=0$

$\therefore$ There is at least one value of $\alpha\in\left(0,\frac{\pi}{2}\right)$ such that

$$g'(\alpha)=f'(\alpha)-\frac{2}{\pi}=0$$

i.e., $f'(\alpha)=\frac{2}{\pi}$ for at least one value of α but may not be for all $\alpha\in\left(0,\frac{\pi}{2}\right)$

$\therefore$ False

(c) Consider $g(x)=(f(x))^2-\frac{2x}{\pi}$

Since $g(0)=0, g\left(\frac{\pi}{2}\right)=0$

$\therefore$ There is at least one value of $\alpha\in\left(0,\frac{\pi}{2}\right)$ such that

$$g'(\alpha)=2f(\alpha)f'(\alpha)-\frac{2}{\pi}=0$$

$$\therefore\ f(\alpha)f'(\alpha)=\frac{1}{\pi}$$

$\therefore$ True

(d) Consider $g(x)=f(x)-\frac{4x^2}{\pi^2}$

Since $g(x)=0,\ g\left(\frac{\pi}{2}\right)=0$

$\therefore$ There is at least one value of $\alpha\in\left(0,\frac{\pi}{2}\right)$ such that

$$g'(\alpha)=f'(\alpha)-\frac{8\alpha}{\pi^2}=0$$

$$\therefore\ f'(\alpha)=\frac{8\alpha}{\pi^2}$$

$\therefore$ True

13. (b) : $\displaystyle\sum_{0\le i<j\le m}(i+j)C_iC_j=m\left[2^{2m-1}-{}^{2m-1}C_{m-1}\right]$

14. (b) : $\displaystyle\sum\sum_{0\le i<j\le m} iC_j=(C_2+C_3+\dots+C_m)+2.(C_3+C_4+\dots+C_m)+3(C_4+C_5+\dots+C_m)+(m-1)\,C_m$

$$=\sum_{r=2}^{m}\frac{r(r-1)}{2}C_r=m(m-1)2^{m-3}$$

15. (c) : $\displaystyle\sum\sum_{0\le i<j\le m} C_i^{\,2}=\sum_{r=0}^{m}(m-r)C_r^{\,2}$

$$=\sum_{r=0}^{m}r\cdot C_r^{\,2}=m\,{}^{2m-1}C_{m-1}$$

16. (A) → (p); (B) → (q); (C) → (s); (D) → (r)

$\sin A=\frac{4}{5},\ AB=AC=5\sqrt{2}$

So point B will be (0, 10), (10, 0)

and point C will be (4, –2), (6, 12)

Centroid will be (3, 13/3)

Circumcentre will be $\left(\frac{-5}{2},\frac{5}{2}\right)$

17. (A) → (q); (B) → (p); (C) → (r); (D) → (s)

Use the slope form of equation of tangents of each curve and then compare, we get equation of common tangent.

◆

SAMURAI SUDOKU

ANSWER - APRIL 2019

4	1	8	7	6	3	9	2	5				2	4	3	9	7	8	1	5	6
3	2	6	5	9	8	4	7	1				1	8	7	5	3	6	9	2	4
5	9	7	1	2	4	6	8	3				9	6	5	1	4	2	8	7	3
7	8	3	4	1	5	2	9	6				7	9	4	8	2	1	6	3	5
2	6	1	3	7	9	8	5	4				8	3	6	7	5	4	2	1	9
9	5	4	6	8	2	3	1	7				5	2	1	3	6	9	7	4	8
6	7	2	9	3	1	5	4	8	1	3	9	6	7	2	4	9	3	5	8	1
8	3	5	2	4	7	1	6	9	4	2	7	3	5	8	6	1	7	4	9	2
1	4	9	8	5	6	7	3	2	8	5	6	4	1	9	2	8	5	3	6	7
						4	5	6	2	9	8	1	3	7						
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						2	8	7	3	6	1	5	9	4						
5	9	3	7	1	6	8	2	4	5	7	3	9	6	1	8	2	3	7	4	5
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4	3	8	6	2	1	5	9	7				1	7	6	2	5	8	4	9	3
2	5	7	4	3	9	1	8	6				3	9	4	7	6	1	5	8	2
1	6	9	8	5	7	4	3	2				2	5	8	9	3	4	6	7	1

JEE WORK OUTS

JEE Advanced on 27th May, 2019

One or More Than One Option(s) Correct Type

1. If the angles of a triangle ABC satisfy the equation $81^{\sin^2 x} + 81^{\cos^2 x} = 30$, then the triangle can not be

(a) equilateral (b) isosceles

(c) obtuse angled (d) right angled

2. Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then

(a) $A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix} \forall n \in N$ (b) $\lim\limits_{n\to\infty} \frac{A^{-n}}{n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

(c) $\lim\limits_{n\to\infty} \frac{A^{-n}}{n^2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) none of these

3. Number of ways in which the letters of the word “TOMATO” can be arranged if two alike vowels are separated, is not equal to

(a) number of ways in which letters of the word “KARNATAKA” can be arranged if no two alike letters are separated.

(b) number of 3 digit numbers with at least one 3 and at least one 2.

(c) number of ways in which Ram and Rama can exchange their maps if Ram has 3 and Rama has 7 maps, all maps being different, maintaining their original number of maps at the end.

(d) number of ways in which 2 alike apples and 4 alike oranges can be distributed in three children if each child get none, one or more fruits.

4. Let three matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$, then $tr(A) + tr\left(\frac{ABC}{2}\right) + tr\left(\frac{A(BC)^2}{4}\right) + tr\left(\frac{A(BC)^3}{8}\right) + \ldots\ldots + \infty =$

(a) 6 (b) 9

(c) 12 (d) none of these

5. If the function

$$f(x) = \begin{cases} x + a^2\sqrt{2}\sin x, & 0 \le x < \pi/4 \\ x\cot x + b, & \pi/4 \le x < \pi/2 \\ b\sin 2x - a\cos 2x, & \pi/2 \le x \le \pi \end{cases}$$

is continuous in the interval $[0, \pi]$, then the values of (a, b) are

(a) $(-1, -1)$ (b) $(0, 0)$ (c) $(-1, 1)$ (d) $(1, 1)$

6. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{-6}$ is

(a) 1 unit (b) 2 unis

(c) 4 units (d) none of these

7. If α is the fifth root of unity, then

(a) $|1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4| = 0$

(b) $|1 + \alpha + \alpha^2 + \alpha^3| = 1$

(c) $|1 + \alpha + \alpha^2| = 2\cos\frac{\pi}{5}$

(d) $|1 + \alpha| = 2\cos\frac{\pi}{10}$

8. $\lim\limits_{x\to 0} \frac{(1+x)^{1/x} - e + \frac{ex}{2}}{x^2} =$

(a) $\frac{5e}{24}$ (b) $\frac{7e}{24}$ (c) $\frac{11e}{24}$ (d) $\frac{13e}{24}$

9. The value of λ with $|\lambda| < 16$ such that $2x^2 - 10xy + 12y^2 + 5x + \lambda y - 3 = 0$ represents a pair of straight lines is

(a) -10 (b) -9 (c) 10 (d) 9

10. If $f(x)=\sqrt{x^2-|x|}, g(x)=\frac{1}{\sqrt{9-x^2}}$, then D_{f+g} contains

(a) $(-3, -1)$ (b) $[1, 3)$

(c) $[-3, 3]$ (d) $\{0\} \cup [1, 3)$

11. The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. From these observations three are 1, 2 and 6 and $\lambda = |x_1 - x_2| + 8$ where x_1 and x_2 are remaining observations. Then number of solution of equation $10 - x^2 - 2x = \lambda$ are

(a) 1 (b) 2 (c) 3 (d) 4

12. The value(s) of 'a' for which the area of the triangle included between the axes and any tangent to the curve $x^a y = \lambda^a$ is constant, is/are

(a) $-\frac{1}{2}$ (b) -1 (c) $\frac{1}{2}$ (d) 1

13. A curve $y = f(x)$ has the property that if the tangent drawn at any point $P(x,y)$ on the curve meets the coordinate axes at A and B, then P is the midpoint of AB. If $y(1) = 1$, then $y(2) =$

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) -2

14. Let $R = (6\sqrt{6}+14)^{2n+1}$ and $f = R - [R]$, where $[\cdot]$ denotes the greatest integer function. The value of Rf, $n \in N$ is

(a) $(25)^{2n+1}$ (b) $(20)^{2n+1}$ (c) $(16)^{2n+1}$ (d) $(14)^{2n+1}$

15. If $(\sin y)^{\sin\frac{\pi x}{2}} + \frac{\sqrt{3}}{2} \sec^{-1} 2x + 2^x \tan \ln(x + 2) = 0$, then $y'(-1) =$

(a) $\frac{1}{\pi^2-3}$ (b) $\frac{1}{\pi\sqrt{\pi^2-3}}$

(c) $\frac{3}{2\pi\sqrt{\pi^2-3}}$ (d) $\frac{-3}{\pi\sqrt{\pi^2-3}}$

16. If $\Delta = \begin{vmatrix} \sin x & \sin(x+h) & \sin(x+2h) \\ \sin(x+2h) & \sin x & \sin(x+h) \\ \sin(x+h) & \sin(x+2h) & \sin x \end{vmatrix}$, then value of $\lim_{h\to 0}\left(\frac{\Delta}{h^2}\right)$ is

(a) $6 \sin x \cos^2 x$ (b) $6 \cos x \sin^2 x$

(c) $9 \sin x \cos^2 x$ (d) $9 \cos x \sin^2 x$

17. If $2f(x)+xf\left(\frac{1}{x}\right)-2f\left(\left|\sqrt{2}\sin\pi\left(x+\frac{1}{4}\right)\right|\right)$ $=4\cos^2\left(\frac{\pi x}{2}\right)+x\cos\frac{\pi}{x}, \forall x \in R-\{0\}$, then which of the following are true?

(a) $f(2)+f\left(\frac{1}{2}\right)=1$ (b) $f(2) + f(1) = 0$

(c) $f(2)+f(1)=f\left(\frac{1}{2}\right)$ (d) $f(1)\,f\left(\frac{1}{2}\right)f(2)=1$

18. If all the three roots of $az^3 + bz^2 + cz + d = 0$ have negative real parts $(a, b, c \in R)$, then

(a) $ab > 0$ (b) $bc > 0$

(c) $ad > 0$ (d) $bc - ad > 0$

19. Value(s) of x for which the fourth term in the expansion of $\left(\sqrt{x}^{1/(\log_2 x+1)}+x^{1/12}\right)^6$ is 40, is (are)

(a) 1/7 (b) 3 (c) 1/16, 2 (d) 1/8, 4

20. How many numbers between 5000 and 10000 can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit appearing not more than once in each number?

(a) $5 \times {}^8P_3$ (b) $5 \times {}^8C_8$

(c) $5! \times {}^8P_3$ (d) $5 \times {}^8C_3 \times 3!$

Comprehension Type

Paragraph for Q. No. 21 to 23

In a G.P., the sum of the first and last term is 66, the product of the second and the last before one is 128 and the sum of the terms is 126.

21. If an increasing G.P. is considered, then the number of terms in G.P. is

(a) 9 (b) 8 (c) 12 (d) 6

22. If the decreasing G.P. is considered, then the sum of infinite terms is

(a) 64 (b) 128 (c) 256 (d) 729

23. In any case, the difference of the least and greatest terms is

(a) 78 (b) 135

(c) 126 (d) none of these

Paragraph for Q. No. 24 to 26

Consider the function $f(x)$, a fourth degree polynomial such that $\lim_{x\to -1}\frac{f(x)}{(x+1)^3} = 1$ and $f'''(0) = 0$.

24. The maximum value of $f(x)$ is

(a) $\frac{13}{4}$ (b) $\frac{17}{4}$ (c) $\frac{23}{4}$ (d) $\frac{27}{4}$

25. The subnormal of the curve $y = f(x)$ where it cuts the y-axis is

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{1}{2}$

26. The number of inflectional points on the curve $y = f(x)$ is

(a) 0 (b) 1 (c) 2 (d) 3

Paragraph for Q. No. 27 to 29

$\vec{a}=2\hat{i}+\hat{j}+2\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and non-zero vector $\vec{c}$ are such that $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$.

27. The vector $\vec{c}$ may be given as

(a) $4\hat{i}+2\hat{j}+4\hat{k}$ (b) $4\hat{i}-2\hat{j}+4\hat{k}$

(c) $\hat{i}+\hat{j}+\hat{k}$ (d) $\hat{i}-4\hat{j}+\hat{k}$

28. Volume (in cubic units) of parallelepiped whose adjacent sides are given by $\vec{a},\vec{b},\vec{b}\times\vec{c}$ is

(a) 18 (b) 54 (c) 12 (d) 36

29. A vector along the bisector of angle between the vectors $\vec{b}$ and $\vec{c}$ is

(a) $(2+\sqrt{3})\hat{i}+(1-\sqrt{3})\hat{j}+(2+\sqrt{3})\hat{k}$

(b) $(2-\sqrt{3})\hat{i}+(\sqrt{3}+1)\hat{j}+(2-\sqrt{3})\hat{k}$

(c) $(2+\sqrt{3})\hat{i}+(1-\sqrt{3})\hat{j}-(2+\sqrt{3})\hat{k}$

(d) $(2+\sqrt{3})\hat{i}-(1-\sqrt{3})\hat{j}+(2+\sqrt{3})\hat{k}$

Matrix Match Type

30. Match the greatest and least value of functions in column-I with column-II.

Column-I		Column-II	
(A)	$x + 2\sqrt{x}$ on [0, 4]	(p)	13 and 4
(B)	$\dfrac{x-1}{x+1}$ on [0, 4]	(q)	8 and 0
(C)	$x^4 - 2x^2 + 5$ on [–2, 2]	(r)	3/5 and –1

(a) (A) → (q) ; (B) → (r) ; (C) → (q)

(b) (A) → (q) ; (B) → (r) ; (C) → (p)

(c) (A) → (r) ; (B) → (p) ; (C) → (q)

(d) (A) → (r) ; (B) → (q) ; (C) → (p)

Directions (31 - 32) : Given that

$$\int_0^{\pi/2}\log\sin x\,dx=\frac{\pi}{2}\log\frac{1}{2}=\alpha$$

Then match the following columns for their values:

Column-I		Column-II		Column-III	
(A)	$\int_0^{\pi/2}\log\cos x\,dx$	(1)	$\frac{1}{\pi}\int_0^{\pi}x\log\sin x\,dx$	(i)	-2α
(B)	$\int_0^{\infty}\log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2}$	(2)	$-\int_0^{\pi}\log(1+\cos x)dx$	(ii)	$-\alpha$
(C)	$\int_0^{\pi/2}x\cot x\,dx$	(3)	$\int_0^{\pi}\log\sqrt{1-\cos x}\,dx$	(iii)	α

31. Which of the following is the correct combination?

(a) (A) (3) (iii) (b) (B) (2) (i)

(c) (C) (1) (ii) (d) none of these

32. Which of the following is not the correct combination?

(a) (A) (1) (iii) (b) (C) (3) (ii)

(c) (B) (2) (i) (d) none of these

Numerical Answer Type

33. The tangents drawn from a point P to the ellipse $9x^2 + 16y^2 = 144$ make angles θ_1, θ_2 with the major axis. If $\theta_1 + \theta_2 = 2\alpha$, then the locus of P is $x^2 - 2xy\cot2\alpha - y^2 = \lambda$, where λ is

34. If $\int\frac{(\sqrt{x})^5}{(\sqrt{x})^7+x^6}dx = a\log\left(\frac{x^\lambda}{x^\lambda+1}\right)+C$, then $a\lambda =$

35. The probability of at least one double-six being thrown in n throws with two ordinary dice is greater than 99 percent. Calculate the least numerical value of n.

36. Let $f(x)$ be a polynomial satisfying $\lim\limits_{x\to\infty}\frac{x^2f(x)}{2x^5+3}=6$, $f(1)=3, f(3)=7$ and $f(7)=15$. Then the value of $\left[\frac{|f(0)|}{30}\right]$, where [.] represents the greatest integer function, is

37. The chord of the curve $y = -a^2x^2 + 5ax - 4$ touches the curve $y=\frac{1}{1-x}$ at the point $x = 2$ and is bisected by that point. Then the value of a is

38. If $\sum\limits_{r=1}^{10}\tan^{-1}\left(\frac{3}{9r^2+3r-1}\right)=\cot^{-1}\left(\frac{m}{n}\right)$ (where m and n are coprime), then find $(2m + n)$.

39. The number of values of θ in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ such that $\theta\neq\frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$, is

40. The curve passing through the point (1, 1) satisfies the differential equation $\frac{dy}{dx}+\frac{\sqrt{(x^2-1)(y^2-1)}}{xy}=0$. If the curve passes through the point $(\sqrt{2},k)$, then the value of $[k]$ is (where [.] represents greatest integer function)

SOLUTIONS

1. (d) : Let $81^{\sin^2 x} = y$, then $81^{1-\cos^2 x} = y$

$\Rightarrow$ $81^{\cos^2 x} = 81y^{-1}$ and the given equation can be written as $y + 81y^{-1} = 30$

$\Rightarrow$ $y^2 - 30y + 81 = 0$ $\Rightarrow$ $y = 3$ or $y = 27$

When, $y = 3$ $\Rightarrow$ $81^{\sin^2 x} = 3$

$\Rightarrow$ $3^{4\sin^2 x} = 3$ $\Rightarrow$ $\sin x = \pm 1/2$

$\Rightarrow$ $x = 30°$ or $150°$

When $y = 27$ $\Rightarrow$ $3^{4\sin^2 x} = 3^3$ $\Rightarrow$ $\sin x = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow x = 60°$ or $120°$

$\therefore$ The possible values of the angles A, B, C are 30° or 60° or 120° or 150°, so we get the set of angles of the triangle as 30°, 30°, 120° or 60°, 60°, 60° and thus the triangle cannot be right angled.

2. (a, b, c) : $A^2 = A \cdot A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ and so on

$\therefore$ $A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$ and $A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

$$\text{and} \lim_{n\to\infty} \frac{A^{-n}}{n} = \begin{bmatrix} \lim\limits_{n\to\infty} \frac{1}{n} & 0 \\ \lim\limits_{n\to\infty} \frac{-n}{n} & \lim\limits_{n\to\infty} \frac{1}{n} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$$

$$\text{and} \lim_{n\to\infty} \frac{A^{-n}}{n^2} = \begin{bmatrix} \lim\limits_{n\to\infty} \frac{1}{n^2} & 0 \\ \lim\limits_{n\to\infty} \frac{-1}{n} & \lim\limits_{n\to\infty} \frac{1}{n^2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

3. (b, c, d) : Arrangement should be × T × M × A × T ×

Number of ways $= {}^5C_2 \times \frac{4!}{2!} = 10 \times 12 = 120.$

(a) The given word is KARNATAKA

K = 2, A = 4, R = 1, N = 1, T = 1

$\boxed{K\ \ K}$ $\boxed{A\ \ A\ \ A\ \ A}$ $\boxed{R}$ $\boxed{N}$ $\boxed{T}$

$\therefore$ Number of ways = 5! = 120.

(b) Possible ways are 2, 3, 3 or 3, 2, 2 or 2, 3, non-zero digit or 2, 3, 0

$\therefore$ Number of ways $= \frac{3!}{2!} + \frac{3!}{2!} + ({}^7C_1 \times 3!) + (2 \times 2)$

$= 3 + 3 + 42 + 4 = 52.$

(c) $M_1, M_2, ..., M_7\, |N_1, N_2, N_3$

Total ways $= 3({}^7C_1) + 3({}^7C_2) + 1({}^7C_3)$

$= 21 + 63 + 35 = 119.$

(d) Let x, y and z be the no. of fruits obtained by three children in which 2 alike apples and 4 alike oranges distributed.

$\therefore$ $x + y + z = 6$

Number of ways in which each child get none, one or more fruits $= {}^{6+3-1}C_{3-1} = {}^8C_2 = 28.$

4. (a) : $BC = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix} \Rightarrow BC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

$\therefore$ Given expression $= tr(A) + tr\left(\frac{A}{2}\right) + tr\left(\frac{A}{2^2}\right) + \ldots + \infty$

$= tr(A) + \frac{1}{2}tr(A) + \frac{1}{2^2}tr(A) + \ldots + \infty = \frac{tr(A)}{1-(1/2)}$

$= 2tr(A) = 2(2 + 1) = 6$

5. (b, d) : At $x = \frac{\pi}{4}$,

$\text{LHL} = \lim\limits_{x\to(\pi/4)^-} f(x) = \lim\limits_{x\to\pi/4} (x + a^2\sqrt{2}\sin x) = a^2 + \frac{\pi}{4}$

$\text{RHL} = \lim\limits_{x\to(\pi/4)^+} f(x) = \lim\limits_{x\to\pi/4} (x\cot x + b) = b + \frac{\pi}{4}$

Since, $f(x)$ is continuous at $x = \frac{\pi}{4}$.

$\therefore a^2 + \frac{\pi}{4} = b + \frac{\pi}{4} \Rightarrow a^2 = b$ ...(i)

At $x = \frac{\pi}{2}$,

$\text{LHL} = \lim\limits_{x\to(\pi/2)^-} f(x) = \lim\limits_{x\to\pi/2} (x\cot x + b) = b$

$\text{RHL} = \lim\limits_{x\to(\pi/2)^+} f(x) = \lim\limits_{x\to\pi/2} (b\sin 2x - a\cos 2x) = a$

Since, $f(x)$ is continuous at $x = \pi/2$.

$\therefore$ $a = b$ ...(ii)

From (i) and (ii), $a = 0, 1$ and $b = 0, 1$

$\therefore$ $(a, b) = (0, 0), (1, 1)$

6. (a) : The equation of the line passing through $P(1, -2, 3)$ and parallel to the given line is

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6}$

Suppose it meets the plane $x - y + z = 5$ at the point Q given by $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$

i.e., $(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

This lies on $x - y + z = 5$. Therefore,

$2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5 \Rightarrow -7\lambda = -1 \Rightarrow \lambda = \frac{1}{7}$

So, the co-ordinates of Q are $(9/7, -11/7, 15/7)$.

Hence, required distance $= PQ = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$ unit.

7. (a, b, c) : We have $\alpha = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$

and $1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 = 0$

$\therefore \quad |1 + \alpha + \alpha^2 + \alpha^3| = |-\alpha^4| = |\alpha|^4 = 1$

Also, $|1 + \alpha + \alpha^2| = |-\alpha^3(1 + \alpha)| = |1 + \alpha|$

$$= \left|1+\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}\right| = \left|2\cos\frac{\pi}{5}\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right)\right| = 2\cos\frac{\pi}{5}$$

8. (c) : $(1 + x)^{1/x} - e + \frac{ex}{2} = e^{\frac{1}{x}\ln(1+x)} - e + \frac{ex}{2}$

We expand the numerator in series

$$= e^{\left(1-\frac{x}{2}+\frac{x^2}{3}\dots\right)} - e + \frac{ex}{2} = e\cdot e^{\left(-\frac{x}{2}+\frac{x^2}{3}\dots\right)} - e + \frac{ex}{2}$$

$$= e\left[1+\left(-\frac{x}{2}+\frac{x^2}{3}-\dots\right)+\frac{\left(-\frac{x}{2}+\frac{x^2}{3}-\dots\right)^2}{2!}\right] - e + \frac{ex}{2}$$

$$= e\left(\frac{1}{3}+\frac{1}{8}\right)x^2 + \dots$$

$\therefore$ The limit $= e\left(\frac{1}{3}+\frac{1}{8}\right) = \frac{11e}{24}$.

9. (b) : Given equation is

$2x^2 - 10xy + 12y^2 + 5x + \lambda y - 3 = 0$

Here $a = 2$, $h = -5$, $b = 12$, $g = \frac{5}{2}$, $f = \frac{\lambda}{2}$, $c = -3$

For pair of lines $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2 & -5 & 5/2 \\ -5 & 12 & \lambda/2 \\ 5/2 & \lambda/2 & -3 \end{vmatrix} = 0$

$\Rightarrow \quad \lambda^2 + 25\lambda + 144 = 0 \quad \Rightarrow \quad \lambda = -9$

10. (a, b, d) : $g(x)$ is defined if $9 - x^2 > 0 \Rightarrow x \in (-3, 3)$

$f(x)$ is defined if $x^2 - |x| \ge 0 \Rightarrow x^2 \ge |x|$

$\Rightarrow \quad x \notin (-1, 0) \cup (0, 1)$

$\therefore \quad D_{f+g} = (-3, 3) - ((-1, 0) \cup (0, 1))$

$\qquad\qquad = (-3, -1] \cup \{0\} \cup [1, 3)$

It contains $(-3, -1)$, $[1, 3)$, $\{0\} \cup [1, 3)$.

11. (a) : Mean $(\bar{x}) = 4$, variance $= 5.2$

Let x_1, x_2 be the remaining values.

Mean, $\bar{x} = \frac{1+2+6+x_1+x_2}{5} \Rightarrow x_1 + x_2 = 11$ ...(i)

Variance, $\sigma^2 = 5.2 = \frac{1^2+2^2+6^2+x_1^2+x_2^2}{5} - (\bar{x})^2$

$\Rightarrow \quad x_1^2 + x_2^2 = 65$ ...(ii)

$\Rightarrow \quad |x_1 - x_2| = 3$

Now, $\lambda = |x_1 - x_2| + 8 \quad \Rightarrow \quad \lambda = 11$

$\Rightarrow \quad 10 - x^2 - 2x = 11 \Rightarrow (x + 1)^2 = 0$ one solution

12. (b, d) : Given curve $x^a y = \lambda^a$ ...(i)

$(\lambda, 1)$ is a point on the given curve.

Now, differentiating (i) w.r.t. x, we get

$$ax^{a-1}y + x^a\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-ax^{a-1}y}{x^a} = -\frac{ay}{x}$$

At $(\lambda, 1)$, $\frac{dy}{dx} = -\frac{a}{\lambda}$

Equation of tangent at $(\lambda, 1)$ is

$$y - 1 = -\frac{a}{\lambda}(x - \lambda)$$

Now, $x = 0 \Rightarrow y = 1 + a$, $y = 0 \Rightarrow x = \frac{\lambda}{a} + \lambda = \frac{\lambda(1+a)}{a}$

$\therefore \quad$ Area, $A = \frac{1}{2}\times(1+a)\frac{(a+1)\lambda}{a}$

Now, $\frac{dA}{da} = \frac{1}{2}\lambda\left[\frac{a\cdot 2(1+a)-(1+a)^2}{a^2}\right] = 0$

$\Rightarrow \quad (2a - 1 - a)(1 + a) = 0 \Rightarrow (a - 1)(a + 1) = 0$

$\Rightarrow \quad a = 1, a = -1$

13. (c) : Tangent at P is $Y - y = (X - x)y_1$

It meets the x-axis at $A\left(x-\frac{y}{y_1}, 0\right)$ and y-axis at $B(0, y - xy_1)$

P is the midpoint of $AB \quad \Rightarrow \quad x - \frac{y}{y_1} = 2x$

$\Rightarrow \quad \frac{y}{y_1} + x = 0 \Rightarrow ydx + xdy = 0 \quad \Rightarrow \quad xy = c^2$

When $x = 1$, $y = 1 \Rightarrow c^2 = 1$, $xy = 1 \quad \therefore \quad y(2) = \frac{1}{2}$.

14. (b) : Since, $f = R - [R]$

$\therefore \quad R = [R] + f$

$\Rightarrow \quad (6\sqrt{6}+14)^{2n+1} = [R] + f$, where $[R]$ is integer and $0 \le f < 1$.

Now, let $f' = (6\sqrt{6}-14)^{2n+1}$, $0 < f' < 1$

Also $[R] + f - f' = (6\sqrt{6}+14)^{2n+1} - (6\sqrt{6}-14)^{2n+1}$

$= 2\{{}^{2n+1}C_1(6\sqrt{6})^{2n}14 + {}^{2n+1}C_3(6\sqrt{6})^{2n-2}(14)^3 + \dots\}$

$= 2(\text{Integer}) = 2k$ $(k \in N)$ = even integer

Hence $f - f'$ = even integer $- [R]$, but $-1 < f - f' < 1$

$\because \quad$ R.H.S. is integer, hence L.H.S. is also integer.

Therefore $f - f' = 0$

$\therefore \quad f = f'$

Hence $Rf = Rf' = (6\sqrt{6}+14)^{2n+1}(6\sqrt{6}-14)^{2n+1}$

$\qquad = (20)^{2n+1}$

15. (c) : $x = -1 \quad \Rightarrow \quad \sin y = -\frac{\sqrt{3}}{\pi}$, $\cos y = \frac{\sqrt{\pi^2-3}}{\pi}$

Differentiating the given relation,

$$(\sin y)^{\sin\frac{\pi x}{2}}\left[\frac{\pi}{2}\cos\frac{\pi x}{2}\ln\sin y + \sin\frac{\pi x}{2}\cdot\cot y\cdot y'\right] + \frac{\sqrt{3}}{2}$$

$$\cdot\frac{2}{|2x|\sqrt{4x^2-1}} + 2^x\left[\ln 2\tan\ln(x+2) + \frac{\sec^2\ln(x+2)}{x+2}\right] = 0$$

Putting $x = -1$ and using $\sin y(-1) = \dfrac{-\sqrt{3}}{\pi}$,

$\cos y(-1) = \dfrac{\sqrt{\pi^2 - 3}}{\pi}$, we get,

$$y'(-1) = \frac{\sin^2 y(-1)}{2\cos y(-1)} = \frac{3}{2\pi\sqrt{\pi^2-3}}.$$

16. (c) : We apply the following transformations to Δ in sequence: $C_3 \to C_3 - C_2$, $C_2 \to C_2 - C_1$, $C_2 \to \dfrac{C_2}{h}$ and $C_3 \to \dfrac{C_3}{h}$. Now using $\lim\limits_{h\to 0}$ on the resulting determinant, we have

$$\lim_{h\to 0}\frac{\Delta}{h^2} = \begin{vmatrix} \sin x & \cos x & \cos x \\ \sin x & -2\cos x & \cos x \\ \sin x & \cos x & -2\cos x \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, $R_2 \to R_2 + 2R_3$, we get

$$\lim_{h\to 0}\frac{\Delta}{h^2} = \begin{vmatrix} 3\sin x & 0 & 0 \\ 3\sin x & 0 & -3\cos x \\ \sin x & \cos x & -2\cos x \end{vmatrix}$$

$= 9 \sin x \cos^2 x$ [Expanding along R_1]

17. (a, b, c) : Replace x by 2, we get

$$2f(2)+2f\left(\frac{1}{2}\right)-2f(1)=4 \Rightarrow f(2)+f\left(\frac{1}{2}\right)=2+f(1) \quad \text{...(i)}$$

Replace x by 1, $f(1) = -1$ ...(ii)

Replace x by $\dfrac{1}{2}$, $2f\left(\dfrac{1}{2}\right)+\dfrac{1}{2}f(2)-2f(1)=\dfrac{5}{2}$

$$\Rightarrow \quad 2f\left(\frac{1}{2}\right)+\frac{1}{2}f(2)=\frac{5}{2}+2f(1) \quad \text{...(iii)}$$

From (i), (ii) and (iii), we get $f\left(\dfrac{1}{2}\right)=0; f(2)=1$

18. (a, b, c) : Let $z_1 = x_1$; $z_2, z_3 = x_2 \pm iy_2$

$$\Rightarrow \quad z_1 + z_2 + z_3 = -\frac{b}{a}$$

$$\Rightarrow \quad x_1 + 2x_2 = -\frac{b}{a} < 0 \quad \Rightarrow \quad ab > 0$$

Also, $z_1z_2z_3 = x_1[x^2{}_2 + y^2{}_2] = -\dfrac{d}{a} < 0 \quad \Rightarrow \quad ad > 0$

Also, $z_1z_2 + z_2z_3 + z_1z_3 = \dfrac{c}{a}$

$$\Rightarrow \quad x_1(x_2 + iy_2) + x_1(x_2 - iy_2) + x^2{}_2 + y^2{}_2$$

$$= 2x_1x_2 + x^2{}_2 + y^2{}_2 > 0 \quad \Rightarrow \quad \frac{c}{a} > 0$$

$$\Rightarrow \quad \frac{b}{a}\frac{c}{a} > 0 \Rightarrow bc > 0$$

19. (c) : $t_4 = {}^6C_3\left[x^{1/2(\log_2 x+1)}\right]^3 (x^{1/12})^3 = 40$

$$\Rightarrow \quad \left(\frac{3}{2t+2}+\frac{1}{4}\right)t = 1, \text{ where } t = \log_2 x$$

$$\Rightarrow \quad \frac{(6+t+1)t}{4(t+1)} = 1 \Rightarrow 7t + t^2 = 4t + 4$$

$\Rightarrow \quad t^2 + 3t - 4 = 0 \Rightarrow t = -4, 1$

$\therefore \quad x = 1/16, 2$

20. (a, d) : We are to form 4-digit numbers with either 5 or 6 or 7 or 8 or 9 at the thousand's place.

$$\text{4-digit number :} \quad \begin{array}{cccc} \times & \times & \times & \times \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 & 4 \end{array}$$

No. of ways of filling 1st place = 5

After filling 1st place, we have 8 digits to be used.

No. of ways of filling 2nd, 3rd, 4th places = ${}^8P_3$

$\therefore$ Total no. of numbers formed = $5 \times {}^8P_3$.

(21-23) : 21. (d) 22. (b) 23. (d)

Let a be the first term and r be the common ratio of the given G.P. Further, let there be n terms in the given G.P. Then,

$a_1 + a_n = 66 \Rightarrow a + ar^{n-1} = 66$ ...(i)

$a_2 \times a_{n-1} = 128 \Rightarrow ar \times ar^{n-2} = 128 \Rightarrow a^2 r^{n-1} = 128$

$$\Rightarrow \quad a \times (ar^{n-1}) = 128 \Rightarrow ar^{n-1} = \frac{128}{a}$$

Putting this value of ar^{n-1} in (i), we get

$$a + \frac{128}{a} = 66 \Rightarrow a^2 - 66a + 128 = 0$$

$\Rightarrow \quad (a - 2)(a - 64) = 0 \Rightarrow a = 2, 64$

Putting $a = 2$ in (i), we get $2 + 2 \times r^{n-1} = 66 \Rightarrow r^{n-1} = 32$

Putting $a = 64$ in (i), we get $64 + 64r^{n-1} = 66 \Rightarrow r^{n-1} = \dfrac{1}{32}$

For an increasing G.P., $r > 1$.

Now, $S_n = 126$

$$\Rightarrow \quad 2\left(\frac{r^n - 1}{r-1}\right) = 126 \Rightarrow \frac{r^n - 1}{r-1} = 63 \Rightarrow \frac{r^{n-1} \times r - 1}{r-1} = 63$$

$$\Rightarrow \quad \frac{32r - 1}{r-1} = 63 \quad \Rightarrow \quad r = 2$$

$\therefore \quad r^{n-1} = 32 \Rightarrow 2^{n-1} = 32 = 2^5 \Rightarrow n - 1 = 5 \Rightarrow n = 6$

For decreasing G.P., $a = 64$ and $r = 1/2$. Hence, the sum of infinite terms is $64/[1 - (1/2)] = 128$.

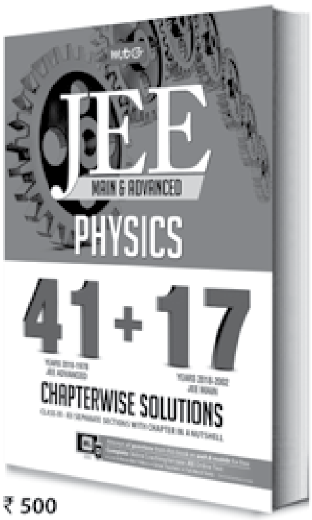
For $a = 2$, $r = 2$, terms are 2, 4, 8, 16, 32, 64.

For $a = 64$, $r = \dfrac{1}{2}$, terms are 64, 32, 16, 8, 4, 2.

$\therefore$ Difference = 62.

24. (d) : $\lim\limits_{x\to -1}\dfrac{f(x)}{(x+1)^3} = 1$

$$\Rightarrow \quad f(-1) = f'(-1) = f''(-1) = 0, f'''(-1) = 6$$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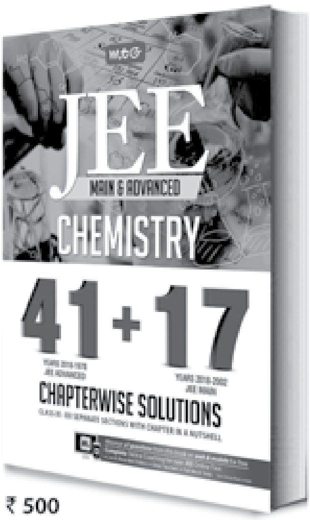
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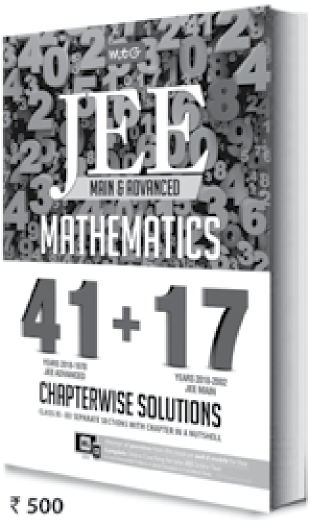
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$\therefore \quad f(x) = A(x+1)^4 + (x+1)^3$

$f'''(x) = 24A(x+1) + 6$

$$f'''(0) = 0 \Rightarrow 24A + 6 = 0,\ A = -\frac{1}{4}$$

$$\therefore \quad f(x) = (x+1)^3 - \frac{1}{4}(x+1)^4$$

$f'(x) = 3(x+1)^2 - (x+1)^3$

$f''(x) = 6(x+1) - 3(x+1)^2$

$f'(x) = 0 \Rightarrow (x+1)^2(2-x) = 0$

$\Rightarrow \quad x = -1, x = 2$ and $f''(2) = -9$

The maximum value is $f(2) = 27 - \frac{81}{4} = \frac{27}{4}$

25. (b) : $f(0) = 1 - \frac{1}{4} = \frac{3}{4}$, $f'(0) = 2$

Subnormal at $x = 0$ is $f(0)\, f'(0) = \frac{3}{4} \times 2 = \frac{3}{2}$

26. (c) : $f''(x) = 3(x+1)(2 - x - 1) = 3(x+1)(1-x)$

$f''(x) = 0 \Rightarrow x = 1, -1$

27. (a) : We have, $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c})$

$\Rightarrow \quad (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$\Rightarrow \quad \vec{a}$ and $\vec{c}$ are parallel.

$\therefore \quad \vec{c}$ may be equal to $4\hat{i} + 2\hat{j} + 4\hat{k}$

28. (d) : We have, $\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 4 & 2 & 4 \end{vmatrix} = -6\hat{i} + 6\hat{k}$

$$\therefore \quad [\vec{a}\ \ \vec{b}\ \ \vec{b}\times\vec{c}] = \begin{vmatrix} 2 & 1 & 2 \\ 1 & -1 & 1 \\ -6 & 0 & 6 \end{vmatrix} = -36$$

$\therefore$ Volume of parallelepiped whose adjacent sides are $\vec{a}, \vec{b}, \vec{b}\times\vec{c}$ is 36 cubic units.

29. (a) : Vector along the bisectors is

$$\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}} + \frac{4\hat{i} + 2\hat{j} + 4\hat{k}}{6}$$

$$= \frac{1}{3}((2+\sqrt{3})\hat{i} + (1-\sqrt{3})\hat{j} + (2+\sqrt{3})\hat{k})$$

$\therefore \quad$ Vector is $(2+\sqrt{3})\hat{i} + (1-\sqrt{3})\hat{j} + (2+\sqrt{3})\hat{k}$

30. (b) : (A) Let $f(x) = x + 2\sqrt{x}$, then $f'(x) = 1 + x^{-1/2}$. $f'(x) \neq 0$ for any $x \in [0, 4]$. Thus, the least value of $f(x)$ is $f(0) = 0$ and the greatest value is $f(4) = 4 + 2\sqrt{4} = 8$.

(B) Let $f(x) = \frac{x-1}{x+1}$, then $f'(x) = \frac{2}{(x+1)^2} > 0$ for any $x \neq -1$. Thus, the least value of $f(x) = f(0) = -1$ and the greatest value of $f(x) = f(4) = 3/5$.

(C) Let $f(x) = x^4 - 2x^2 + 5$

Then, $f'(x) = 4x^3 - 4x$, $f''(x) = 12x^2 - 4$

Now, $f'(x) = 0 \ \Rightarrow\ 4x^3 - 4x = 0$

$\Rightarrow \quad 4x(x^2 - 1) = 0 \ \Rightarrow\ x = 0, 1, -1$

$f''(0) = 12(0) - 4 = -4 < 0$

So, $x = 0$ is a point of local maxima.

$f''(1) = 12(1)^2 - 4 = 8 > 0$

So, $x = 1$ is a point of local minima.

$f''(-1) = 12(-1)^2 - 4 = 8 > 0$

So, $x = -1$ is a point of local minima.

Now, $f(0) = 5$, $f(1) = 4 = f(-1)$, $f(-2) = f(2) = 13$.

Thus, greatest value of $f(x)$ is 13 and least value of $f(x)$ is 4.

(31-32) : 31. (b) **32. (b)**

(A) Let $I = \int_0^{\pi/2} \log\sin x\, dx$

$$\Rightarrow \quad I = \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right)dx = \int_0^{\pi/2} \log\cos x\, dx = \alpha$$

(B) Let $I = \int_0^{\infty} \log\left(x + \frac{1}{x}\right)\frac{dx}{1+x^2}$

Putting $x = \tan\theta \ \Rightarrow\ dx = \sec^2\theta\, d\theta$, we get

$$I = \int_0^{\pi/2} \frac{\log(\tan\theta + \cot\theta)}{1 + \tan^2\theta}\cdot\sec^2\theta\, d\theta$$

$$= \int_0^{\pi/2} \log\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)d\theta$$

$$= \int_0^{\pi/2} \log\left(\frac{1}{\sin\theta\cos\theta}\right)d\theta$$

$$= -\int_0^{\pi/2} \log\sin\theta\, d\theta - \int_0^{\pi/2} \log\cos\theta\, d\theta$$

$$= -2\left(-\frac{1}{2}\pi\log 2\right) = \pi\log 2 = -2\alpha$$

(C) $I = \int_0^{\pi/2} x\cot x\, dx$

Integrating by parts taking cot x as second function, we get

$$I = [x\log\sin x]_0^{\pi/2} - \int_0^{\pi/2} \log\sin x\, dx$$

$$= 0 - \lim_{x\to 0}(x\log\sin x) - \int_0^{\pi/2} \log\sin x\, dx = \frac{1}{2}\pi\log 2$$

as $\lim\limits_{x\to 0} x\log\sin x = \lim\limits_{x\to 0}\left(\frac{\log\sin x}{1/x}\right)$

$$= \lim_{x\to 0}\left(\frac{\cot x}{-1/x^2}\right) \qquad \text{(Applying L' Hospital Rule)}$$

$$= \lim_{x\to 0}\left(\frac{-x^2}{\tan x}\right) = \lim_{x\to 0}\left(-x\times\frac{x}{\tan x}\right) = 0\times 1 = 0$$

(1) Let $I = \int_0^{\pi} x\log\sin x\, dx$ ...(i)

$$\Rightarrow \quad I = \int_0^{\pi}(\pi - x)\log\sin(\pi - x)\,dx$$

$$\Rightarrow \quad I = \int_0^{\pi}(\pi - x)\log\sin x\,dx \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \pi\int_0^{\pi}\log\sin x\,dx \quad \Rightarrow \quad 2I = 2\pi\int_0^{\pi/2}\log\sin x\,dx$$

$$\Rightarrow \quad I = \pi\int_0^{\pi/2}\log\sin x\,dx = \pi\alpha$$

$$\therefore \quad \frac{1}{\pi}\int_0^{\pi}x\log\sin x\,dx = \frac{1}{\pi}\cdot\pi\alpha = \alpha$$

(2) Let $I = -\int_0^{\pi}\log(1+\cos x)dx = -\int_0^{\pi}\log\left(2\cos^2\frac{x}{2}\right)dx$

$$= -\int_0^{\pi}\left(\log 2 + 2\log\cos\frac{x}{2}\right)dx$$

$$= -\left[\pi\log 2 + 2\int_0^{\pi}\log\cos\frac{x}{2}dx\right]$$

$$= -\left[\pi\log 2 + 2\times 2\int_0^{\pi/2}\log\cos t\,dt\right]$$

$$\left(\text{where } t = \frac{x}{2} \text{ and } dx = 2dt\right)$$

$$= -\left[\pi\log 2 + 4\times\left(-\frac{\pi}{2}\right)\log 2\right] = \pi \log 2 = -2\alpha$$

(3) $\int_0^{\pi}\log\sqrt{1-\cos x}\;dx = \int_0^{\pi}\log\sqrt{2\sin^2\frac{x}{2}}\;dx$

$$= \int_0^{\pi}\log\sqrt{2}dx + \int_0^{\pi}\log\sin\frac{x}{2}dx$$

$$= \pi\log\sqrt{2} + 2\int_0^{\pi/2}\log\sin y\,dy$$

$$\left(\text{where } \frac{x}{2} = y \text{ and } dx = 2dy\right)$$

$$= \pi\log\sqrt{2} + 2\left(-\frac{\pi}{2}\log 2\right) = \frac{\pi}{2}\log 2 - \pi\log 2$$

$$= -\frac{\pi}{2}\log 2 = \alpha$$

33. (7) : The tangent with slope m to $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is $y = mx \pm \sqrt{16m^2+9}$

$\Rightarrow \quad (mx - y)^2 = 16m^2 + 9$

$\Rightarrow \quad (16 - x^2)m^2 + 2xym + 9 - y^2 = 0$

$$\tan\theta_1 + \tan\theta_2 = \frac{-2xy}{16-x^2},\ \tan\theta_1\tan\theta_2 = \frac{9-y^2}{16-x^2}$$

Now, $\theta_1 + \theta_2 = 2\alpha \Rightarrow \tan 2\alpha = \frac{-2xy}{7-x^2+y^2}$

$\Rightarrow \quad 7 - x^2 + y^2 + 2xy\cot 2\alpha = 0$

$\Rightarrow \quad x^2 - 2xy\cot 2\alpha - y^2 = 7 \therefore \lambda = 7$

34. (1) : Let $I = \int\frac{(\sqrt{x})^5}{(\sqrt{x})^7 + x^6}dx$

$$= \int\frac{dx}{(\sqrt{x})^2 + (\sqrt{x})^7} = \int\frac{1}{(\sqrt{x})^7\left(1+\frac{1}{(\sqrt{x})^5}\right)}dx$$

Let $1 + \frac{1}{(\sqrt{x})^5} = t$. Then $-\frac{5}{2}x^{-7/2}\,dx = dt$

$$\therefore \quad I = -\frac{2}{5}\int\frac{dt}{t} = -\frac{2}{5}\log|t| + C = \frac{2}{5}\log\left(\frac{x^{5/2}}{1+x^{5/2}}\right) + C$$

On comparing a log $\left(\frac{x^{\lambda}}{x^{\lambda}+1}\right) + C$ with

$\frac{2}{5}\log\left(\frac{x^{5/2}}{1+x^{5/2}}\right) + C$, we get $a = \frac{2}{5}$ and $\lambda = \frac{5}{2}$

$\therefore \quad a\lambda = 1.$

35. (164): The probability of getting a double-six in one throw with two dice $= \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$.

$$\therefore \quad p = \frac{1}{36}, q = 1 - p = 1 - \frac{1}{36} = \frac{35}{36}.$$

Now, $(p + q)^n = p^n + {}^nC_1\,p^{n-1}q + {}^nC_2\,p^{n-2}\,q^2 + \ldots + {}^nC_{n-1}pq^{n-1} + q^n.$

The probability of getting atleast one double-six in n throws with two dice $= (p + q)^n - q^n = 1 - q^n$

$$= 1 - \left(\frac{35}{36}\right)^n \quad \therefore 1 - \left(\frac{35}{36}\right)^n > 0.99$$

$$\Rightarrow \quad \left(\frac{35}{36}\right)^n < 0.01 \Rightarrow n[\log_{10}35 - \log_{10}36] < \log_{10}0.01$$

$\Rightarrow \quad n[1.5441 - 1.5563] < -2.0000 \Rightarrow -0.0122n < -2$

$\Rightarrow \quad 0.0122n > 2 \quad \Rightarrow \quad n > \frac{2}{0.0122} \quad \Rightarrow n > 163.9$

$\therefore$ The least value of n is 164.

36. (8) : Since $\lim_{x\to\infty}\frac{x^2 f(x)}{2x^5+3} = 6, f(x)$ must be of degree 3.

Also from $f(1) = 3$, $f(3) = 7$ and $f(7) = 15$, we get, $f(x) = \lambda(x - 1)(x - 3)(x - 7) + 2x + 1$

$$\therefore \quad \lim_{x\to\infty}\frac{x^2\cdot(\lambda(x-1)(x-3)(x-7)+2x+1)}{2x^5+3} = 6$$

$\Rightarrow \quad \frac{\lambda}{2} = 6 \quad \Rightarrow \lambda = 12$

$\therefore \quad f(x) = 12(x - 1)(x - 3)(x - 7) + 2x + 1$

$\therefore \quad f(0) = -12 \times 21 + 1 = -251$

$$\left[\frac{|f(0)|}{30}\right] = \left[\frac{251}{30}\right] = [8.366] = 8$$

37. (1) : $y=\frac{1}{1-x}$

At $x = 2$, $y = -1$

Now, $\frac{dy}{dx}=\frac{1}{(1-x)^2}$ $\quad\therefore\quad \left.\frac{dy}{dx}\right|_{(2,-1)}=1$

So, equation of tangent to the curve is

$$y + 1 = 1(x - 2) \quad\text{or}\quad y = x - 3$$

Solving this line with $y = -a^2x^2 + 5ax - 4$, we get

$$x - 3 = -a^2x^2 + 5ax - 4$$

or $\quad a^2x^2 + x(1 - 5a) + 1 = 0 \qquad$ ...(i)

Above equation has two roots x_1 and x_2 which are abscissas of point of intersection P and Q.

Mid point of PQ lies on $x = 2$

$$\therefore\quad \frac{x_1+x_2}{2}=2 \quad\Rightarrow\quad x_1 + x_2 = 4$$

From equation (i), $x_1 + x_2 = -\left(\frac{1-5a}{a^2}\right) \Rightarrow 4=\frac{5a-1}{a^2}$

$\Rightarrow\ 4a^2 - 5a + 1 = 0 \quad\Rightarrow\quad a = 1/4, 1$

But for $a = 1/4$, eq. (i) gives imaginary roots.

38. (32) : We have, $a_n = \tan^{-1}\left(\frac{3}{9n^2+3n-1}\right)$

$$=\tan^{-1}\left(\frac{3}{1+(3n+2)(3n-1)}\right)$$

$$=\tan^{-1}\left(\frac{(3n+2)-(3n-1)}{1+(3n+2)(3n-1)}\right)$$

$= \tan^{-1}(3n + 2) - \tan^{-1}(3n - 1)$

$\therefore\quad$ Sum of first 10 terms

$$=\sum_{r=1}^{10}a_r=\sum_{r=1}^{10}[\tan^{-1}(3r+2)-\tan^{-1}(3r-1)]$$

$= (\tan^{-1} 5 - \tan^{-1} 2) + (\tan^{-1} 8 - \tan^{-1} 5) + \ldots$
$\qquad + (\tan^{-1} 32 - \tan^{-1} 29)$

$$= \tan^{-1} 32 - \tan^{-1} 2 = \tan^{-1}\left(\frac{32-2}{1+32\cdot 2}\right)$$

$$=\tan^{-1}\left(\frac{30}{65}\right)=\tan^{-1}\left(\frac{6}{13}\right)=\cot^{-1}\left(\frac{13}{6}\right)=\cot^{-1}\left(\frac{m}{n}\right)$$

[Given]

$\Rightarrow\quad m = 13$ and $n = 6$

Hence, $(2m + n) = 32$

39. (3) : $\tan\theta = \cot 5\theta = \tan\left(\frac{\pi}{2}-5\theta\right)$

$$\therefore\quad \theta=n\pi+\frac{\pi}{2}-5\theta \Rightarrow \theta=(2n+1)\frac{\pi}{12}$$

$\theta=\frac{\pi}{12},\frac{\pi}{4},\frac{5\pi}{12},\frac{-\pi}{12},\frac{-\pi}{4},\frac{-5\pi}{12} \qquad$ ...(i)

$$\cos 4\theta = \sin 2\theta = \cos\left(\frac{\pi}{2}-2\theta\right)$$

$$\therefore\quad 4\theta=2n\pi\pm\left(\frac{\pi}{2}-2\theta\right)$$

$\Rightarrow\quad \theta=(4n+1)\frac{\pi}{12},(4n-1)\frac{\pi}{4} \qquad$ ...(ii)

$$\theta=\frac{\pi}{12},\frac{5\pi}{12},\frac{-\pi}{4}$$

The common values of (i) and (ii) are in (ii). The number of values of θ is 3.

40. (1) : We have, $\frac{dy}{dx}=-\frac{\sqrt{(x^2-1)(y^2-1)}}{xy}$

$$\Rightarrow\quad \int\frac{y}{\sqrt{y^2-1}}dy=-\int\frac{\sqrt{x^2-1}}{x}dx=-\int\frac{x^2-1}{x\sqrt{x^2-1}}dx$$

$$\Rightarrow\quad \sqrt{y^2-1}=-\int\frac{x}{\sqrt{x^2-1}}dx+\int\frac{1}{x\sqrt{x^2-1}}dx$$

$$\Rightarrow\quad \sqrt{y^2-1}=-\sqrt{x^2-1}+\sec^{-1}x+c$$

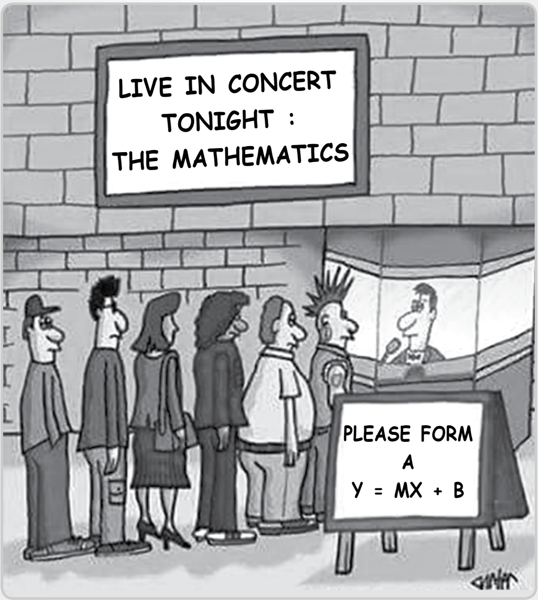
Curve passes through the point (1, 1). Then the value of $c = 0$.

Hence, the curve is $\sqrt{y^2-1}=-\sqrt{x^2-1}+\sec^{-1}(x)$

Curve passes through $(\sqrt{2},k)$

$$\therefore\quad \sqrt{k^2-1}=-1+\frac{\pi}{4} \Rightarrow k = 1$$

COMIC CAPSULE



PRACTICE PAPER

BITSAT

Exam date:
16th to 26th
May 2019

1. If a, b, c are roots of $y^3 - 3y^2 + 3y + 26 = 0$ and ω is cube root of unity, then the value of $\frac{a-1}{b-1}+\frac{b-1}{c-1}+\frac{c-1}{a-1}$ equals
(a) $-3\omega^2$ (b) $3\omega^2$ (c) ω^2 (d) $2\omega^2$

2. If $z_n = \cos\left(\frac{n\pi}{10}\right) + i\sin\left(\frac{n\pi}{10}\right)$, then $\prod_{n=1}^{4} z_n$ equals
(a) 0 (b) 1
(c) -1 (d) none of these

3. The value of x satisfying the equation
$|x-1|^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$ is
(a) 27 (b) 81 (c) 9 (d) 3

4. The roots of the equation $|x^2 - x - 6| = x + 2$ are
(a) $-2, 2, 4$ (b) $0, 1, 4$ (c) $0, 2, 4$ (d) $-2, 1, 4$

5. If $\sum_{r=0}^{\infty} a_r x^r = e^x(1+e^{4x})e^{-3x}$, then
$\sum_{p=1}^{\infty}(3^{2p-1}a_{2p-1}) =$
(a) e^x (b) $e^2 - e^{-2}$
(c) 0 (d) none of these

6. 28 is divided into 4 parts which are in A.P. The ratio of the product of the first and third to the product of the second and fourth is 8 : 15. The largest part is
(a) 6 (b) 8 (c) 10 (d) 12

7. The number of ways you can find to pack 9 different books into five parcels if four of the parcels must contain two books each, is
(a) 945 (b) ${}^9C_2 \times {}^7C_2 \times {}^5C_2 \times {}^3C_2$
(c) ${}^9C_2 \times {}^7C_2 \times {}^5C_2 \times {}^3C_2 \times 5!$
(d) none of these

8. The number of permutations of letters A, B, C, D, E, F, G so that neither the pattern BEG nor CAD appears is
(a) $\frac{7!}{3!3!}$ (b) $\frac{7!}{2!3!3!}$
(c) 4806 (d) none of these

9. Numerically the greatest term in the expansion of $(2 + 3x)^{12}$, when $x = \frac{5}{6}$ is
(a) ${}^{12}C_7\, 2^4\left(\frac{5}{2}\right)^7$ (b) ${}^{12}C_5\, 2^5\left(\frac{5}{2}\right)^7$
(c) ${}^{12}C_5\, 2^5\left(\frac{5}{4}\right)^7$ (d) ${}^{12}C_5\, 2^3\left(\frac{5}{2}\right)^5$

10. Let t be a positive integer and
$\Delta_t = \begin{vmatrix} 2t-1 & m^2-1 & \cos^2(m^2) \\ {}^mC_t & 2^m & \cos^2(m) \\ 1 & m+1 & \cos(m^2) \end{vmatrix}$, then the value of
$\sum_{t=0}^{m} \Delta_t$ is equal to
(a) 2^m (b) 0
(c) $2^m\cos^2(2^m)$ (d) m^2

11. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has
(a) no real roots (b) exactly one real root
(c) exactly four real roots (d) infinite real roots

12. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$, then a_n is
(a) $\frac{b^n - a^n}{b-a}$ (b) $\frac{a^n - b^n}{b-a}$
(c) $\frac{a^{n+1} - b^{n+1}}{b-a}$ (d) $\frac{b^{n+1} - a^{n+1}}{b-a}$

13. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$,
then $f(100)$ equals
(a) 0 (b) 1 (c) 100 (d) -100

14. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then $x^2 + y^2$ is

(a) 2 (b) 1 (c) 3 (d) 4

15. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its centre is

(a) $2ax + 2by - (a^2 + b^2 + k^2) = 0$
(b) $2ax + 2by - (a^2 - b^2 + k^2) = 0$
(c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$
(d) $x^2 + y^2 - 2ax - 3by + (a^2 + b^2 - k^2) = 0$

16. The equation of the plane through the line of intersection of planes $ax + by + cz + d = 0$, $a'x + b'y + c'z + d' = 0$ and parallel to the lines $y = 0 = z$ is

(a) $(ab' - a'b)\,x + (bc' - b'c)\,y + (ad' - a'd) = 0$
(b) $(ab' - a'b)\,y + (ac' - a'c)\,z + (ad' - a'd) = 0$
(c) $(ab' - a'b)\,x + (bc' - b'c)\,z + (ad' - a'd) = 0$
(d) none of these

17. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where p is a constant.

Then $\dfrac{d^3}{dx^3} f(x)$ at $x = 0$ is

(a) p (b) $p + p^2$
(c) $p + p^3$ (d) independent of p

18. $\displaystyle\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt =$

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{2}$ (d) π

19. If $y^2 = P(x)$, a polynomial of degree 3, then $2\dfrac{d}{dx}\left(y^3 \dfrac{d^2y}{dx^2}\right)$ equals

(a) $P'''(x) + P'(x)$ (b) $P''(x)\,P'''(x)$
(c) $P(x)\,P'''(x)$ (d) a constant

20. The probability that a teacher will given a unannounced test is $\dfrac{1}{4}$. If a student was absent twice, then probability that he will missed at least one test is

(a) $\dfrac{1}{2}$ (b) $\dfrac{7}{16}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{3}{4}$

21. If a unit vector $\vec{c}$ makes an angle $\dfrac{\pi}{6}$ with $\hat{i} \times \hat{j}$, then the maximum value of $(\hat{i} \times \hat{j}) \cdot \vec{c}$ is

(a) 0 (b) $\dfrac{\sqrt{3}}{2}$
(c) 1 (d) none of these

22. If $x_1, x_2, \ldots, x_{18}$ are observations such that $\displaystyle\sum_{j=1}^{18}(x_j - 8) = 9$ and $\displaystyle\sum_{j=1}^{18}(x_j - 8)^2 = 45$, then the standard deviation of these observations is

(a) $\sqrt{\dfrac{81}{34}}$ (b) 5 (c) $\sqrt{5}$ (d) $\dfrac{3}{2}$

23. The maximum value of $Z = 3x + 2y$, subjected to $x + 2y \le 2$, $x + 2y \ge 8$; $x, y \ge 0$ is

(a) 32 (b) 24
(c) 40 (d) none of these

24. Let $\theta \in \left(0, \dfrac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then

(a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$
(c) $t_3 > t_1 > t_2 > t_4$ (d) $t_2 > t_3 > t_1 > t_4$

25. The focal chord to $y^2 = 16x$ is tangent to $(x - 6)^2 + y^2 = 2$, then the possible values of the slope of this chord, are

(a) $\{-1, 1\}$ (b) $\{-2, 2\}$
(c) $\{-2, 1/2\}$ (d) $\{2, -1/2\}$

26. The position vector of the point where the line $\vec{r} = \hat{i} - \hat{j} + \hat{k} + t(\hat{i} + \hat{j} - \hat{k})$ meets the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 5$ is

(a) $5\hat{i} + \hat{j} - \hat{k}$ (b) $5\hat{i} + 3\hat{j} - 3\hat{k}$
(c) $2\hat{i} + \hat{j} + 2\hat{k}$ (d) $5\hat{i} + \hat{j} + \hat{k}$

27. The value of $f(0)$ so that the function $f(x) = \dfrac{\sqrt{1+x} - \sqrt[3]{1+x}}{x}$ becomes continuous, is equal to

(a) $\dfrac{1}{6}$ (b) $\dfrac{1}{4}$ (c) 2 (d) $\dfrac{1}{3}$

28. The differential equation of all 'Simple Harmonic Motions' of given period $\dfrac{2\pi}{n}$, where $x = a\cos(nt + b)$ is

(a) $\dfrac{d^2x}{dt^2} + nx = 0$ (b) $\dfrac{d^2x}{dt^2} + n^2x = 0$
(c) $\dfrac{d^2x}{dt^2} - n^2x = 0$ (d) $\dfrac{d^2x}{dt^2} + \dfrac{1}{n^2}x = 0$

29. The integral $\int_{\frac{-1}{2}}^{\frac{1}{2}}\left[[x]+\ln\left(\frac{1+x}{1-x}\right)\right]dx$ equals

(a) $\frac{-1}{2}$ (b) 0

(c) 1 (d) $2\ \ln\left(\frac{1}{2}\right)$

30. The statement $\sim(p \leftrightarrow \sim q)$ is

(a) equivalent to $\sim p \leftrightarrow q$

(b) a tautology

(c) a fallacy

(d) equivalent to $p \leftrightarrow q$

31. If q is the angle between the lines AB and AC where A, B and C are the three points with coordinates $(1, 2, -1)$, $(2, 0, 3)$, $(3, -1, 2)$ respectively, then $\sqrt{462}\cos\theta$ is equal to

(a) 20 (b) 10 (c) 30 (d) 40

32. The area of the smaller portion enclosed by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$, (in sq. units) is

(a) $\frac{\sqrt{2}}{3}+\frac{9\pi}{4}-\frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)$

(b) $2\left(\frac{\sqrt{2}}{3}+\frac{9\pi}{4}-\frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right)$

(c) $2\left(\frac{\sqrt{2}}{3}+\frac{9\pi}{4}+\frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right)$

(d) $\frac{\sqrt{2}}{3}+\frac{9\pi}{4}+\frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)$

33. If a and b are positive numbers such that $a > b$, then the minimum value of $a\sec\theta - b\tan\theta \left(0<\theta<\frac{\pi}{2}\right)$, is

(a) $\frac{1}{\sqrt{a^2-b^2}}$ (b) $\frac{1}{\sqrt{a^2+b^2}}$

(c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$

34. The shortest distance between the parabolas $y^2 = 4x$ and $y^2 - 2x + 6 = 0$ is

(a) $\sqrt{2}$ units (b) $\sqrt{5}$ units

(c) $\sqrt{3}$ units (d) none of these

35. The mean of five observations is 4 and their variance is 5.2. If three of these observations are 2, 4 and 6, then the other two observations are

(a) 3 and 5 (b) 2 and 6 (c) 4 and 4 (d) 1 and 7

36. If $\int\frac{\cos 8x+1}{\tan 2x-\cot 2x}dx = a\cos 8x + c$, then a =

(a) $-\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $-\frac{1}{8}$

37. If $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$, then x is

(a) $\pm\frac{1}{2}$ (b) $0, \frac{1}{2}$ (c) $0, -\frac{1}{2}$ (d) $0, \pm\frac{1}{2}$

38. The ellipse $E_1: \frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

39. The shortest distance between the skew lines

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}+3\hat{j}+2\hat{k})$ and

$\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$ is

(a) 3 units (b) $2\sqrt{3}$ units

(c) $\sqrt{3}$ units (d) $\sqrt{6}$ units

40. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (in sq. units) is

(a) ab (b) $2ab$ (c) a/b (d) $\sqrt{ab}$

41. The solution of the differential equation

$$\frac{x\frac{dy}{dx}-y}{\sqrt{x^2-y^2}}=10x^2 \text{ is}$$

(a) $\sin^{-1}\left(\frac{y}{x}\right)-5x^2=C$ (b) $\sin^{-1}\left(\frac{y}{x}\right)=10x^2+C$

(c) $\frac{y}{x}=5x^2+C$

(d) $\sin^{-1}\left(\frac{y}{x}\right)=10x^2+Cx$

42. E and F are two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happens is 1/2, then

(a) $P(E) = 1/3, P(F) = 1/4$

(b) $P(E) = 1/2, P(F) = 1/6$

(c) $P(E) = 1/6, P(F) = 1/2$

(d) $P(E) = 1/2, P(F) = 1/3$

43. In a survey, it is found that 70% of employees like bananas and 64% like apples. If x % like both bananas and apples, then

(a) $x \geq 34$ (b) $x \leq 64$

(c) $34 \leq x \leq 64$ (d) all of these

44. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ =

(a) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{3}}(-\hat{i}-\hat{j}-\hat{k})$

(c) $\frac{1}{\sqrt{5}}(-\hat{i}-2\hat{j})$ (d) $\frac{1}{\sqrt{3}}(-\hat{j}-\hat{k})$

45. If $y = e^x \cdot e^{x^2} \cdot e^{x^3} \ldots e^{x^n}$ ...., for $0 < x < 1$, then $\frac{dy}{dx}$ at $x = \frac{1}{2}$ is

(a) e (b) $4e$ (c) $2e$ (d) $3e$

SOLUTIONS

1. (b) : From given equation $(y - 1)^3 = -27$

$$\Rightarrow \left(\frac{y-1}{-3}\right)^3 = 1 \quad \Rightarrow \quad \frac{y-1}{-3} = (1)^{1/3}$$

$$\therefore \frac{y-1}{-3} = 1, \omega, \omega^2$$

As a, b, c are roots,

$\therefore$ $a - 1 = -3$, $b - 1 = -3\omega$, $c - 1 = -3\omega^2$

$$\therefore \frac{a-1}{b-1}+\frac{b-1}{c-1}+\frac{c-1}{a-1}=\frac{1}{\omega}+\frac{1}{\omega}+\frac{\omega^2}{\omega^3}$$

$$= \omega^2 + \omega^2 + \omega^2 = 3\omega^2$$

2. (c) : $z_1\, z_2\, z_3\, z_4 = \prod_{n=1}^{4} z_n$

$$\therefore z_1 z_2 z_3 z_4 = \left(\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}\right)\left(\cos\frac{2\pi}{10}+i\sin\frac{2\pi}{10}\right)$$

$$\times\left(\cos\frac{3\pi}{10}+i\sin\frac{3\pi}{10}\right)\left(\cos\frac{4\pi}{10}+i\sin\frac{4\pi}{10}\right)$$

$$= \cos\left(\frac{\pi}{10}+\frac{2\pi}{10}+\frac{3\pi}{10}+\frac{4\pi}{10}\right)+i\sin\left(\frac{\pi}{10}+\frac{2\pi}{10}+\frac{3\pi}{10}+\frac{4\pi}{10}\right)$$

$= \cos \pi + i \sin \pi = -1$

3. (b) : We know that

$\log_a b$ hold good if $a > 0$, $a \neq 1$, $a > 1$ *i.e.* $a - 1 > 0$

$\therefore$ $|a - 1| = a - 1$

And $a^b > 0 \ \forall \ b \in R$

Now for $2 \log_x 9$, $(x - 1) > 0$ $[\because x > 0, x \neq 1, x > 1]$

$\Rightarrow$ $|x - 1| = x - 1$

We have, $|x-1|^{\log_3 x^2 - 2\log_x 9} = (x - 1)^7$

$\Rightarrow$ $(x-1)^{\log_3 x^2 - 2\log_x 9} = (x - 1)^7$

$\Rightarrow$ $2 \log_3 x - 2\log_x 9 = 7$

(Taking log at base $(x - 1)$ both sides)

$\Rightarrow$ $2t^2 - 7t - 4 = 0$ [where $t = \log_3 x$ $\therefore$ $3^t = x$]

$\Rightarrow$ $(2t + 1)(t - 4) = 0$ $\Rightarrow$ $t = 4$ or $t = -1/2$

$\therefore$ $x = 3^4, 3^{-1/2}$ $\Rightarrow$ $x = 81$ $[\therefore x > 1]$

4. (a) : $|x^2 - x - 6| = x + 2 \Rightarrow |(x + 2)(x - 3)| = x + 2$

$$\text{Now, } x + 2 = \begin{cases}(x+2)(x-3) & \forall \quad x \leq -2 \text{ and } x \geq 3\\ -(x^2-x-6) & \forall \quad -2 < x < 3\end{cases}$$

If $(x + 2) = (x + 2)(x - 3)$, then $x = -2$, $x = 4$

and $(x + 2) = -(x + 2)(x - 3)$, then $x = -2$, $x = 2$

$\therefore$ $x = -2, 2, 4$

5. (c) : $a_0 + a_1 x + a_2 x^2 + \ldots + \text{to } \infty = e^{2x} + e^{-2x}$

$$= 2\left\{1+\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}+\ldots\text{to } \infty\right\}$$

$\therefore$ $a_1 = a_3 = a_5 = \ldots = 0$

$$\therefore \sum_{p=1}^{\infty} 3^{2p-1} \ a_{2p-1} = 3\cdot a_1 + 3^3\cdot a_3 + 3^5\cdot a_5 + \ldots = 0$$

6. (c) : Since the sum is 28, we take the A.P. as $7 - 3d$, $7 - d$, $7 + d$, $7 + 3d$.

Now, $\frac{(7-3d)(7+d)}{(7-d)(7+3d)} = \frac{8}{15}$ [Given]

$\Rightarrow$ $15[49 - 14d - 3d^2] = 8[49 + 14d - 3d^2]$

$\Rightarrow$ $3d^2 + 46d - 49 = 0$ $\Rightarrow$ $d = 1$

$\therefore$ The parts are $7 - 3$, $7 - 1$, $7 + 1$, $7 + 3$

or 4, 6, 8, 10.

7. (a) : Since 9 things are to be divided into 5 groups out of which four groups each have equal number 2 each and one group has only one element

Required number of ways $= \frac{9!}{2!2!2!2!4!} = 945$.

8. (c) : Total number of permutations = 7!

Let A be the property that 'BEG' occurs.

B be the property that 'CAD' occurs.

Number of permutations with A = 5! = That of with B

And $(A \cap B) = 3!$

$\therefore$ $n(A \cup B) = 5! + 5! - 3! = 234$

$\therefore$ Required number = 7! − 234 = 4806.

9. (b) : Let r^{th} and $(r + 1)^{\text{th}}$ term be denoted by T_r, T_{r+1} respectively, we know that in the expansion $(x + a)^n$, we have

$$\frac{T_{r+1}}{T_r} = \frac{n-r+1}{r}\frac{a}{x}$$

$\therefore$ In $(2 + 3x)^{12}$, we have

$$\frac{T_{r+1}}{T_r} = \frac{12-r+1}{r}\left(\frac{3x}{2}\right) = \frac{12-r+1}{r}\left(\frac{3}{2}\times\frac{5}{6}\right) = \frac{65-5r}{4r}$$

Now, $T_{r+1} > T_r$ if $\frac{65-5r}{4r} > 1$

i.e., if $r < \frac{65}{9} = 7\frac{2}{9}$

Again, $T_{r+1} = T_r$ if $r = 7\frac{2}{9}$ and $T_{r+1} < T_r$ if $r > 7\frac{2}{9}$

So upto 7 for all values of r, $T_{r+1} > T_r$ and for all values of $r \geq 8$, $T_{r+1} < T_r$.

Hence, the greatest term is T_{r+1} when $r = 7$

$\therefore$ T_8 is greatest term.

Now, $T_8 = T_{7+1} = {}^{12}C_7\, 2^{12-7}\, (3x)^7$

$$= {}^{12}C_7\, 2^5\left(\frac{3\times 5}{6}\right)^7 \qquad \left(\because x = \frac{5}{6}\right)$$

$$= {}^{12}C_5\, 2^5\left(\frac{5}{2}\right)^7$$

10. (b): Applying the property of summation of determinant to column C_1, we get

$$\sum_{t=0}^{m}\Delta_t = \begin{vmatrix} \sum_{t=0}^{m}(2t-1) & m^2-1 & \cos^2(m^2) \\ \sum_{t=0}^{m}{}^{m}C_t & 2^m & \cos^2(m) \\ \sum_{t=0}^{m}1 & m+1 & \cos(m^2) \end{vmatrix}$$

Now, $C_1 \sim C_2$ *i.e.* C_1, C_2 are identical

$$\therefore \quad \sum_{t=0}^{m}\Delta_t = 0$$

11. (a) : Given, $e^{\sin x} - e^{-\sin x} - 4 = 0$

Let $e^{\sin x} = y$ $\qquad [y > 0\ \forall\ x \in R]$

then, $y - 1/y - 4 = 0$

$$\Rightarrow y^2 - 4y - 1 = 0 \quad \Rightarrow \quad y = \frac{4-\sqrt{16+4}}{2} = 2-\sqrt{5}$$

y can never be negative, so $2-\sqrt{5}$ can not be accepted.

Now, $2+\sqrt{5} > e$ and maximum value of $e^{\sin x} = e$

Hence, $e^{\sin x} \neq 2+\sqrt{5}$ *i.e.*, there is no solution (no real roots).

12. (d) : We have, $\frac{1}{(1-ax)(1-bx)} = (1-ax)^{-1}(1-bx)^{-1}$

$$\frac{1}{(1-ax)}\cdot\frac{1}{(1-bx)} = (a_0 + a_1x + \ldots.. + a_nx^n + \ldots..)$$

$\Rightarrow$ $(1 + ax + a^2x^2 + \ldots\ldots. + a^{n-1}x^{n-1} + a^nx^n + \ldots\ldots)$ $(1 + bx + b^2x^2 \ldots\ldots. + b^nx^n + \ldots\ldots.) = (a_0 + a_1x + \ldots\ldots. + a_nx^n + \ldots\ldots.)$

$\Rightarrow$ $1 + x\,(a + b) + x^2\,(a^2 + ab + b^2) + x^3\,(a^3 + a^2b + ab^2 + b^3) + \ldots. + x^n\,(a^n + a^{n-1}\,b + a^{n-2}\,b^2 + \ldots. + ab^{n-1} + b^n) + \ldots. = (a_0 + a_1x + a_2x^2 + \ldots\ldots. + a_nx^n + \ldots\ldots.)$

On comparing the coefficient of x^n both sides, we have

$$a_n = a^n + a^{n-1}\,b + a^{n-2}\,b^2 + \ldots\ldots + a\,b^{n-1} + b^n$$

$$= \frac{(a^n + a^{n-1}b + a^{n-2}b^2 + \ldots.. + ab^{n-1} + b^n)(b-a)}{b-a}$$

(Multiplying and dividing by $(b-a)$)

$$= \frac{b^{n+1}-a^{n+1}}{b-a}.$$

13. (a) : Taking x common from R_2 and $x(x-1)$ from R_3, we get

$$f(x) = x^2(x-1)\begin{vmatrix} 1 & x & x+1 \\ 2 & x-1 & x+1 \\ 3 & x-2 & x+1 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 + C_1$, we get

$$f(x) = x^2(x-1)\begin{vmatrix} 1 & x+1 & x+1 \\ 2 & x+1 & x+1 \\ 3 & x+1 & x+1 \end{vmatrix} = x^2\,(x-1)\,(0) = 0$$

$\Rightarrow$ $f(100) = 0.$

14. (b) : $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ $\qquad$ ....(i)

and $x\sin\theta = y\cos\theta$ $\quad \therefore\ y = x\frac{\sin\theta}{\cos\theta}$ $\qquad$ ....(ii)

Putting this value of y in (i), we get

$$x\sin^3\theta + x\frac{\sin\theta}{\cos\theta}\cdot\cos^3\theta = \sin\theta\cos\theta$$

$\Rightarrow$ $x\sin^3\theta\cos\theta + x\sin\theta\cos^3\theta = \sin\theta\cos^2\theta$

$$\Rightarrow \quad x = \frac{\cos\theta}{\sin^2\theta + \cos^2\theta} = \cos\theta$$

Putting value of x in (ii), we get

$y = \sin\theta$ $\therefore$ $x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1$

15. (a) : Given circle is $x^2 + y^2 = k^2$ $\qquad$ ...(i)

Let the equation of the other circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ $\qquad$ ...(ii)

Since, (i) and (ii) are orthogonal.

$\Rightarrow$ $2g_1\,g_2 + 2\,f_1\,f_2 = c_1 + c_2$

$\Rightarrow$ $2g(0) + 2\,f\,(0) = c - k^2 \Rightarrow c = k^2$ $\qquad$ ...(iii)

Now the circle (ii) is passing through (a, b)

$\therefore$ $a^2 + b^2 + 2ga + 2fb + k^2 = 0$ $\qquad$ (Using (iii) also)

$\Rightarrow$ $2(-g)\,a + 2(-f)\,b = a^2 + b^2 + k^2$

Locus of the centre $(-g, -f)$ of (ii) is

$2ax + 2by - (a^2 + b^2 + k^2) = 0.$

16. (b) : Let equation of plane passes through the intersection of the planes $ax + by + cz + d = 0$ and $a'x + b'y + c'z + d' = 0$ is

$(a'x + b'y + c'z + d') + \lambda\,(ax + by + cz + d) = 0$ ...(*)
which is parallel to $y = 0 = z$ means parallel to x-axis.

$$\Rightarrow\ a' + a\lambda = 0\ \Rightarrow\ a\lambda = -a'\ \Rightarrow\ \lambda = -\frac{a'}{a}$$

Putting $\lambda = -\frac{a'}{a}$ in (*), we have

$\therefore\ a(a'x + b'y + c'z + d') - a'(ax + by + cz + d) = 0$

$\Rightarrow\ (ab' - a'b)y + (ac' - a'c)z + ad' - a'd = 0$

17. (d) : $\because \frac{d}{dx}\{f(x)\} = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

$$+\begin{vmatrix} x^3 & \sin x & \cos x \\ 0 & 0 & 0 \\ p & p^2 & p^3 \end{vmatrix} + \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

(Using row wise differentiation)

Differentiating 2 more times,

$$\frac{d^3}{dx^3}\{f(x)\} = \begin{vmatrix} 6 & -\cos x & \sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}_{\text{at } x=0} = 0$$

Hence, $\frac{d^3}{dx^3}\{f(x)\}$ is independent of p.

18. (a) : Let $t = \sin^2 u$ in the first integral and $t = \cos^2 u$ in the second integral.

$$I_1 + I_2 = \int_0^x u\sin 2u\,du + \int_x^{\frac{\pi}{2}} u\sin 2u\,du = \int_0^{\frac{\pi}{2}} u\sin 2u\,du$$

$$= \left[-\frac{u}{2}\cos 2u + \frac{1}{4}\sin 2u\right]_0^{\pi/2} = \frac{\pi}{4}.$$

19. (c) : Given $y^2 = P(x)$ ...(i)

Differentiating (i), we get $2yy' = P'(x)$ ...(ii)

$$\Rightarrow\ (y')^2 = \frac{P'(x)^2}{4y^2} = \frac{P'(x)^2}{4P(x)}$$

Differentiating (ii) again we get , $2yy'' + 2(y')^2 = P''(x)$

$\Rightarrow\ 2yy'' = P''(x) - 2(y')^2$

Using (i), $2yy'' = P''(x) - 2\left[\frac{P'(x)^2}{4P(x)}\right] = P''(x) - \frac{P'(x)^2}{2P(x)}$

$$\therefore\ 2P(x)yy'' = P(x)P''(x) - \frac{P'(x)^2}{2}$$

Now, $P(x) = y^2$

$$\therefore\ 2y^3y'' = P(x)P''(x) - \frac{P'(x)^2}{2}$$

Differentiating again, we get

$$2\frac{d}{dx}(y^3y'') = P(x)P'''(x) + P'(x)P''(x) - P'(x)P''(x)$$

$$\Rightarrow\ 2\frac{d}{dx}(y^3y'') = P(x)P'''(x).$$

20. (b) : The probability that only one test was held in those days in which the student was absent

$$= \frac{1}{4}\times\frac{3}{4} + \frac{3}{4}\times\frac{1}{4} = 2\times\frac{1}{4}\times\frac{3}{4} = \frac{3}{8}$$

Now the probability that the test was held on both the days $= \frac{1}{4}\times\frac{1}{4} = \frac{1}{16}$

$\therefore$ The probability that student missed at least one test

$$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$$

21. (b) : $(\hat{i}\times\hat{j})\cdot\vec{c} \le |\hat{i}\times\hat{j}||\vec{c}|\cos\frac{\pi}{6} = |\hat{k}||\vec{c}|\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

$$\Rightarrow\ (\hat{i}\times\hat{j})\cdot\vec{c} \le \frac{\sqrt{3}}{2}$$

22. (d) : $\sum_{j=1}^{18}(x_j - 8) = 9 \Rightarrow \sum_{j=1}^{18} x_j = 153$

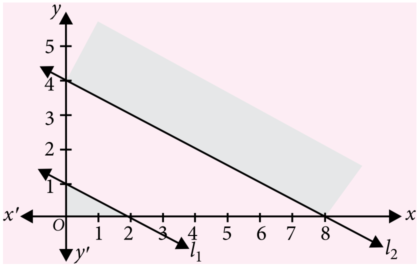
Also, $\sum_{j=1}^{18}(x_j - 8)^2 = 45 \Rightarrow \sum_{j=1}^{18}(x_j^2 - 16x_j + 64) = 45$

$$\Rightarrow \sum_{j=1}^{18} x_j^2 = 45 - 64\times 18 + 16\sum_{j=1}^{18} x_j = 45 - 1152 + 2448 = 1341$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x_j^2}{n} - \left(\frac{\Sigma x_j}{n}\right)^2}$$

$$= \sqrt{\frac{1341}{18} - \left(\frac{153}{18}\right)^2} = \sqrt{74.5 - 72.25} = 1.5$$

23. (d) : Let $l_1 : x + 2y = 2$, $l_2 : x + 2y = 8$,
$l_3 : x = 0$, $l_4 : y = 0$



Clearly, it has no solution because the constraints are inconsistent.

24. (b) : Since, $\theta \in \left(0, \frac{\pi}{4}\right) \Rightarrow \tan\theta < 1$ and $\cot\theta > 1$

Let $\tan\theta = 1 - x$ and $\cot\theta = 1 + y$ where $x, y > 0$ and are very small, then

$t_1 = (1 - x)^{1 - x}$, $t_2 = (1 - x)^{1 + y}$

$t_3 = (1 + y)^{1 - x}$, $t_4 = (1 + y)^{1 + y}$

Clearly, $t_4 > t_3$ and $t_1 > t_2$. Also $t_3 > t_1$.

Thus $t_4 > t_3 > t_1 > t_2$.

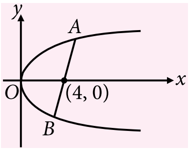
25. (a) : For parabola $y^2 = 16x$, focus = (4, 0)

Let m be the slope of focal chord then equation is

$y = m(x - 4)$ ...(i)

But according to question above is a tangent to the circle $(x - 6)^2 + y^2 = 2$ which has $C(6, 0)$, $r = \sqrt{2}$

∴ Length of ⊥ from (6, 0) to (i) = r



$$\Rightarrow \frac{6m - 4m}{\sqrt{m^2 + 1}} = \sqrt{2}$$

$$\Rightarrow 2m = \sqrt{2(m^2 + 1)}$$

$$\Rightarrow 2m^2 = m^2 + 1 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1.$$

26. (b) : Given equation of line is

$$\vec{r} = \hat{i} - \hat{j} + \hat{k} + t(\hat{i} + \hat{j} - \hat{k})$$

$$\Rightarrow \vec{r} = (1 + t)\hat{i} + (t - 1)\hat{j} + (1 - t)\hat{k} \quad \text{...(i)}$$

Given, equation of plane is $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 5$

Using (i), we get $(1 + t) + (t - 1) + (1 - t) = 5$

$\Rightarrow t = 4$

Substituting this value in (i), we get

$\vec{r} = 5\hat{i} + 3\hat{j} - 3\hat{k}$

27. (a) : The function $f(x)$ is continuous except possibly at $x = 0$. For $f(x)$ to be continuous at $x = 0$, we must have

$$f(0) = \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{(1 + x)^{\frac{1}{2}} - (1 + x)^{\frac{1}{3}}}{x}$$

$$= \lim_{x \to 0} \frac{\left[1 + \frac{1}{2}x + \frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{2}x^2 + \cdots\right] - \left[1 + \frac{1}{3}x + \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2}x^2 + \cdots\right]}{x}$$

$$= \lim_{x \to 0} \frac{x\left[\frac{1}{2} + \frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{2}x + \cdots\right] - x\left[\frac{1}{3} + \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2}x + \cdots\right]}{x}$$

$$= \lim_{x \to 0}\left[\left(\frac{1}{2} - \frac{1}{3}\right) + \text{terms containing } x\right] = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

28. (b) : Given $x = a\cos(nt + b)$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\frac{dx}{dt} = -a\sin(nt + b) \cdot n \quad \text{...(ii)}$$

Again differentiating (ii) w.r.t. x, we get

$$\frac{d^2x}{dt^2} = -n^2 a\cos(nt + b)$$

$$\Rightarrow \frac{d^2x}{dt^2} = -n^2x \Rightarrow \frac{d^2x}{dt^2} + n^2x = 0.$$

29. (a) : Let $I = \int_{\frac{-1}{2}}^{\frac{1}{2}}\left[[x] + \ln\left(\frac{1 + x}{1 - x}\right)\right]dx$

$$= \int_{\frac{-1}{2}}^{\frac{1}{2}}[x]dx + \int_{\frac{-1}{2}}^{\frac{1}{2}}\ln\left(\frac{1 + x}{1 - x}\right)dx$$

Now, $\int_{\frac{-1}{2}}^{\frac{1}{2}}[x]dx = \int_{\frac{-1}{2}}^{0}[x]dx + \int_{0}^{\frac{1}{2}}[x]dx = \int_{\frac{-1}{2}}^{0}(-1)dx + 0$

$$= [-x]_{\frac{-1}{2}}^{0} = \frac{-1}{2}.$$

And $\int_{\frac{-1}{2}}^{\frac{1}{2}}\ln\left(\frac{1 + x}{1 - x}\right)dx = 0$

$$\left(\because \ln\left(\frac{1 + x}{1 - x}\right) \text{is an odd function}\right)$$

$$\therefore I = \frac{-1}{2} + 0 = \frac{-1}{2}.$$

30. (d) :

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

From the truth table, we have the statement $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

31. (a) : Since, the coordinates of A, B and C are (1, 2, –1), (2, 0, 3) and (3, –1, 2) respectively, then

$$\overrightarrow{AB} = (2 - 1)\hat{i} + (0 - 2)\hat{j} + (3 + 1)\hat{k} = \hat{i} - 2\hat{j} + 4\hat{k}$$

and $\overrightarrow{AC} = (3 - 1)\hat{i} + (-1 - 2)\hat{j} + (2 + 1)\hat{k} = 2\hat{i} - 3\hat{j} + 3\hat{k}$

$$\therefore \cos\theta = \frac{(\hat{i} - 2\hat{j} + 4\hat{k}) \cdot (2\hat{i} - 3\hat{j} + 3\hat{k})}{\sqrt{1 + 4 + 16}\sqrt{4 + 9 + 9}}$$

$$= \frac{2 + 6 + 12}{\sqrt{21}\sqrt{22}} = \frac{20}{\sqrt{462}}$$

$$\Rightarrow \sqrt{462}\cos\theta = 20.$$

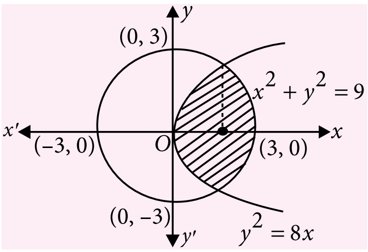
32. (b) : $x^2 + y^2 = 9$,

$x^2 + 8x - 9 = 0$

$$\Rightarrow x = \frac{-8 \pm \sqrt{64+36}}{2}$$

$$\Rightarrow x = \frac{-8 \pm 10}{2} = -9, 1$$

$\Rightarrow x = 1$



$$\text{Area enclosed} = 2\left[\int_0^1 2\sqrt{2}\sqrt{x}dx + \int_1^3 \sqrt{9-x^2}dx\right]$$

$$= 2\left[2\sqrt{2}\int_0^1 \sqrt{x}dx + \int_1^3 \sqrt{9-x^2}dx\right]$$

On simplifying, we get

$$\text{Area} = 2\left[\frac{\sqrt{2}}{3} + \frac{9\pi}{4} - \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right] \text{sq. units}$$

33. (d) : Let $a \sec\theta - b \tan\theta = x$

$\Rightarrow \ a^2 \sec^2\theta = (x + b \tan\theta)^2$

$\Rightarrow \ a^2 (1 + \tan^2\theta) = x^2 + 2bx \tan\theta + b^2 \tan^2\theta$

$\Rightarrow \ \tan^2\theta (a^2 - b^2) - 2bx \tan\theta + a^2 - x^2 = 0$

$$\Rightarrow \left(\tan\theta - \frac{bx}{a^2-b^2}\right)^2 = \frac{a^2[x^2+b^2-a^2]}{(a^2-b^2)^2}$$

Since L.H.S. is perfect square, so it is always +ve.

So, R.H.S. will be +ve.

When $x^2 + (b^2 - a^2) \ge 0 \ \Rightarrow \ x^2 \ge a^2 - b^2$

Thus, the minimum value of x is $\sqrt{a^2 - b^2}$.

34. (b) : Shortest distance between two curves exist along the common normal, we need the equation of normals to both the curve.

Now normal equation to the curve $y^2 = 4x$ at $(m^2, 2m)$ is taken as

$$y - y_1 = -\left(\frac{1}{\frac{dy}{dx}}\right)_{(x,y)=(m^2,2m)}(x - x_1)$$

$\therefore \ (y - 2m) = -m(x - m^2)$

$\Rightarrow \ y + mx - 2m - m^3 = 0$ ...(i)

Similarly, normal to $y^2 - 2x + 6 = 0$ at $\left(\frac{1}{2}t^2 + 3, t\right)$ is

$y + t(x - 1) - 3t - \frac{1}{2}t^3 = 0$ ...(ii)

(i) and (ii) represents same line.

$\Rightarrow \ -2m - m^3 = -4m - \frac{1}{2}m^3$

$\Rightarrow \ 2m = m^3 - \frac{1}{2}m^3 \ \Rightarrow \ 2m = \frac{1}{2}m^3$

$\Rightarrow \ m\left(2 - \frac{1}{2}m^2\right) = 0 \ \Rightarrow \ m = 0, m = \pm 2$

$\therefore$ Points on the parabola are $(m^2, 2m) = (4, 4)$

and $\left(\frac{1}{2}m^2 + 3, m\right) = (5, 2)$

$\therefore$ Shortest distance $= \sqrt{(5-4)^2 + (2-4)^2} = \sqrt{5}$ units

35. (d) : Let the unknown data be a and b.

$$\text{Variance} = \frac{1}{n}\sum x^2 - \bar{x}^2 = 5.2$$

$$\Rightarrow \ \frac{1}{5}(4+16+36+a^2+b^2) - 16 = 5.2$$

$\Rightarrow \ a^2 + b^2 = 50$ ...(i)

Also, $2 + 4 + 6 + a + b = 20 \ \Rightarrow \ a + b = 8$ ...(ii)

It is clear that the two observations are 1 and 7 which satisfies both (i) and (ii).

36. (c) : L.H.S. $= \int \frac{\cos 8x + 1}{\tan 2x - \cot 2x} dx$

$$= \int \frac{2\cos^2 4x}{\left(\frac{\sin^2 2x - \cos^2 2x}{\sin 2x \cos 2x}\right)} dx = -\int \frac{\cos^2 4x(2\sin 2x \cos 2x)}{(\cos^2 2x - \sin^2 2x)} dx$$

$$= -\int \frac{\cos^2 4x \times \sin 4x}{\cos 4x} dx = -\frac{1}{2}\int 2 \sin 4x \cos 4x \ dx$$

$$= -\frac{1}{2}\int \sin 8x \ dx = \frac{1}{2} \times \frac{\cos 8x}{8} + c$$

Now, $\frac{1}{2} \frac{\cos 8x}{8} + c = a \cos 8x + c \ \Rightarrow \ a = \frac{1}{16}$

37. (d) : $\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1} 3x$

$\Rightarrow \ \tan^{-1}(x - 1) + \tan^{-1} x = \tan^{-1} 3x - \tan^{-1}(x + 1)$

$$\Rightarrow \ \tan^{-1}\left[\frac{(x-1)+x}{1-(x-1)x}\right] = \tan^{-1}\left[\frac{3x-(x+1)}{1+3x(x+1)}\right]$$

$\Rightarrow \ (1 + 3x^2 + 3x)(2x - 1) = (1 - x^2 + x)(2x - 1)$

$\Rightarrow \ (2x - 1)(4x^2 + 2x) = 0 \ \Rightarrow \ x = 0, \pm\frac{1}{2}$

38. (c) : Let the required ellipse be $\frac{x^2}{\alpha} + \frac{y^2}{\beta} = 1$

It passes through (0, 4) $\Rightarrow \ \frac{16}{\beta} = 1 \ \Rightarrow \ \beta = 16$

It passes through (3, 2) $\Rightarrow \ \frac{9}{\alpha} + \frac{4}{\beta} = 1 \ \Rightarrow \ \alpha = 12$

Also $\alpha = \beta(1 - e^2) \ \Rightarrow \ 12 = 16(1 - e^2)$

$\Rightarrow \ \frac{3}{4} = 1 - e^2 \ \Rightarrow \ e^2 = \frac{1}{4} \ \Rightarrow \ e = \frac{1}{2}$

39. (c) : Shortest distance $= \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 3 & 2\\ 2 & 3 & 1\end{vmatrix}=\hat{i}(-3)-\hat{j}(-3)+\hat{k}(-3)=-3\hat{i}+3\hat{j}-3\hat{k}$$

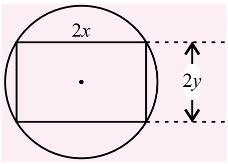
$$\therefore\quad |\vec{b}_1\times\vec{b}_2|=\sqrt{9+9+9}=\sqrt{27}=3\sqrt{3}$$

$$(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)=(3\hat{i}+3\hat{j}+3\hat{k})\cdot(-3\hat{i}+3\hat{j}-3\hat{k})$$

$= -9 + 9 -9 = -9$

$\therefore$ Shortest distance $=\dfrac{9}{3\sqrt{3}}=\sqrt{3}$ units

40. (b) : Any point on the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is $(a\cos\theta,\ b\sin\theta)$.



So the area of rectangle inscribed in the ellipse is given by

$A = (2a\cos\theta)\ (2b\sin\theta)$

$$\therefore\ A = 2ab\sin 2\theta \Rightarrow \frac{dA}{d\theta}=4ab\cos 2\theta$$

Now for maximum or minimum area,

$$\frac{dA}{d\theta}=0\Rightarrow \theta=\frac{\pi}{4}\text{ and }\left(\frac{d^2A}{d\theta^2}\right)_{\theta\,=\,\pi/4}=-8ab\sin 2\theta$$

As $\dfrac{d^2A}{d\theta^2}<0\quad\therefore$ Area is maximum for $\theta = \pi/4$.

$\therefore$ Sides of rectangle are $\dfrac{2a}{\sqrt{2}},\ \dfrac{2b}{\sqrt{2}}$

Required area $= 2ab$ sq. units

41. (a) : We have, $\dfrac{x\dfrac{dy}{dx}-y}{\sqrt{x^2-y^2}}=10x^2$

$$\Rightarrow\ \frac{x\,dy-y\,dx}{x^2}=10x\cdot\sqrt{1-\frac{y^2}{x^2}}\,dx \qquad \text{...(i)}$$

Put $\dfrac{y}{x}=t\ \Rightarrow\ \dfrac{x\,dy-y\,dx}{x^2}=dt$

$\therefore$ Equation (i) becomes, $\dfrac{dt}{\sqrt{1-t^2}}=10x\,dx$

Integrating both sides, we get $\displaystyle\int\frac{dt}{\sqrt{1-t^2}}=10\int x\,dx$

$$\Rightarrow\ \sin^{-1}t=\frac{10x^2}{2}+C\Rightarrow \sin^{-1}\left(\frac{y}{x}\right)=5x^2+C$$

42. (a) : For two independent events E and F $P(E\cap F) = P(E)\cdot P(F)$

Given $P(E)\cdot P(F)=\dfrac{1}{12}$ ....(i)

and $P(E^c)\cdot P(F^c)=\dfrac{1}{2}$

Since E and F are independent

$\therefore\ P(E^c\cap F^c) = P(E^c)\ P(F^c)$

$$\Rightarrow\ 1-P(E\cup F)=\frac{1}{2}\qquad\left[\text{since } P(E^c)\cdot P(F^c)=\frac{1}{2}\right]$$

$$\Rightarrow\ 1-[P(E)+P(F)-P(E)\cdot P(F)]=\frac{1}{2}$$

$$\Rightarrow\ 1-P(E)-P(F)+\frac{1}{12}=\frac{1}{2}$$

$$\Rightarrow\ P(E)+P(F)=\frac{7}{12}\qquad\text{....(ii)}$$

From equation (i) and (ii), we have

$$P(E)=\frac{1}{3},\ P(F)=\frac{1}{4}\text{ or }P(E)=\frac{1}{4},\ P(F)=\frac{1}{3}$$

43. (d) : Let A and B denote the set of employees who like bananas and apples respectively and let total number of employees is 100.

$\therefore\ n(A) = 70,\ n(B) = 64,\ n(U) = 100$ and $n(A\cap B) = x$

Now $n(A\cup B)\le 100$

$\therefore\ n(A) + n(B) - n(A\cap B)\le 100$

$\Rightarrow\ 70 + 64 - x\le 100\ \Rightarrow\ x\ge 34$ ...(i)

Again $A\cap B\subseteq B$

$\therefore\ n(A\cap B)\le n(B)\ \Rightarrow\ x\le 64$ ...(ii)

By (i) and (ii), we have $34\le x\le 64$

44. (a) : Since $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, therefore $\vec{c}=\vec{a}\alpha+\beta\vec{b}$, where α, β are scalars. ... (i)

Since $\vec{c}$ is perpendicular to $\vec{a}$, therefore $\vec{c}\cdot\vec{a}=0$

From (i), $\alpha\vec{a}\cdot\vec{a}+\beta\vec{b}\cdot\vec{a}=0$

$\Rightarrow\ \alpha\ (6) + \beta\ (2 + 2 - 1) = 3\ (2\alpha + \beta) = 0$

$\therefore\ 2\alpha + \beta = 0\ \Rightarrow\ \beta = -2\alpha$

$$\therefore\ \vec{c}=\alpha(\vec{a}-2\vec{b})=\alpha(-3\hat{j}+3\hat{k})=3\alpha(-\hat{j}+\hat{k})$$

or $|\vec{c}|^2 = 9\alpha^2\ (1 + 1) = 18\alpha^2$

$$\Rightarrow\ 1 = 18\alpha^2\ \Rightarrow\ \alpha=\pm\frac{1}{3\sqrt{2}}\qquad\therefore\ c=\pm\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$$

Thus, we have $c=\dfrac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$

45. (b) : $y = e^x\cdot e^{x^2}\cdot e^{x^3}\cdot e^{x^4}\ldots e^{x^n}\ldots$

$$\Rightarrow\ y = e^{x+x^2+x^3+x^4+\ldots+x^n\ldots}$$

$$\Rightarrow\ y=e^{\frac{x}{1-x}}\qquad[\because\ 0<x<1]$$

$$\therefore\ \frac{dy}{dx}=e^{\frac{x}{1-x}}\times\frac{(1-x)(1)-x(-1)}{(1-x)^2}=e^{\frac{x}{1-x}}\frac{1}{(1-x)^2}$$

$$\therefore\ \left(\frac{dy}{dx}\right)_{x=\frac{1}{2}}=e^{\frac{1/2}{1-1/2}}\left(\frac{1}{\left(1-\frac{1}{2}\right)^2}\right)=4e$$

◆◆

MATH archives

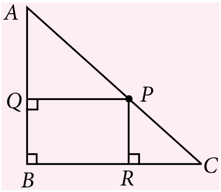
Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main & Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main & Advanced). In every issue of MT, challenging problems are offered with detailed solution. The reader's comments and suggestions regarding the problems and solutions offered are always welcome.

1. If $x\cos\alpha + y\sin\alpha = x\cos\beta + y\sin\beta = a \left(0 < \alpha, \beta < \frac{\pi}{2}\right)$, then the value of $\cos(\alpha + \beta)$ is

(a) $\frac{4axy}{x^2+y^2}$ (b) $\frac{2a^2-x^2-y^2}{x^2+y^2}$

(c) $\frac{x^2-y^2}{x^2+y^2}$ (d) $\frac{a^2-x^2}{a^2-y^2}$

2. ABC is an isosceles right angled triangle with $AB = BC = 1$. If P is any point on AC, then min{max{area APQ, (area $PQBR$), area PRC}} =



(a) $\frac{2}{9}$ (b) $\frac{1}{9}$

(c) $\frac{1}{3}$ (d) none of these

3. Number of ordered pair (x, y) satisfying $x^2 + 1 = y$ and $y^2 + 1 = x$ $(x, y \in R)$, is

(a) 0 (b) 1 (c) 2 (d) 4

4. If a, b, c and d are distinct integers such that $a + b + c + d = 17$, then the real roots of $(x - a)(x - b)(x - c)(x - d) = 4$ are

(a) all rational (b) all integers

(c) irrational (d) nothing can be said

5. Let $f(x) = x + \sin x$, then $\int_a^1 f^{-1}(x)\,dx$ is equal to, $\left(\text{where } a = \lim_{x\to 0^+}\left[\sec^{-1}\left(\frac{1}{x}\right) - \sec^{-1}\left(-\frac{1}{x}\right)\right]\right.$, [.] denotes the greatest integer function)

(a) $-2 + \sin 1$ (b) 0

(c) $2(1 + \sin 1)$ (d) 3

6. $\lim_{n\to\infty}\int_0^1 \frac{nx^{n-1}}{1+x^2}dx =$

(a) 0 (b) 1 (c) 2 (d) 1/2

7. If a, b, c are non zero real numbers and the equation $ax^2 + bx + c + i = 0$ has purely imaginary roots, then

(a) $a = bc$ (b) $a = b^2c$

(c) $a = \sqrt{bc}$ if $b > 0$ (d) none of these

8. Let u, v, w, z be complex numbers such that $|u| < 1$, $|v| = 1$ and $w = \frac{v(u-z)}{\bar{u}z - 1}$, then

(a) $|w| \le 1 \Rightarrow |z| < 1$ (b) $|w| \ge 1 \Rightarrow |z| \ge 1$

(c) $|w| \le 1 \Rightarrow |z| \ge 1$ (d) $|w| \ge 1 \Rightarrow |z| \le 1$

9. P and Q are any two points on the circle $x^2 + y^2 = 4$ such that PQ is a diameter. If α and β are the lengths of perpendiculars from P and Q on $x + y = 1$, then the maximum value of $\alpha\beta$ is

(a) 1/2 (b) 7/2 (c) 1 (d) 2

10. There is a point $P(3, 0)$ inside the circle $x^2 + y^2 = r^2$, a chord AB of the circle passes through point P such that $AP = 2$ and $BP = 8$, then the radius of the circle is

(a) 3 (b) 4 (c) 5 (d) $\sqrt{3+4+5}$

SOLUTIONS

1. (c) : α and β are roots of $x\cos\theta + y\sin\theta = a$ ...(i)

$\Rightarrow (a - x\cos\theta)^2 = (y\sin\theta)^2$

$\Rightarrow (x^2 + y^2)\cos^2\theta - 2ax\cos\theta + a^2 - y^2 = 0$

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$$\therefore \quad \cos\alpha\cos\beta = \frac{a^2 - y^2}{x^2 + y^2}$$

Similarly expressing (i) as quadratic in sinθ, we get

$$\sin\alpha \, \sin\beta = \frac{a^2 - x^2}{x^2 + y^2}$$

2. (a) : Let $B \equiv (0, 0)$, $P \equiv (h, k)$

$\Rightarrow$ Area of $\Delta APQ = \frac{1}{2}h^2$, area of $PQBR = hk$,

area of $\Delta PRC = \frac{1}{2}k^2$

Clearly, max{area ΔAPQ, area $PQBR$, area ΔPRC}

$= \text{area } APQ = \frac{1}{2}h^2 \left[\text{for } \frac{h}{k} \geq 2\right] \geq \frac{2}{9}$,

$= \text{area } PQBR = hk \left[\text{for } \frac{1}{2} \leq \frac{h}{k} < 2\right] > \frac{2}{9}$

3. (a) : $x^2 + 1 = y$ and $y^2 + 1 = x$

Subtracting the above equations, we get, $x^2 - y^2 = y - x$

$\Rightarrow (x - y)(x + y + 1) = 0$

$\Rightarrow x = y$ or $x + y + 1 = 0$

If $x = y$ then $x^2 + 1 = x \Rightarrow x^2 - x + 1 = 0$,

which has no real roots.

If $x + y + 1 = 0$, then $x + y = -1$

Adding given equations, we have $x^2 + y^2 + 2 = x + y$

$\Rightarrow x^2 + y^2 + 2 = -1 \quad \Rightarrow \quad x^2 + y^2 + 3 = 0$,

which is not possible.

4. (c) : Roots of $(x - a)(x - b)(x - c)(x - d) = 4$ are integers if they are rational.

For integral roots

$(x - a)(x - b)(x - c)(x - d) = (1)(-1)(2)(-2)$

$\Rightarrow x - a = 1, x - b = -1, x - c = 2, x - d = -2$

$$\Rightarrow \quad x = \frac{a+b+c+d}{4} \quad \text{(not an integer)}$$

$\Rightarrow x$ is irrational.

5. (b) : $f(x)$ is an odd function.

6. (d) : $$\int_0^1 \frac{nx^{n-1}}{1+x^2}dx = \left[\frac{x^n}{1+x^2}\right]_0^1 + 2\int_0^1 \frac{x^{n+1}}{(1+x^2)^2}dx$$

$$= \frac{1}{2} + 2\int_0^1 \frac{x^{n+1}}{(1+x^2)^2}dx = \frac{1}{2}$$

$$\left(\text{As, } \lim_{n\to\infty} \frac{x^{n+1}}{(1+x^2)^2} = 0 \text{ (as } 0 < x < 1)\right)$$

7. (b) : Let ki be the root of the given equation and $k \in R$

$\Rightarrow (-k^2a + c) + (bk + 1)i = 0$

$$\Rightarrow \quad k = -\frac{1}{b}, \ \frac{a}{b^2} = c \Rightarrow a = b^2c$$

8. (b) : $$|w| = \frac{|v||u-z|}{|\bar{u}z - 1|} = \frac{|u-z|}{|\bar{u}z-1|} \qquad [\because |v| = 1]$$

Let $|w| \leq 1 \Rightarrow |u - z| \leq |\bar{u}z - 1|$

This simplifies to

$$(|u|^2 - 1)(|z|^2 - 1) \geq 0 \Rightarrow |z|^2 - 1 \leq 0$$

9. (b) : Let P be $(2\cos\theta, 2\sin\theta)$ and Q be $(-2\cos\theta, -2\sin\theta)$

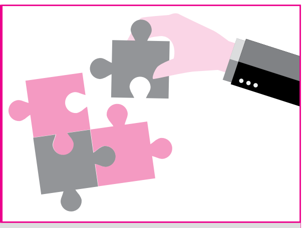
$$\Rightarrow \quad \alpha\beta = \frac{|2\cos\theta + 2\sin\theta - 1| \cdot |-2\cos\theta - 2\sin\theta - 1|}{2}$$

$$= \frac{|4(\cos\theta + \sin\theta)^2 - 1|}{2} \leq \frac{7}{2}$$

10. (c) : $PA \cdot BP = r^2 - OP^2$ (where O is centre of circle)

◆ ◆

PUZZLE CORNER

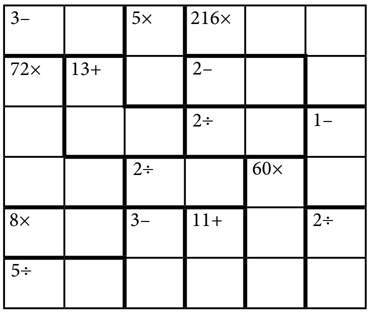


MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is multiple of its numbers. For example, if that value is 3 for a two-box cluster, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



3–		5×	216×		
72×	13+		2–		
			2÷		1–
		2÷		60×	
8×		3–	11+		2÷
5÷					

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

CONCEPT
MAP

RELATIONS AND FUNCTIONS

Class XI

Cartesian Product of Sets

- Cartesian product of two sets A & B is denoted and defined as, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$
- Cartesian product of two sets is not commutative.
- **Note : (i)** If $n(A) = p$, $n(B) = q$, then $n(A \times B) = p \times q$.
  (ii) $(a, b) = (p, q) \Leftrightarrow a = p \;\&\; b = q$.
  (iii) Ordered triplet : $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$

Relations

- R is a relation from A to B (where $A, B \neq \phi$) if $R \subseteq A \times B \Leftrightarrow R \subseteq \{(a, b) : a \in A, b \in B\}$
- Dom $(R) = \{a : (a, b) \in R\}$
- Range $(R) = \{b : (a, b) \in R, \forall\, a \in A\}$
- If $n(A) = p$ and $n(B) = q$, then total number of relations is 2^{pq}.

Functions

- A relation $(f : A \to B)$ where every element of set A has only one image in set B.
- Dom $(f) = \{a : (a, b) \in f\}$
- Codomain $(f) = \{b : \forall\, b \in B\}$
- Range $(f) = \{b : (a, b) \in f, \forall\, a \in A\}$

Algebra of Real Functions

For $f : X \to R$ and $g : X \to R$

- $(f + g)\,(x) = f(x) + g(x)$ for all $x \in X$
- $(f - g)\,(x) = f(x) - g(x)$ for all $x \in X$
- $(\alpha f)\,(x) = \alpha\, f(x)$ for all $x \in X$, α is a constant
- $(f g)\,(x) = f(x) \cdot g(x)$ for all $x \in X$
- $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$, provided $g(x) \neq 0$, $x \in X$

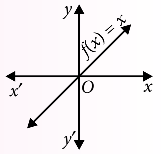
Types of Functions

- **Identity Function**

The function $f : R \to R$ defined by $f(x) = x$, $\forall\, x \in R$

Domain : R

Range : R

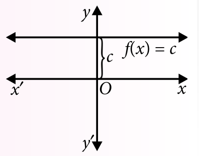


- **Constant Function**

The function $f : R \to R$ defined by $f(x) = c$, $\forall\, x \in R$

Domain : R

Range : $\{c\}$



- **Exponential Function**

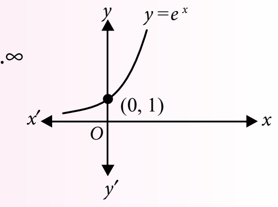
A function f defined by $f(x) = e^x$ as $e^x = f(x)$

$= 1 + x + \dfrac{x^2}{2!} + \dfrac{x}{3!} + \ldots .\infty$

is called exponential function

Domain : R

Range : $(0, \infty)$



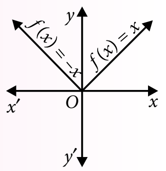
- **Modulus Function**

The function $f : R \to R$ defined by

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Domain : R

Range : $[0, \infty)$



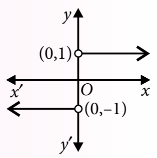
- **Signum Function**

The function $f : R \to R$ defined by

$$f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \\ 0, & x = 0 \end{cases}$$

Domain : R

Range : $\{-1, 0, 1\}$



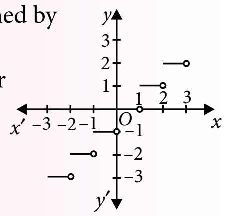
- **Greatest Integer Function**

The function $f : R \to R$ defined by

$$f(x) = [x] = \begin{cases} x, \; x \in Z \\ \text{greatest integer less than equal to } x, \; x \notin Z \end{cases}$$

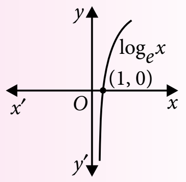
Domain : R

Range : Z



- **Logarithmic Function**

A function f defined by $f(x) = \log_e x$, $x > 0$ is called logarithmic function.



Domain : $(0, \infty)$

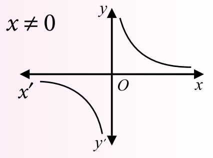
Range : R

- **Reciprocal Function**

A function f defined by

$f(x) = \dfrac{1}{x}$, $x \neq 0$

is called reciprocal function.



Domain : $R - \{0\}$

Range : $R - \{0\}$

MATRICES

Class XII

CONCEPT
MAP

Matrix

A set of mn numbers arranged in the form of a rectangular array of m rows and n columns is called a matrix.

Types of Matrices

- Row Matrix : Matrix having only one row.
- Column Matrix : Matrix having only one column.
- Square Matrix : Matrix having equal number of rows and columns.
- Diagonal Matrix : A square matrix is called diagonal matrix, if all its non-diagonal elements are zero. The diagonal elements may or may not be zero.
- Zero Matrix : Matrix whose each and every element is zero.
- Identity Matrix : A diagonal matrix whose all diagonal elements is equal to 1.
- Involutary Matrix : $A^2 = I$
- Orthogonal Matrix : $AA^T = A^TA = I$
- Idempotent Matrix : $A^2 = A$
- Unitary Matrix : $AA^\theta = A^\theta A = I$
- Symmetric Matrix : $A^T = A$
- Skew-Symmetric Matrix : $A^T = -A$

Inverse of a Matrix

A unique matrix B for A such that $AB = BA = I$.

- Inverse of a square matrix, if it exists, is unique.
- $(A^{-1})^{-1} = A$
- $(kA)^{-1} = A^{-1}/k$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(A^T)^{-1} = (A^{-1})^T$

Order of a Matrix

The number of rows and columns that a matrix has is called its order.

Trace of a Matrix

Sum of principal diagonal elements of a square matrix A is called trace of a matrix.

- $tr(A) + tr(B) = tr(A + B)$
- $tr(kA) = k \cdot tr(A)$

Transpose of a Matrix

Transpose of a matrix is obtained by interchanging rows and columns. If $A = [a_{ij}]_{m \times n}$, then A' or $A^T = [a_{ji}]_{n \times m}$

- $(A')' = A$
- $(kA)' = kA'$
- $(AB)' = B'A'$
- $(A \pm B)' = A' \pm B'$
- $(ABC)' = C'\,B'\,A'$

Comparable Matrices

Two matrices are said to be comparable if they have same orders.

Equal Matrices : Two comparable matrices A and B are said to be equal iff all of their corresponding elements are equal.

Elementary Operations

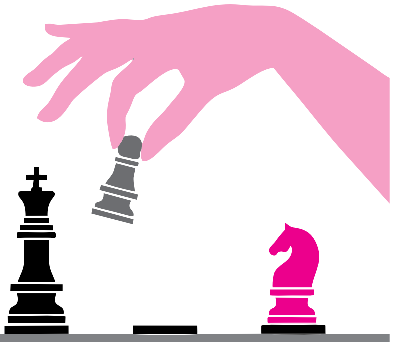
- $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$
- $R_i \to kR_i$ or $C_i \to kC_i$
- $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$

Operations on Matrices

<table>
<tr><th>Addition and Subtraction</th><th>Multiplication</th><th>Scalar Multiplication</th></tr>
<tr><td>$A \pm B = C$
i.e., $[a_{ij}]_{m \times n} \pm [b_{ij}]_{m \times n}$
$= [a_{ij} \pm b_{ij}]_{m \times n} = [c_{ij}]_{m \times n}$</td><td>$A_{m \times k} \times B_{k \times q} = C_{m \times q}$ i.e., $\left[\sum_{r=1}^{k} a_{ir} b_{rj}\right] = [c_{ij}]$</td><td>$kA = B$ i.e., $k[a_{ij}]_{m \times n} = [ka_{ij}]_{m \times n} = [b_{ij}]_{m \times n}$</td></tr>
<tr><th colspan="3">Properties</th></tr>
<tr><td>• $A + B = B + A$
• $A + (B + C) = (A + B) + C$
• Additive inverse of $A = -A$
• Additive identity $= O$</td><td>• AB exist $\not\Rightarrow$ BA exists
• AB may or may not be equal to BA
• $(AB)C = A(BC)$
• $I_m \times A_{m \times m} = A_{m \times m} = A_{m \times m} \times I_m$
• $A(B + C) = AB + AC$; $(B + C)\,A = BA + CA$</td><td>• $k(A + B) = kA + kB$
• $(k + m)A = kA + mA$</td></tr>
</table>

Challenging PROBLEMS

for JEE Advanced



Single Option Correct Type

1. The number of real solutions x of the equation $x^{10} - x^8 + 8x^6 - 24x^4 + 32x^2 - 48 = 0$ is/ are

(a) 0 (b) 1 (c) 2 (d) 4

2. For complex number z with $|z| = 1$, the maximum value of $|z^3 - z + 2|$ is

(a) 13 (b) 16 (c) 20 (d) 24

3. Let $a, b, c \in R^+$ and $a + b + c = abc$, then minimum value of $\sqrt{1+\frac{1}{a^2}}+\sqrt{1+\frac{1}{b^2}}+\sqrt{1+\frac{1}{c^2}}$ is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $2\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

4. The number of values of m for which the system of equations $x^2 + y^2 = 1$, $x^2 = 2^{|x|} + |x| - y - m$ has a unique solution is/are

(a) 0 (b) 1 (c) 2 (d) 4

5. $\sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)(2k+1)3^k} = 3\log\left(\frac{3}{4}\right)+\lambda$ where λ is

(a) $\frac{\pi}{\sqrt{3}}$ (b) $\frac{\pi}{3}$ (c) $\pi\sqrt{3}$ (d) 3π

6. The locus of point P in the plane of square $ABCD$ (side length 1 cm) such that maximum $\{PA, PC\}$ $= \frac{1}{\sqrt{2}}(PB + PD)$ encloses an area of

(a) $\frac{1}{2}$ (b) $\frac{\pi}{2}$ (c) 2 (d) 2π

7. $\int\frac{(2\sin x+1)dx}{(2+\sin x)^2} =$

(a) $\frac{\cos x}{2+\sin x}+c$ (b) $\frac{-\cos x}{2+\sin x}+c$

(c) $\frac{\sin x}{2+\sin x}+c$ (d) $\frac{-\sin x}{2+\sin x}+c$

8. O is the centre, AB and BC are two diagonals of the adjacent faces of a rectangular box. If angles AOB, BOC and COA and α, β, θ then $\cos\alpha + \cos\beta + \cos\theta =$

(a) 1.5 (b) 1

(c) 0 (d) −1

9. The number of non-empty subsets of $\{1, 2, 3, ..., 12\}$ having the property that the sum of the largest element and the smallest element is 13, is

(a) 2540 (b) 1365

(c) 1250 (d) 1100

10. We call a number, T-number if its middle digit is larger than any other digit. For example 254 is a T-number. How many such 3-digit T-numbers are there?

(a) 120 (b) 125

(c) 240 (d) 250

Comprehension Type

Paragraph for Q. No. 11 and 12

Let M be the centroid of ΔABC with $\angle AMB = 2\angle ACB$

11. $AC^4 + BC^4 - AB^4 =$

(a) $AC^2 \cdot BC^2$ (b) $AC^2 \cdot AB^2$

(c) $AB^2 \cdot BC^2$ (d) $BC^2 \cdot AM^2$

12. $\angle ACB$

(a) $\geq 60°$ (b) $\leq 60°$

(c) $\geq 90°$ (d) $< 90°$

Integer Answer Type

13. If $|x + y| + |x - y| = 2$ then maximum possible value of $x^2 - 6x + y^2$ is

14. Let $x, y \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ and $x^2 + \sin x = 2a$ and $4y^3+\frac{1}{2}\sin 2y+a=0$ then $\cos(x + 2y) =$

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15. If $\frac{\tan 1}{\cos 2}+\frac{\tan 2}{\cos 4}+....+\frac{\tan 1024}{\cos 2048}=\tan\lambda-\tan\mu$, then $\left[\frac{\lambda-\mu}{1000}\right]$ ([.] denotes greatest integer function) =

16. For complex number z, $\arg(z^2-4)=\frac{5\pi}{6}$ and $\arg(z^2+4)=\frac{\pi}{3}$, then $|z|$ =

17. In ΔABC, H is its orthocentre then $\frac{AH^2+BH^2+BC^2+AC^2}{CH^2+AB^2}=$

18. Let $y = x^4 + 9x^3 + ax^2 + 9x + 4$ contains four collinear points then maximum value of $\left[\frac{a}{5}\right]$ is([.] denotes greatest integer function)

SOLUTIONS

1. **(c) :** Putting $x^2 = y$, the given equation becomes
$y^5 - y^4 + 8y^3 - 24y^2 + 32y - 48 = 0$
i.e., $(y - 2)(y^4 + y^3 + 10y^2 - 4y + 24) = 0$
$\Rightarrow (y - 2)(y^4 + y^3 + 9y^2 + (y - 2)^2 + 20) = 0$
Hence, $y = 2$ is only required solution $\Rightarrow x=\pm\sqrt{2}$

2. **(a) :** Putting $z = x + iy \Rightarrow x^2 + y^2 = 1$, $x, y \in [-1, 1]$
$f(z) = |z^3 - z + 2|$ becomes $g(x) = 16x^3 - 4x^2 - 16x + 8$
For maximum, put $g'(x) = 0 \Rightarrow x=\frac{-1}{2},\frac{2}{3}$
$\therefore\ g\left(\frac{-1}{2}\right)=13$ is maximum

3. **(c) :** Let $a = \tan x$ etc. Then, according to question
$\Sigma\tan x = \Pi\tan x \Rightarrow x + y + z = \pi$
So, given expression = $\operatorname{cosec}x + \operatorname{cosec}y + \operatorname{cosec}z$

$$\geq 3\operatorname{cosec}\left(\frac{x+y+z}{3}\right)$$

[According to Jensen's in eq. cosec is convex in $(0, \pi)$]

$$=2\sqrt{3}$$

4. **(b) :** Notice that if (x, y) is a solution, then $(-x, y)$ is also a solution, hence, for unique solution $x = 0$.
Hence $y = 1, -1 \Rightarrow m = 0, 2$
But for $m = 2$, there are two solutions $(x, y) = (1, 0)$, $(0, -1)$.
Hence, only for $m = 0$, there exists a unique solution.

5. **(a) :** Given sum, $S=\int_{k=0}^{\infty}\frac{(-1)^k}{3^k}\left[\frac{2}{2k+1}-\frac{1}{k+1}\right]$

$$=2\sum_{k=0}^{\infty}\frac{(-1)^k}{3^k}\left(\int_0^1 x^{2k}\,dx\right)-\sum_{k=0}^{\infty}\frac{(-1)^k}{3^k}\left(\int_0^1 x^k\,dx\right)$$

$$=2\cdot\int_0^1\sum_{k=0}^{\infty}\left(\frac{-x^2}{3}\right)^k dx-\int_0^1\sum_{k=0}^{\infty}\left(\frac{-x}{3}\right)^k dx$$

$$=2\cdot\int_0^1\frac{1}{1+\frac{x^2}{3}}dx-\int_0^1\frac{1}{1+\frac{x}{3}}dx \quad \text{(sum of infinite G.P.)}$$

$$S=\frac{\pi}{\sqrt{3}}+3\log(3/4)$$

6. **(b) :** Let max. $\{PA, PC\} = PA$, then according to question, $\sqrt{2}PA=PB+PD$
i.e. $BD \cdot PA = AD \cdot PB + AB \cdot PD$
Hence, by converse of ptolemy's theorem, it follows that $PDAB$ is cyclic quadrilateral. *i.e.* locus of P is circumcircle of square $ABCD$ with radius $=\frac{1}{\sqrt{2}}$.

7. **(b) :** Given integral = $\int\frac{(2\tan x\sec x+\sec^2 x)}{(\tan x+2\sec x)^2}dx$
Now, take $\tan x + 2\sec x = t$

8. **(d) :** Let $O(0, 0, 0)$ and edges be along co-ordinate axes and let $A(a, -b, -c)$, $B(a, b, c)$, $C(-a, b, -c)$ then $\cos\alpha + \cos\beta + \cos\theta$

$$=\frac{(a^2-b^2-c^2)+(-a^2+b^2-c^2)+(-a^2-b^2+c^2)}{a^2+b^2+c^2}$$

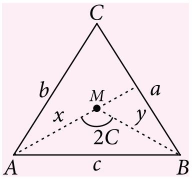
$= -1$

9. **(b) :** Let t be the smallest element then $(13 - t)$ is the largest element *i.e.* $13 - t \geq t \Rightarrow t \leq 6$.
Now, the set can contain any combination of elements from $(t + 1)$ to $(12 - t)$. There are $(12 - 2t)$ such elements.
So, there are $2^{(12-t)-(t+1)+1}=2^{12-2t}$ such combinations.
Hence, total required number of solution sets

$$=\sum_{t=1}^{6}2^{12-2t}=1365$$

10. **(c) :** Following 3 types of T-numbers are possible.
(i) xyz form $(x \neq 0) \rightarrow {}^9C_2$
(ii) xyz form $(x \neq z, z \neq 0) \rightarrow 2 \times {}^9C_3$
(iii) $xy0$ form $(x \neq 0, y \neq 0) \rightarrow {}^9C_2$
$\therefore$ Required 3-digit T-numbers = ${}^9C_2 + 2\cdot{}^9C_3 + {}^9C_2$
= 240

(11-12) : 11. (a) **12. (a)**



In ΔAMB,

$$\cos 2C = \frac{x^2+y^2-c^2}{2xy} \quad \text{...(i)}$$

and area of $\Delta AMB = \frac{1}{3}$ area of ΔABC

$$i.e. \ \frac{1}{2}xy\cdot\sin 2C = \frac{1}{3}\Delta \ \Rightarrow \ xy = \frac{2\Delta}{3\sin 2C} \quad \text{...(ii)}$$

$$\text{Now, } x = \frac{2}{3}(\text{length of median from } A)$$

$$= \frac{2}{3}\cdot\frac{1}{2}\sqrt{2b^2+2c^2-a^2} \quad \text{...(iii)}$$

$$\text{Similarly, } y = \frac{2}{3}\cdot\frac{1}{2}\sqrt{2c^2+2a^2-b^2} \quad \text{...(iv)}$$

Simplifying (i), (ii), (iii), (iv), we have

$$c^4 = a^4 + b^4 - a^2b^2$$

And, it is clear that, $a^4 + b^4 - a^2b^2 \geq (a^2 + b^2 - ab)^2$

i.e. $a^2 + b^2 - 2ab\cos C = c^2 \geq a^2 + b^2 - ab$

$$i.e., \ \cos C \leq \frac{1}{2} \ \Rightarrow \ \angle C \geq 60°$$

13. (8) : $|x+y| + |x-y| = 2 \Rightarrow$ It is square with vertices at $(\pm1, \pm1)$ $x^2 - 6x + y^2 = (x-3)^2 + y^2 - 9$ is the distance of any point on the square from (3, 0).

14. (1) : Replacing $2y$ with z and taking $f(x) = x^3 + \sin x$, the given equation becomes

$f(x) = f(-z) \ \Rightarrow \ x = -z$ as $f(x)$ is odd

and increasing in $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

So, $x = -2y$

$\therefore \ \cos(x + 2y) = \cos(0) = 1$

15. (2) : Using, $\frac{\tan\theta}{\cos 2\theta} = \tan 2\theta - \tan\theta$, the given sum becomes

$(\tan 2 - \tan 1) + (\tan 4 - \tan 2) + ... + (\tan 2048 - \tan 1024)$

$= \tan 2048 - \tan 1$

16. (2) : Let $O(0, 0)$, $B(z^2)$, $A(z^2 - 4)$, $C(z^2 + 4)$, then $AB = BC = 4$ and $\angle OAX = \frac{5\pi}{6}$, $\angle OCX = \frac{\pi}{3}$, then ΔOBC is equilateral.

$$\text{So, } z^2 = 4\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]$$

$$i.e., z = \pm(1 + i\sqrt{3}) \Rightarrow |z| = 2$$

17. (2) : If R is the circumradius of the triangle, then $AH^2 + BC^2 = 4R^2 = BH^2 + AC^2 = CH^2 + AB^2$

18. (6) : According to question, $y = 0$ has four distinct roots. So, $\frac{d^2y}{dx^2} = 12x^2 + 54x + 2a$ should have 2 real and distinct roots.

$$\Rightarrow \ D > 0 \Rightarrow a < \frac{243}{8}$$

$$\text{So, } \left[\frac{a}{5}\right]_{\max} = 6$$

◆ ◆

CBSE Introduces 20 Marks Practical for Class 12 Mathematics

CBSE has revised its evaluation and assessment practices to improve learning outcomes and to encourage critical and creative thinking among students. The Human Resource Development Ministry (MHRD) has taken this step after deciding to participate in PISA (Program for International Student Assessment) in 2021. "It is now a requirement that 'learning to assess' should be enhanced with the *evaluation of learning* and *evaluation for learning*", the CBSE says.

According to the assessment plan, for the academic year 2019-20 CBSE will present internal assessment component for Mathematics class 12. So far, CBSE conducted a 100 marks examination for the subject Mathematics. But under the new evaluation plan, there will also be an internal assessment component of at least 20 marks in subjects such as Mathematics. According to the new scheme, in the board exam question papers, there will also be objective components in the question paper from 2019-20 academic year. Minimum 25% marks will be allocated for objective type questions, including multiple choice questions. To give sufficient time to give students 'analytical and constructive feedback', the number of subjective questions will also be reduced.

Practical experiments are as important as theoretical knowledge to students. The human mind learns better when it sees things. Hence, practical knowledge is very important for all students. The practical experiments also help students to learn in a better way as they can visualize the concepts and principles.

Students must know that the practical exam of Mathematics Class 11 and Class 12 consists of 20 marks. They must aim to get full marks in the practical exam to increase their overall marks in the exam. Students are advised not to overlook and ignore the practical exam and prepare all experiments along with all viva questions.

Some tips to prepare for the practical exams are given below.

- Graphs and formulas are an integral part of the exam so students must practice them thoroughly.
- Practicing diagrams can help students to visualize and prepare better for the exam.
- It is very important that students know each and every theory, laws, principles behind the topics.
- Before the exam, students must clear all their doubts from their teachers.
- Important viva questions must be noted and revised properly before the exam.

Exam on 26th May, 2019

WB JEE
PRACTICE PAPER 2019

CATEGORY-I (Q. 1 to Q. 50)

Only one answer is correct. Correct answer will fetch full marks 1. Incorrect answer or any combination of more than one answer will fetch –¼ marks. No answer will fetch 0 marks.

1. If $\log_e 5$, $\log_e(5^x - 1)$ and $\log_e\left(5^x - \frac{11}{5}\right)$ are in A.P., then the values of x are

(a) $\log_5 4, \log_5 3$ (b) $\log_3 4, \log_4 3$
(c) $\log_3 4, \log_3 5$ (d) 12, 6

2. Let x_1 and y_1 be real numbers. If z_1 and z_2 are complex numbers such that $|z_1| = |z_2| = 4$, then $|x_1z_1 - y_1z_2|^2 + |y_1z_1 + x_1z_2|^2 =$

(a) $32(x_1^2 + y_1^2)$ (b) $16(x_1^2 + y_1^2)$
(c) $4(x_1^2 + y_1^2)$ (d) none of these

3. If a, b, c are distinct integers and $\omega \neq 1$ is a cube root of unity, then minimum value of $x = |a + b\omega + c\omega^2| + |a + b\omega^2 + c\omega|$ is

(a) $2\sqrt{3}$ (b) 3 (c) $4\sqrt{2}$ (d) 2

4. Normals are drawn at points A, B and C on the parabola $y^2 = 4x$ which intersect at $P(h, k)$. The locus of the point P if the slope of the line joining the feet of two of them is 2, is

(a) $x + y = 1$ (b) $x - y = 3$
(c) $y^2 = 2(x - 1)$ (d) $y^2 = 2\left(x - \frac{1}{2}\right)$

5. If both roots of $x^2 - 2ax + a^2 - 1 = 0$ lies in $(-2, 1)$ then $[a]$, where $[\cdot]$ denotes greatest integer function is

(a) –1 (b) 0 (c) 1 (d) 2

6. If the roots of $x^2 + 2ax + b = 0$ are real and they differ by almost $2m$, then b lies in the interval

(a) $(a^2 - m^2, a^2)$ (b) $(a^2, a^2 + m^2)$
(c) $[a^2 - m^2, a^2]$ (d) $(a^2 - m^2, a^2 + m^2)$

7. $f(x) = \begin{cases} 3[x] - 5\dfrac{|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$. Then $\int_{-3/2}^{2} f(x)dx =$

([·] denotes the greatest integer function)

(a) $-\frac{11}{2}$ (b) $-\frac{7}{2}$ (c) –6 (d) $-\frac{17}{2}$

8. Let $f(x) = e^{\cos^{-1}\sin\left(x + \frac{\pi}{3}\right)}$, then $f\left(\frac{8\pi}{9}\right) =$

(a) $e^{\frac{7\pi}{12}}$ (b) $e^{\frac{13\pi}{18}}$ (c) $e^{\frac{5\pi}{18}}$ (d) $e^{\frac{\pi}{12}}$

9. By mathematical induction, for all natural numbers n, $a^{2n-1} - 1$ is divisible by

(a) $a + 1$ (b) $a - 1$
(c) $-(a + 1)$ (d) none of these

10. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$G(y) = \begin{bmatrix} \cos y & 0 & \sin y \\ 0 & 1 & 0 \\ -\sin y & 0 & \cos y \end{bmatrix}$, then $[F(x)G(y)]^{-1} =$

(a) $F(x)G(y)$ (b) $F(x^{-1})G(y^{-1})$
(c) $G(-y)F(-x)$ (d) $G(-y^{-1})F(-x^{-1})$

11. The probability that a scheduled flight departs on time is 0.9, the probability that it arrives on time is 0.8 and the probability that it departs and arrives on time is 0.7. Then the probability that a plane arrives on time, given that it departs on time, is

(a) 0.72 (b) 8/9 (c) 7/9 (d) 0.56

12. In a bakery, four types of biscuits are available. In how many ways a person can buy 10 biscuits if he decide to take at least one biscuit of each variety?

(a) 84 (b) 94 (c) 86 (d) 96

13. Let $f: I \to N$ be defined as $f(n)=\begin{cases} 2n, & \text{if} \quad n>0 \\ 1-2n, & \text{if} \quad n\le 0 \end{cases}$. Then $f^{-1}(n)$ is defined as

(a) $f^{-1}(n)=\begin{cases} -2n, & \text{if} \quad n \text{ is even} \\ 2n-1, & \text{if} \quad n \text{ is odd} \end{cases}$

(b) $f^{-1}(n)=\begin{cases} n/2, & \text{if} \quad n \text{ is even} \\ \dfrac{1-n}{2}, & \text{if} \quad n \text{ is odd} \end{cases}$

(c) $f^{-1}(n)=(-1)^n\dfrac{n}{2}$

(d) $f^{-1}(n)=(-1)^n\dfrac{n+1}{2}$

14. If $y=\sin\left(\dfrac{\pi}{6}e^{xy}\right)$, then the value of $\dfrac{dy}{dx}$ at $x = 0$, is

(a) $\dfrac{\pi\sqrt{3}}{6}$ (b) $\dfrac{\pi\sqrt{3}}{12}$ (c) $\dfrac{\pi\sqrt{3}}{18}$ (d) $\dfrac{\pi\sqrt{3}}{24}$

15. If $y = \sin(2\sin^{-1}x)$, then it satisfies the differential equation

(a) $(1-x^2)y_2-xy_1+4y=0$

(b) $(1+x^2)y_2-xy_1+4y=0$

(c) $(1-x^2)y_2-xy_1+y=0$

(d) $(1+x^2)y_2-xy_1+y=0$

16. The mean and S.D. of 63 children in an arithmetic test are respectively 27.6 and 7.1. To them are added a new group of 26 students who had less training and whose mean is 19.2 and S.D. 6.2. The values of the combined group differ from the original as to (i) the mean and (ii) the S.D. is

(a) 2.5, 0.7 (b) 2.3, 0.8

(c) 1.5, 0.9 (d) none of these

17. The constant c of Cauchy's mean value theorem for $f(x) = e^x$, $g(x) = e^{-x}$ in $[a, b]$ is

(a) $\sqrt{ab}$ (b) $\dfrac{a+b}{2}$ (c) $\dfrac{2ab}{a+b}$ (d) $\dfrac{1}{a}+\dfrac{1}{b}$

18. If the coefficients of the r^{th} term and $(r + 1)^{\text{th}}$ term in the expansion of $(1 + x)^{20}$ are in the ratio 1 : 2, then r is equal to

(a) 6 (b) 7 (c) 8 (d) 9

19. If P is a point in space such that $OP = 12$ and $\overrightarrow{OP}$ is inclined at angles of 45° and 60° with OX and OY respectively, then the position vector of P is

(a) $6\hat{i}+6\hat{j}\pm 6\sqrt{2}\,\hat{k}$ (b) $6\hat{i}+6\sqrt{2}\,\hat{j}\pm 6\hat{k}$

(c) $6\sqrt{2}\,\hat{i}+6\hat{j}\pm 6\hat{k}$ (d) none of these

20. If α, β, γ are acute angles and $\cos\theta = \sin\beta/\sin\alpha$, $\cos\phi = \sin\gamma/\sin\alpha$ and $\cos(\theta - \phi) = \sin\beta\sin\gamma$, then the value of $\tan^2\alpha - \tan^2\beta - \tan^2\gamma$ is equal to

(a) -1 (b) 0 (c) 1 (d) 2

21. The equation of the circle passing through the foci of the ellipse $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1$ and having centre at (0, 3) is

(a) $x^2 + y^2 - 6y - 7 = 0$ (b) $x^2 + y^2 - 6y + 7 = 0$

(c) $x^2 + y^2 - 6y - 5 = 0$ (d) $x^2 + y^2 - 6y + 5 = 0$

22. The coordinates of the foot of perpendicular drawn from the point $A(2, 4, -1)$ on the line $\dfrac{x+5}{1}=\dfrac{y+3}{4}=\dfrac{z-6}{-9}$ is

(a) (1, –3, 4) (b) (–4, 1, –3)

(c) (4, 1, 3) (d) none of these

23. The value of $\lim\limits_{n\to\infty}\left[\dfrac{n!}{n^n}\right]^{1/n}=$

(a) e^{-1} (b) 1 (c) e (d) e^2

24. A particle's velocity v at time t is given by $v = 2e^{2t}\cos\dfrac{\pi t}{3}$. The least value of t at which the acceleration becomes zero is

(a) 0 (b) $\dfrac{3}{2}$

(c) $\dfrac{3}{\pi}\tan^{-1}\left(\dfrac{6}{\pi}\right)$ (d) $\dfrac{3}{\pi}\cot^{-1}\left(\dfrac{6}{\pi}\right)$

25. If the area bounded by the curves $y = 2^x$, $y = \ln x$, $x = \dfrac{1}{2}$, $x = 2$ is $a + b\ln 2 + \dfrac{c}{\ln 2}$, then $a + b + c =$

(a) $3+\sqrt{2}$ (b) $3-\sqrt{2}$

(c) $4-\sqrt{2}$ (d) $4+\sqrt{2}$

26. If the eccentricity of the hyperbola $x^2 - y^2\sec^2\theta = 4$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\theta + y^2 = 16$, then the value of θ equals

(a) $\pi/6$ (b) $3\pi/4$ (c) $\pi/3$ (d) $\pi/2$

27. If $\dfrac{5z_2}{11z_1}$ is purely imaginary, then the value of $\left|\dfrac{2z_1+3z_2}{2z_1-3z_2}\right|$ is

(a) $\dfrac{37}{33}$ (b) $\dfrac{11}{5}$ (c) 1 (d) $\dfrac{5}{11}$

28. $\lim\limits_{x \to 1} \frac{x^{2^{32}} - 2^{32}x + 4^{16} - 1}{(x-1)^2}$ is equal to

(a) $2^{63} - 2^{31}$ (b) $2^{64} - 2^{31}$
(c) $2^{62} - 2^{31}$ (d) $2^{65} - 2^{33}$

29. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, then

$\frac{[\vec{a}+2\vec{b} \;\; \vec{b}+2\vec{c} \;\; \vec{c}+2\vec{a}]}{[\vec{a}\,\vec{b}\,\vec{c}]} =$

(a) 2 (b) 6 (c) 9 (d) 3

30. If the function $f(x) = 4x^3 + ax^2 + bx - 1$ satisfies all the conditions of Rolle's theorem in $-\frac{1}{4} \le x \le 1$ and if $f'\left(\frac{1}{2}\right) = 0$, then find the values of a and b.

(a) $a = 2, b = -3$ (b) $a = 1, b = -4$
(c) $a = -1, b = 4$ (d) $a = -4, b = -1$

31. The area bounded by the parabola $y = 4x^2$, $y = \frac{x^2}{9}$ and the line $y = 2$ is

(a) $\frac{5\sqrt{2}}{3}$ sq. units (b) $\frac{10\sqrt{2}}{3}$ sq. units
(c) $\frac{15\sqrt{2}}{3}$ sq. units (d) $\frac{20\sqrt{2}}{3}$ sq. units

32. A plane passes through (1, –2, 1) and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$. The distance of the plane from the point (1, 2, 2) is

(a) 0 unit (b) 1 unit
(c) $\sqrt{2}$ units (d) $2\sqrt{2}$ units

33. A book writer writes a good book with probability $\frac{1}{2}$. If it is a good book, the probability that it will be published is $\frac{2}{3}$, otherwise it is $\frac{1}{4}$. If he writes 2 books, the probability that at least one book will be published is

(a) $\frac{407}{576}$ (b) $\frac{411}{576}$
(c) $\frac{405}{576}$ (d) $\frac{307}{576}$

34. $\vec{a}, \vec{b}, \vec{c}$ are three vectors of equal magnitude. The angle between each pair of vectors is π/3 such that $|\vec{a}+\vec{b}+\vec{c}| = \sqrt{6}$ then $|\vec{a}|$ is equal to

(a) 2 (b) –1 (c) 1 (d) $\frac{\sqrt{6}}{3}$

35. Let $f(x) = \frac{\sin^{101} x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}}$, where $[x]$ denotes the integral part of x, is

(a) an odd function
(b) an even function
(c) neither odd nor even function
(d) none of these

36. Let the population of rabbits surviving at a time be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t) =$

(a) $400 - 300e^{t/2}$ (b) $300 - 200e^{-t/2}$
(c) $600 - 500e^{t/2}$ (d) $400 - 300e^{-t/2}$

37. Form the differential equation of $\sin^{-1}x + \sin^{-1}y = c$.

(a) $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

(b) $(1-x^2)\frac{d^2y}{dx^2} + y^2 = 0$

(c) $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

(d) none of these

38. The intersection of all the intervals having the form $\left[1+\frac{1}{n}, 6-\frac{2}{n}\right]$, where n is a positive integer is

(a) [1, 6] (b) (1, 6) (c) [2, 4] (d) [3/2, 5]

39. Solution set of inequality $\log_{10}(x^2 - 2x - 2) \le 0$ is

(a) $[-1, 1-\sqrt{3}]$ (b) $[1+\sqrt{3}, \sqrt{3}]$
(c) $[-1, 1-\sqrt{3}) \cup (1+\sqrt{3}, 3]$
(d) none of these

40. If two arithmetic means A_1, A_2, two geometric means G_1, G_2 and two harmonic means H_1, H_2 are inserted between any two numbers, then $\frac{A_1 + A_2}{H_1 + H_2}$ is

(a) $\frac{G_1G_2}{H_1H_2}$ (b) $\sqrt{G_1G_2}$
(c) $\frac{H_1H_2}{G_1G_2}$ (d) none of these

41. The line $(p + 2q)\,x + (p - 3q)\,y = p - q$ for different values of p and q passes through the fixed point

(a) (3/2, 5/2) (b) (2/5, 2/5)
(c) (3/5, 3/5) (d) (2/5, 3/5)

42. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be the reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then focus of the hyperbola is at

(a) (2, 0) (b) (0, 2) (c) (3, 0) (d) (0, 3)

43. If $h(x) = f(x) - (f(x))^2 + (f(x))^3$ and $f'(x) > 0\ \forall\ x$, then

(a) $h(x)$ is an increasing function of x in some specified interval
(b) $h(x)$ is an increasing function $\forall\ x \in R$
(c) $h(x)$ is a decreasing function $\forall\ x \in R$
(d) $h(x)$ is a decreasing function of x in some specified interval

44. Let $f: N \to N$ defined by

$$f(n)=\begin{cases}\frac{n+1}{2}, & \text{if } n \text{ is odd}\\ \frac{n}{2}, & \text{if } n \text{ is even}\end{cases}, \text{ then } f \text{ is}$$

(a) onto but not one-one
(b) one-one and onto
(c) neither one-one nor onto
(d) one-one but not onto

45. Find c so that the distance between the points (7, 1, –3) and (4, 5, c) be 13 units.

(a) 9 (b) –15
(c) Both (a) and (b) (d) None of these

46. Let $f: R-\left\{\frac{3}{2}\right\} \to R,\ f(x)=\frac{3x+5}{2x-3}$.

Let $f_2(x) = f(f(x)), f_3(x) = f(f_2(x)),\ldots,$ $f_n(x) = f(f_{n-1}(x))$. Then, $f_{2008}(x) + f_{2009}(x) =$

(a) $\frac{2x^2+5}{2x-3}$ (b) $\frac{x^2+5}{2x-3}$
(c) $\frac{2x^2-5}{2x-3}$ (d) $\frac{x^2-5}{2x-3}$

47. $\lim\limits_{x\to 0}\frac{f(x)}{x}=2$, where $f(x)=\min\left\{\sin\sqrt{[m]}x, |x|\right\}$ and $[\cdot]$ greatest integer, then

(a) $m \in \{4\}$ (b) $m \in [4, 5]$
(c) $m \in [4, 5)$ (d) $m \in \{5\}$

48. The absolute value of $\int_{10}^{19}\frac{\cos x}{1+x^8}dx$, is

(a) less than 10^{-7} (b) more than 10^{-7}
(c) less than 10^{-8} (d) none of these

49. If the product of the matrices

$$\begin{bmatrix}1&1\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}1&3\\0&1\end{bmatrix}\cdots\begin{bmatrix}1&n\\0&1\end{bmatrix}=\begin{bmatrix}1&378\\0&1\end{bmatrix}, \text{ then } n$$

is equal to

(a) 27 (b) 26 (c) 376 (d) 378

50. The derivative of $(x + \sec x)(x - \tan x)$ is $A(x + \sec x) + B(x - \tan x)$, where A and B respectively are

(a) $\sec^2 x - 1, 1 + \sec x \tan x$
(b) $1 - \sec^2 x, 1 - \sec x \tan x$
(c) $1 - \sec^2 x, 1 + \sec x \tan x$
(d) $\sec^2 x - 1, 1 - \sec x\ \tan x$

CATEGORY-II (Q. 51 to Q. 65)

Only one answer is correct. Correct answer will fetch full marks 2. Incorrect answer or any combination of more than one answer will fetch –½ marks. No answer will fetch 0 marks.

51. The set of values of x which satisfy the inequations

$5x + 2 < 3x + 8$ and $\frac{x+2}{x-1}<4, x \neq 1$ is

(a) $(-\infty, 1)$ (b) $(2, 3)$
(c) $(-\infty, 3)$ (d) $(-\infty, 1) \cup (2, 3)$

52. $3^{\frac{1}{4}}\times 9^{\frac{1}{8}}\times 27^{\frac{1}{16}}\times \ldots\ \infty$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

53. $\int\frac{dx}{x^{22}(x^7-6)}=A\{\ln(p)^6+9p^2-2p^3-18p\}+c,$

then

(a) $A=\frac{1}{9072}, p=\left(\frac{x^7-6}{x^7}\right)$

(b) $A=\frac{1}{54432}, p=\left(\frac{x^7-6}{x^7}\right)$

(c) $A=\frac{1}{54432}, p=\left(\frac{x^7}{x^7-6}\right)$

(d) $A=\frac{1}{9072}, p=\left(\frac{x^7-6}{x^7}\right)^{-1}$

54. The smallest positive value of p for which the equation $\cos(p \sin x) = \sin(p \cos x)$ has a solution in $0 \le x \le 2\pi$, is

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{2\sqrt{2}}$ (d) $\frac{\pi}{4}$

55. A circle of radius $\sqrt{8}$ is passing through origin and the point (4, 0). If the centre lies on the line $y = x$, then the equation of the circle is

(a) $(x-2)^2+(y-2)^2=8$
(b) $(x+2)^2+(y+2)^2=8$
(c) $(x-3)^2+(y-3)^2=8$
(d) $(x+3)^2+(y+3)^2=8$

56. Let $f(x)=\min\{x+1,\sqrt{(1-x)}\}\ \forall\ x\in[-1,1]$, then area bounded by $f(x)$ and x-axis is

(a) $\frac{1}{6}$ sq. unit (b) $\frac{5}{6}$ sq. unit
(c) $\frac{7}{6}$ sq. units (d) $\frac{11}{6}$ sq. units

57. If the tangent from a point P to the circle $x^2+y^2=1$ is perpendicular to the tangent from P to the circle $x^2+y^2=3$, then the locus of P is

(a) a circle of radius 2 (b) a circle of radius 4
(c) a circle of radius 3 (d) none of these

58. If the line $ax+by+c=0$ is a tangent to the curve $xy=4$, then

(a) $a<0, b>0$ (b) $a\le 0, b>0$
(c) $a<0, b<0$ (d) $a\le 0, b<0$

59. $\int \frac{x^2-1}{x^3\cdot\sqrt{2x^4-2x^2+1}}\,dx=$

(a) $\frac{1}{x^2}\cdot\sqrt{2x^4-2x^2+1}+C$
(b) $\frac{1}{x^3}\cdot\sqrt{2x^4-2x^2+1}+C$
(c) $\frac{1}{x}.\sqrt{2x^4-2x^2+1}+C$
(d) $\frac{1}{2x^2}.\sqrt{2x^4-2x^2+1}+C$

60. Let C be the curve $y=x^3$ (where x takes all real values). The tangent at A meets the curve again at B. If the gradient at B is K times the gradient at A, then K is equal to

(a) 4 (b) 2 (c) –2 (d) 1/4

61. P has 2 children. He has a son, Jatin. What is the probability that Jatin's sibling is a brother?

(a) 1/3 (b) 1/4 (c) 2/3 (d) 1/2

62. If $\frac{1+3p}{4},\frac{1-p}{3},\frac{1-3p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is

(a) $\left[-\frac{1}{3},\frac{1}{3}\right]$ (b) $\left[-\frac{1}{3},1\right]$
(c) $\left[\frac{1}{13},1\right]$ (d) $\left[\frac{1}{13},\frac{1}{3}\right]$

63. What is the equation of the plane which passes through the z-axis and is perpendicular to the line $\frac{x-a}{\cos\theta}=\frac{y+2}{\sin\theta}=\frac{z-3}{0}$?

(a) $x+y\tan\theta=0$ (b) $y+x\tan\theta=0$
(c) $x\cos\theta-y\sin\theta=0$ (d) $x\sin\theta-y\cos\theta=0$

64. If three vectors $\vec{a},\vec{b},\vec{c}$ are such that $\vec{a}\ne\vec{0}$ and $\vec{a}\times\vec{b}=2(\vec{a}\times\vec{c}), |\vec{a}|=|\vec{c}|=1, |\vec{b}|=4$ and the angle between $\vec{b}$ and $\vec{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$, then $\vec{b}-2\vec{c}=\lambda\vec{a}$, where λ is equal to

(a) ±4 (b) –2 (c) ±3 (d) 2

65. If $f'(2)=1$, then $\lim\limits_{h\to 0}\frac{f(2+h^2)-f(2-h^2)}{2h^2}=$

(a) 0 (b) 1 (c) 2 (d) 1/2

CATEGORY-III (Q. 66 to Q. 75)

One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also no answers will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers.

66. A function $f(x)$ is defined in the interval [1, 4] as follows:

$$f(x)=\begin{cases}\log_e[x], & 1\le x<3\\ |\log_e x|, & 3\le x<4\end{cases}$$

The graph of the function $f(x)$

(a) is broken at two points
(b) is broken at exactly at one point
(c) does not have a definite tangent at two points
(d) does not have a definite tangent at more than two points.

67. If the 6th term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is $3x^{-4}$, then

(a) $a=\frac{1}{2}$ (b) $n=8$ (c) $a=\frac{2}{3}$ (d) $n=6$

68. If $x^2+2hxy+y^2=0$ $(h\ne 1)$ represents the equation of the pair of straight lines through the origin (having slopes m_1 and m_2) which make an angle α with the straight line $y+x=0$, then

(a) $\sec 2\alpha=h$ (b) $\cos\alpha=\sqrt{\frac{1+h}{2h}}$
(c) $m_1+m_2=-2\sec 2\alpha$ (d) $\cot\alpha=\sqrt{\frac{h+1}{h-1}}$

69. The resolved part of the vector $\vec{a}$ along the vector $\vec{b}$ is $\vec{\lambda}$ and that perpendicular to $\vec{b}$ is $\vec{\mu}$. Then

(a) $\vec{\lambda}=\frac{(\vec{a}\cdot\vec{b})\vec{a}}{\vec{a}^2}$ (b) $\vec{\lambda}=\frac{(\vec{a}\cdot\vec{b})\vec{b}}{\vec{b}^2}$

(c) $\vec{\mu}=\frac{(\vec{b}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{b})\vec{b}}{\vec{b}^2}$ (d) $\vec{\mu}=\frac{\vec{b}\times(\vec{a}\times\vec{b})}{\vec{b}^2}$

70. If $f(x)=\begin{vmatrix} 3 & 3x & 3x^2+2a^2 \\ 3x & 3x^2+2a^2 & 3x^3+6a^2x \\ 3x^2+2a^2 & 3x^3+6a^2x & 3x^4+12a^2x^2+2a^4 \end{vmatrix}$, then

(a) $f'(x) = 0$ (b) $f(x)$ is independent of x

(c) $\int_0^1 f(x)dx=4a^6$ (d) none of these

71. Let $f_n(\theta)=\tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)....$ $(1 + \sec 2^n\theta)$, then

(a) $f_5\left(\frac{\pi}{128}\right)=1$ (b) $f_3\left(\frac{\pi}{32}\right)=1$

(c) $f_4\left(\frac{\pi}{64}\right)=1$ (d) $f_2\left(\frac{\pi}{16}\right)=1$

72. $f(x)$ is a polynomial of third degree which has a local maxima at $x = -1$. If $f(1) = -1$, $f(2) = 18$ and $f'(x)$ has a local minimum at $x = 0$, then
(a) $f(0) = 5$
(b) $f(x)$ has local minimum at $x = 1$
(c) $f(x)$ is increasing in $[1, 2\sqrt{5}]$
(d) The distance between $(-1, 2)$ and $(a, f(a))$ is $2\sqrt{5}$, where a is a point of local minimum.

73. If $f(x)=\begin{cases} x\tan^{-1}(1/x), & x\neq 0 \\ 0, & x=0 \end{cases}$, then
(a) $f(x)$ is discontinuous at $x = 0$
(b) $f(x)$ is continuous at $x = 0$
(c) $f(x)$ is differentiable at $x = 0$
(d) $f(x)$ is non-differentiable at $x = 0$

74. Let $\frac{dy}{dx}+y=f(x)$, where y is a continuous function of x with $y(0) = 1$ and $f(x)=\begin{cases} e^{-x} & , \text{if } 0\le x\le 2 \\ e^{-2} & , \text{if } x>2 \end{cases}$.
Which of the following hold(s) good?
(a) $y(1) = 2e^{-1}$ (b) $y'(1) = -e^{-1}$
(c) $y(3) = -2e^{-3}$ (d) $y'(3) = -2e^{-3}$

75. Let $f(x) = ax^3 + bx^2 + cx$ has relative extrema at $x = 1$ and at $x = 5$. If $\int_{-1}^{1} f(x)\,dx=6$, then

(a) $a = -1$ (b) $b = 9$
(c) $c = 15$ (d) $a = 1$

SOLUTIONS

1. (a) : Given that $\log_e 5$, $\log_e(5^x - 1)$, $\log_e\left(5^x-\frac{11}{5}\right)$ are in A.P.

$\therefore\ 2\log_e(5^x-1)=\log_e 5+\log_e\left(5^x-\frac{11}{5}\right)$

$\Rightarrow\ (5^x-1)^2=5\left(5^x-\frac{11}{5}\right)$

$\Rightarrow\ (5^x)^2 - 2\cdot 5^x + 1 = 5\cdot 5^x - 11$

$\Rightarrow\ (5^x)^2 - 7\cdot 5^x + 12 = 0 \Rightarrow (5^x - 3)(5^x - 4) = 0$

$\Rightarrow\ 5^x - 3 = 0 \ \Rightarrow\ 5^x = 3 \ \therefore\ x = \log_5 3$

and $5^x - 4 = 0 \ \Rightarrow\ 5^x = 4 \ \therefore\ x = \log_5 4$

2. (a) : $|x_1z_1 - y_1z_2|^2 + |y_1z_1 + x_1z_2|^2$

$= |x_1z_1|^2 + |y_1z_2|^2 - 2Re(x_1y_1z_1z_2) + |y_1z_1|^2 + |x_1z_2|^2 + 2Re(x_1y_1z_1z_2)$

$=x_1^2|z_1|^2+y_1^2|z_2|^2+y_1^2|z_1|^2+x_1^2|z_2|^2$

$=2(x_1^2+y_1^2)4^2=32(x_1^2+y_1^2)$

3. (a) : Let $z = a + b\omega + c\omega^2$

Then, $|z|^2=z\bar{z}=(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$

$= a^2 + b^2 + c^2 - bc - ca - ab$

$=\frac{1}{2}[(b-c)^2+(c-a)^2+(a-b)^2]$

Thus, $x=|z|+|\bar{z}|=2|z|$

$=\sqrt{2[(b-c)^2+(c-a)^2+(a-b)^2]}$

As a, b, c are distinct integers, so minimum value of $(b - c)^2 + (c - a)^2 + (a - b)^2$ is $1^2 + 1^2 + 2^2 = 6$.

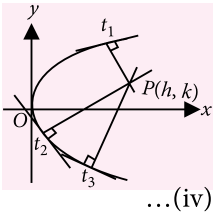
Therefore, minimum value of x is $2\sqrt{3}$.

4. (b) : The equation of normal at $(at^2, 2at)$ is
$y + tx = 2at + at^3$ ...(i)

As (i) passes through $P(h, k)$, so
$at^3 + t(2a - h) - k = 0$ ...(ii)

Here, $a = 1$

$\therefore\ t_1 + t_2 + t_3 = 0$ ...(iii)



Also, $\frac{2}{t_1+t_2}=2$

$\Rightarrow\ t_1 + t_2 = 1$ ...(iv)

From (iii) and (iv), we get
$t_3 = -1$

Putting $t_3 = -1$ in (ii), we get

$-1 - 1(2 - h) - k = 0 \Rightarrow -1 -2 + h - k = 0$

$\therefore$ Locus of $P(h, k)$, is $x - y = 3$

5. (a) : The given equation is

$x^2 - 2ax + a^2 - 1 = 0$

$\Rightarrow x^2 - (a + 1)x - (a - 1)x + (a + 1)(a - 1) = 0$

$\Rightarrow x\{x - (a + 1)\} - (a - 1)\{x - (a + 1)\} = 0$

$\Rightarrow (x - a + 1)(x - a - 1) = 0$

$\Rightarrow x = a - 1$ or $x = a + 1$

Since, both the roots lies in $(-2, 1)$

$\therefore -2 < a -1 < 1$ and $-2 < a + 1 < 1$

$\Rightarrow -1 < a < 2$ and $-3 < a < 0$

$\therefore -1 < a < 0 \Rightarrow [a] = -1$

6. (c) : Let the roots are α and β

$\therefore \alpha + \beta = -2a$ and $\alpha\beta = b$

Also given that $|\alpha - \beta| \le 2m \Rightarrow (\alpha - \beta)^2 \le 4m^2$

Now, $(\alpha + \beta)^2 - 4\alpha\beta = (\alpha - \beta)^2$

$\Rightarrow 4a^2 - 4b \le 4m^2 \Rightarrow a^2 - b \le m^2$

$\Rightarrow a^2 - m^2 \le b$ ...(i)

Also, the discriminant $4a^2 - 4b \ge 0$ (as roots are real)

$\Rightarrow a^2 \ge b$ ...(ii)

From (i) and (ii), we have

$a^2 - m^2 \le b \le a^2$

i.e., $b \in [a^2 - m^2, a^2]$

7. (a) : The given function

$$f(x) = \begin{cases} 3[x] - 5\dfrac{|x|}{x}, & x \ne 0 \\ 2, & x = 0 \end{cases} = \begin{cases} 3[x] - 5, & \text{when } x > 0 \\ 3[x] + 5, & \text{when } x < 0 \\ 2\ , & \text{when } x = 0 \end{cases}$$

Now, $\int_{-3/2}^{2} f(x)dx$

$$= \int_{-3/2}^{-1} f(x)dx + \int_{-1}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx$$

$$= \int_{-3/2}^{-1} -1dx + \int_{-1}^{0} 2dx + \int_{0}^{1} (-5)dx + \int_{1}^{2} (-2)dx$$

$$= -1\left(-1 + \frac{3}{2}\right) + 2(0 + 1) + (-5)(1 - 0) + (-2)(2 - 1)$$

$$= -1\left(\frac{1}{2}\right) + 2 - 5 - 2 = -\frac{1}{2} + 2 - 5 - 2 = -\frac{11}{2}$$

8. (b) : $f(x) = e^{\cos^{-1}\sin\left(x + \frac{\pi}{3}\right)}$

$$\therefore \quad f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\sin\left(\frac{8\pi}{9} + \frac{\pi}{3}\right)}$$

$$= e^{\cos^{-1}\sin\left(\frac{11\pi}{9}\right)} = e^{\cos^{-1}\sin\left(\frac{22\pi}{18}\right)}$$

$$= e^{\cos^{-1}\sin\left(\frac{9\pi}{18} + \frac{13\pi}{18}\right)} = e^{\cos^{-1}\sin\left(\frac{\pi}{2} + \frac{13\pi}{18}\right)}$$

$$= e^{\cos^{-1}\cos\left(\frac{13\pi}{18}\right)} = e^{\frac{13\pi}{18}}$$

9. (b) : Let $P(n) : a^{2n - 1} - 1$

Now $P(1)$ means, $a^{2 \times 1 - 1} - 1 = a - 1$ which is divisible by $a - 1$.

Let us assume that $P(m)$ is true, *i.e.*, $a^{2m - 1} - 1$ is divisible by $a - 1$, $m \in N$

$\Rightarrow a^{2m - 1} - 1 = (a - 1)Q$ ...(i)

where Q is some polynomial in a.

We shall prove that $P(m + 1)$ is true, *i.e.*, $a^{2(m + 1) - 1} - 1$ is divisible by $a - 1$.

Now, $a^{2(m + 1) - 1} - 1 = a^{(2m - 1) + 2} - 1 = a^{2m - 1} a^2 - 1$

$= \{1 + (a - 1)Q\}a^2 - 1$ (using (i))

$= a^2 + a^2(a - 1)Q - 1 = (a^2 - 1) + a^2(a - 1)Q$

$= (a - 1)\{(a + 1) + a^2Q\}$

$\Rightarrow a^{2(m + 1) - 1} - 1$ is divisible by $a - 1$

$\Rightarrow P(m + 1)$ is true.

Hence, by mathematical induction, $P(n)$ is true for all $n \in N$.

10. (c) : We have,

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } G(y) = \begin{bmatrix} \cos y & 0 & \sin y \\ 0 & 1 & 0 \\ -\sin y & 0 & \cos y \end{bmatrix}$$

$$\text{Now, } F(x)F(-x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad F(x)\,F(-x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\Rightarrow F(-x) = [F(x)]^{-1}$ ...(i)

Similarly, $G(-y) = [G(y)]^{-1}$ ...(ii)

Now, $[F(x)\ G(y)]^{-1} = [G(y)]^{-1}[F(x)]^{-1} = G(-y)\ F(-x)$

[Using (i) and (ii)]

11. (c) : Let E_1 be the event that the flight departs on time, E_2 be the event that the flight arrives on time.

$\therefore P(E_1) = 0.9$, $P(E_2) = 0.8$ and $P(E_1 \cap E_2) = 0.7$

$$\text{Now } P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{0.7}{0.9} = \frac{7}{9}$$

12. (a) : Let the person select x biscuits from first variety, y from the second, z from the third and w from the fourth variety. Then the number of ways = number of solutions of the equation $x + y + z + w = 10$, where $x = 1, 2, \ldots\ldots\ldots\ldots, 7$

$y = 1, 2, \ldots\ldots\ldots\ldots, 7$

$z = 1, 2, \ldots\ldots\ldots\ldots, 7$

$w = 1, 2, \ldots\ldots\ldots\ldots, 7$

So, number of ways

$= \text{coefficient of } x^{10} \text{ in } (x + x^2 + \ldots.. + x^7)^4$

$= \text{coefficient of } x^6 \text{ in } (1 + x + \ldots.. + x^6)^4$

$= \text{coefficient of } x^6 \text{ in } (1 - x^7)^4 (1 - x)^{-4}$

$= \text{coefficient of } x^6 \text{ in } (1 - x)^{-4}$

$= {}^{4+6-1}C_6 = {}^9C_6 = 84.$

13. (b) : Let $m = f(n)$

$$\Rightarrow \quad m = \begin{cases} 2n, & \text{if } n > 0 \\ 1-2n, & \text{if } n \le 0 \end{cases}$$

$$\Rightarrow \quad n = \begin{cases} m/2, & \text{if } n > 0,\ m = 2n \\ \dfrac{1-m}{2}, & \text{if } n \le 0,\ m = 1-2n \end{cases}$$

$$\left(\because m = 2n \Leftrightarrow n = \frac{m}{2} \text{ and } m = 1-2n \Leftrightarrow n = \frac{1-m}{2}\right)$$

$$\Rightarrow \quad n = \begin{cases} m/2, & \text{if } m \text{ is even} \\ \dfrac{1-m}{2}, & \text{if } m \text{ is odd} \end{cases}$$

$$\Rightarrow \quad f^{-1}(m) = \begin{cases} m/2, & \text{if } m \text{ is even} \\ (1-m)/2, & \text{if } m \text{ is odd} \end{cases}$$

14. (d) : Here, $y = \sin\left(\dfrac{\pi}{6}e^{xy}\right)$

$$\therefore \quad \frac{dy}{dx} = \cos\left(\frac{\pi}{6}e^{xy}\right) \times \frac{\pi}{6}e^{xy}\left(x\frac{dy}{dx} + y\right)$$

Now putting $x = 0$ in the above equation, we get

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{\pi}{6} \times \frac{\sqrt{3}}{2}\left(0 + y_{(x=0)}\right)$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{x=0} = \frac{\pi}{6} \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\pi\sqrt{3}}{24}$$

$$\left(\because \quad y_{(x=0)} = \sin\frac{\pi}{6} = \frac{1}{2}\right)$$

15. (a) : $y = \sin(2\sin^{-1}x) \Rightarrow \sqrt{1-x^2}\dfrac{dy}{dx} = 2\cos(2\sin^{-1}x)$

$$\Rightarrow \quad (1-x^2)\left(\frac{dy}{dx}\right)^2 = 4\{1 - \sin^2(2\sin^{-1}x)\} = 4 - 4y^2$$

$$\Rightarrow \quad 2(1-x^2)\left(\frac{dy}{dx}\right)\cdot\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = -8y\frac{dy}{dx}$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

16. (a) : Mean and S.D. of the combined group are

$$\text{Mean } (\bar{x}) = \frac{63 \times 27.6 + 26 \times 19.2}{63 + 26} = 25.1$$

Thus mean is decreased by 27.6 – 25.1 = 2.5

$$\text{Variance, } \sigma^2 = \frac{\begin{array}{r} 63 \times (7.1)^2 + 26 \times (6.2)^2 + 63(25.1 - 27.6)^2 \\ +26(25.1 - 19.2)^2 \end{array}}{89}$$

$\Rightarrow \quad \sigma = 7.8$ (approx)

Thus, S.D. is increased by 7.8 – 7.1 = 0.7

17. (b) : Clearly $f(x)$ and $g(x)$ are continuous on $[a, b]$ and derivable on (a, b).

$\therefore$ By Cauchy's mean value theorem $\exists\ c \in (a, b)$ such that

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)} \Rightarrow \frac{e^b - e^a}{e^{-b} - e^{-a}} = \frac{e^c}{-e^{-c}}$$

$$\Rightarrow \quad \frac{e^b - e^a}{(1/e^b - 1/e^a)} = -e^{2c} \Rightarrow e^{2c} = \frac{(e^a - e^b)}{(e^a - e^b)}e^a \cdot e^b = e^{a+b}$$

$$\Rightarrow \quad 2c = a + b \Rightarrow c = \frac{a+b}{2}$$

18. (b) : $t_r = t_{(r-1)+1} = {}^{20}C_{r-1}\, x^{r-1}$ and $t_{r+1} = {}^{20}C_r\, x^r$

According to the problem,

$$\frac{{}^{20}C_{r-1}}{{}^{20}C_r} = \frac{1}{2} \quad \Rightarrow \quad \frac{r}{20 - r + 1} = \frac{1}{2}$$

$\Rightarrow \quad 2r = 20 - r + 1 \ \Rightarrow \ 3r = 21$

$\therefore \quad r = 7$

19. (c) : Let l, m, n be the direction cosines of $\overrightarrow{OP}$.

Then $l = \cos 45° = \dfrac{1}{\sqrt{2}}$, $m = \cos 60° = \dfrac{1}{2}$

Now, $l^2 + m^2 + n^2 = 1 \Rightarrow n = \pm\dfrac{1}{2}$

Let $\overrightarrow{OP} = \vec{r}$

$$\therefore \quad \vec{r} = |\vec{r}|(l\hat{i} + m\hat{j} + n\hat{k}) = 12\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} \pm \frac{1}{2}\hat{k}\right)$$

$$= 6\sqrt{2}\,\hat{i} + 6\hat{j} \pm 6\hat{k}$$

20. (b) : From the third relation, we get

$\cos\theta\cos\phi + \sin\theta\sin\phi = \sin\beta\sin\gamma$

$\Rightarrow \quad \sin^2\theta\sin^2\phi = (\cos\theta\cos\phi - \sin\beta\sin\gamma)^2$

$$\Rightarrow \left(1 - \frac{\sin^2\beta}{\sin^2\alpha}\right)\left(1 - \frac{\sin^2\gamma}{\sin^2\alpha}\right) = \left(\frac{\sin\beta\sin\gamma}{\sin^2\alpha} - \sin\beta\sin\gamma\right)^2$$

$\Rightarrow \quad (\sin^2\alpha - \sin^2\beta)(\sin^2\alpha - \sin^2\gamma) = \sin^2\beta\sin^2\gamma\,(1 - \sin^2\alpha)^2$

$\Rightarrow \quad \sin^4\alpha\,(1 - \sin^2\beta\sin^2\gamma) - \sin^2\alpha\,(\sin^2\beta + \sin^2\gamma - 2\sin^2\beta\sin^2\gamma) = 0$

$$\therefore\ \sin^2\alpha=\frac{\sin^2\beta+\sin^2\gamma-2\sin^2\beta\sin^2\gamma}{1-\sin^2\beta\sin^2\gamma}$$

$$\text{and}\ \cos^2\alpha=\frac{1-\sin^2\beta-\sin^2\gamma+\sin^2\beta\sin^2\gamma}{1-\sin^2\beta\sin^2\gamma}$$

$$\Rightarrow\ \tan^2\alpha=\frac{\sin^2\beta-\sin^2\beta\sin^2\gamma+\sin^2\gamma-\sin^2\beta\sin^2\gamma}{\cos^2\beta-\sin^2\gamma(1-\sin^2\beta)}$$

$$=\frac{\sin^2\beta\cos^2\gamma+\cos^2\beta\sin^2\gamma}{\cos^2\beta\cos^2\gamma}=\tan^2\beta+\tan^2\gamma$$

$\Rightarrow\ \tan^2\alpha - \tan^2\beta - \tan^2\gamma = 0$

21. (a) : For the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, we have $a = 4$ and $b = 3$

$$\therefore\ e^2=1-\frac{b^2}{a^2}=1-\frac{9}{16}=\frac{7}{16}\quad\Rightarrow\ e=\frac{\sqrt{7}}{4}$$

The coordinates of the foci are $S(\sqrt{7},0)$ and $S'(-\sqrt{7},0)$.

The coordinates of the centre C are (0, 3).

Let r be the radius of the circle. Then,

$$r = CS = \sqrt{(0-\sqrt{7})^2+(3-0)^2}=4$$

Hence, the equation of the circle is

$(x - 0)^2 + (y - 3)^2 = 4^2$

$\Rightarrow\ x^2 + y^2 - 6y - 7 = 0$

22. (b) : The general point on the straight line

$$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda\ \text{(say)}\qquad\ldots\text{(i)}$$

is $(\lambda - 5, 4\lambda - 3, -9\lambda + 6)$

Let the foot of perpendicular $L \equiv (\lambda - 5, 4\lambda - 3, -9\lambda + 6)$

∴ The direction ratios of AL are $\lambda - 7, 4\lambda - 7, -9\lambda + 7$

Since AL is perpendicular to the line (i), therefore

$$1(\lambda - 7) + 4(4\lambda - 7) - 9(-9\lambda + 7) = 0\ \Rightarrow\ \lambda = 1$$

Thus, coordinates of foot of perpendicular are (–4, 1, –3)

23. (a) : Let $A=\left(\frac{n!}{n^n}\right)^{\frac{1}{n}}=\left(\frac{1\cdot 2\cdot 3\cdot\ldots\cdot n}{n\cdot n\cdot n\cdot n\ldots\cdot n}\right)^{\frac{1}{n}}$

$$=\left\{\left(\frac{1}{n}\right)\cdot\left(\frac{2}{n}\right)\cdot\left(\frac{3}{n}\right)\ldots\cdot\left(\frac{n}{n}\right)\right\}^{\frac{1}{n}}$$

$$\therefore\ \log A=\frac{1}{n}\left\{\log\left(\frac{1}{n}\right)+\log\left(\frac{2}{n}\right)+\log\left(\frac{3}{n}\right)+\ldots+\log\left(\frac{n}{n}\right)\right\}$$

$$\text{Now,}\ \lim_{n\to\infty}\log A=\lim_{n\to\infty}\frac{1}{n}\left\{\log\left(\frac{1}{n}\right)+\log\left(\frac{2}{n}\right)+\log\left(\frac{3}{n}\right)+\ldots+\log\left(\frac{n}{n}\right)\right\}$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(\frac{r}{n}\right)$$

$$\Rightarrow\ \log\left(\lim_{n\to\infty}A\right)=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(\frac{r}{n}\right)=\int_0^1\log x\,dx$$

$$=\left[x\log x-x\right]_0^1=-1$$

$$\therefore\ \lim_{n\to\infty}A=e^{-1}\Rightarrow\lim_{n\to\infty}\left(\frac{n!}{n^n}\right)^{\frac{1}{n}}=\frac{1}{e}$$

24. (c) : We have, $v=2e^{2t}\cos\frac{\pi t}{3}$

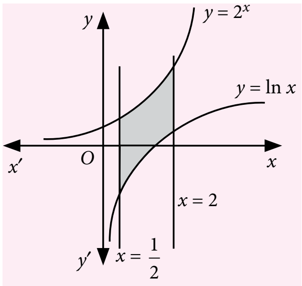
$$\Rightarrow\ \frac{dv}{dt}=4e^{2t}\cos\frac{\pi t}{3}-\frac{2\pi}{3}e^{2t}\sin\frac{\pi t}{3}$$

Now, acceleration = 0 *i.e.*, $\frac{dv}{dt}=0$

$$\Rightarrow\ 4e^{2t}\cos\frac{\pi t}{3}-\frac{2\pi}{3}e^{2t}\sin\frac{\pi t}{3}=0$$

$$\Rightarrow\ \tan\frac{\pi t}{3}=\frac{6}{\pi}\quad\Rightarrow\ t=\frac{3}{\pi}\tan^{-1}\left(\frac{6}{\pi}\right)$$

25. (b) : Required area



$$=\int_{1/2}^{2}(2^x-\ln x)dx$$

$$=\left|\frac{2^x}{\ln 2}\right|_{1/2}^{2}+\left|-x\ln x+x\right|_{1/2}^{2}$$

$$=\frac{4-\sqrt{2}}{\ln 2}-\frac{5}{2}\ln 2+\frac{3}{2}$$

$$=a+b\ln 2+\frac{c}{\ln 2}\quad\text{(Given)}$$

Comparing, we get

$$a=\frac{3}{2},\ b=-\frac{5}{2}\ \text{and}\ c=4-\sqrt{2}$$

$$\therefore\ a+b+c=\frac{3}{2}-\frac{5}{2}+4-\sqrt{2}=3-\sqrt{2}.$$

26. (b) : The equation of the hyperbola is

$$x^2 - y^2\sec^2\theta = 4 \quad \Rightarrow \quad \frac{x^2}{4} - \frac{y^2}{4\cos^2\theta} = 1$$

The equation of the ellipse is

$$x^2\sec^2\theta + y^2 = 16 \quad \Rightarrow \quad \frac{x^2}{16\cos^2\theta} + \frac{y^2}{16} = 1$$

Now, by the problem,

$$1 + \frac{4\cos^2\theta}{4} = 3\left(1 - \frac{16\cos^2\theta}{16}\right)$$

$$\Rightarrow \frac{4+4\cos^2\theta}{4} = 3\left(\frac{16-16\cos^2\theta}{16}\right)$$

$$\Rightarrow 1 + \cos^2\theta = 3(1 - \cos^2\theta)$$

$$\Rightarrow 4\cos^2\theta = 2 \quad \Rightarrow \quad \cos\theta = \pm\frac{1}{\sqrt{2}}$$

$$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

27. (c) : Given that $\frac{5z_2}{11z_1}$ is purely imaginary.

$$\therefore \frac{5z_2}{11z_1} = i\lambda \quad \Rightarrow \quad \frac{z_2}{z_1} = \frac{11}{5}i\lambda$$

Now, $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right| = \left|\frac{1+\frac{3}{2}\cdot\frac{z_2}{z_1}}{1-\frac{3}{2}\cdot\frac{z_2}{z_1}}\right| = \left|\frac{1+\frac{33}{10}i\lambda}{1-\frac{33}{10}i\lambda}\right|$

$= 1 \qquad [\because |a + ib| = |a - ib|]$

28. (a)

29. (c) : We have, $[\vec{a}+2\vec{b} \;\; \vec{b}+2\vec{c} \;\; \vec{c}+2\vec{a}]$

$= (\vec{a}+2\vec{b})\cdot\{(\vec{b}+2\vec{c})\times(\vec{c}+2\vec{a})\}$

$= (\vec{a}+2\vec{b})\cdot\{(\vec{b}\times\vec{c})+2(\vec{b}\times\vec{a})+4(\vec{c}\times\vec{a})\}$

$= [\vec{a}\,\vec{b}\,\vec{c}]+8[\vec{a}\,\vec{b}\,\vec{c}] \;=\; 9[\vec{a}\,\vec{b}\,\vec{c}]$

$$\Rightarrow \frac{[\vec{a}+2\vec{b} \;\; \vec{b}+2\vec{c} \;\; \vec{c}+2\vec{a}]}{[\vec{a}\,\vec{b}\,\vec{c}]} = 9$$

30. (b) : Here, the given function is

$f(x) = 4x^3 + ax^2 + bx - 1$

$\Rightarrow f'(x) = 12x^2 + 2ax + b$

Given that $f'\left(\frac{1}{2}\right) = 0 \quad \Rightarrow \quad 3 + a + b = 0$

$\Rightarrow a + b = -3 \qquad \ldots(\text{i})$

Since $f(x)$ satisfies all the conditions of Rolle's theorem

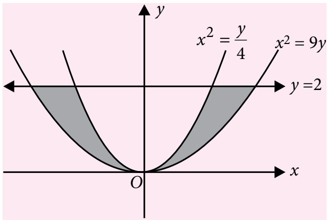
$$\therefore f(1) = f\left(-\frac{1}{4}\right) \Rightarrow 4+a+b-1 = -\frac{1}{16}+\frac{a}{16}-\frac{b}{4}-1$$

$$\Rightarrow a - \frac{a}{16} + b + \frac{b}{4} = -\frac{65}{16} \Rightarrow \frac{15}{16}a + \frac{5}{4}b = -\frac{65}{16}$$

$\Rightarrow 3a + 4b = -13 \qquad \ldots(\text{ii})$

Solving (i) and (ii) we get $a = 1$ and $b = -4$

31. (d) : Given, $y = 4x^2$



$\Rightarrow x^2 = \frac{y}{4} \qquad \ldots(\text{i})$

$x^2 = 9y \qquad \ldots(\text{ii})$

and $y = 2 \qquad \ldots(\text{iii})$

$\therefore$ Area bounded by the above three curves

$$= 2\int_0^2\left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right)dy = 2\int_0^2\frac{5}{2}\sqrt{y}\,dy = 2\times\frac{5}{2}\left[\frac{2y^{3/2}}{3}\right]_0^2$$

$$= \frac{10}{3}[2\sqrt{2}-0] = \frac{20\sqrt{2}}{3} \text{ sq. units}$$

32. (d) : Normal vector of required plane is

$$\vec{n} = (2\hat{i}-2\hat{j}+\hat{k})\times(\hat{i}-\hat{j}+2\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= \hat{i}(-4+1) - \hat{j}(4-1) + \hat{k}(-2+2)$$

$\therefore \vec{n} = -3\hat{i} - 3\hat{j} = -3(\hat{i}+\hat{j})$

$\therefore$ Equation of plane is $1(x - 1) + 1(y + 2) = 0$

$\Rightarrow x + y + 1 = 0$

$\therefore$ Distance from (1, 2, 2) is $d = \frac{3+1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ units

33. (a) : Let G = Event of good book

G' = Event of not a good book

E = Event of published the book

Then $E = (G \cup G') \cap E = (G \cap E) \cup (G' \cap E)$

Now, $P\left(\frac{E}{G}\right) = \frac{2}{3}, \; P\left(\frac{E}{G'}\right) = \frac{1}{4}, \; P(G) = P(G') = \frac{1}{2}$

Therefore, $P(E) = \frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{4} = \frac{11}{24}$

Further, let X denotes the number of books published.

Then, P(at least one book will be published)

$= P(X = 1) + P(X = 2)$

$$= {}^2C_1\left(\frac{11}{24}\right)\left(\frac{13}{24}\right) + {}^2C_2\left(\frac{11}{24}\right)^2\left(\frac{13}{24}\right)^0$$

$$= 2\times\frac{11}{24}\times\frac{13}{24}+\left(\frac{11}{24}\right)^2 = \frac{407}{576}$$

34. (c)

35. (b) : When $x \neq n\pi$, $n \in I$, $\sin x = 0$

and $\left[\frac{x}{\pi}\right] + \frac{1}{2} \neq 0$

$\therefore$ When $x = n\pi$, $f(x) = 0$ and $f(-x) = 0$

$\therefore f(-x) = f(x)$

When $x \neq n\pi,\ n \in I, \frac{x}{\pi}$ is not an integer.

$$\therefore\ \left[\frac{x}{\pi}\right]+\left[-\frac{x}{\pi}\right]=-1\ \therefore\ \left[-\frac{x}{\pi}\right]=-1-\left[\frac{x}{\pi}\right]$$

$$\Rightarrow\ \left[-\frac{x}{\pi}\right]+\frac{1}{2}=-\left[\frac{x}{\pi}\right]-\frac{1}{2}=-\left(\left[\frac{x}{\pi}\right]+\frac{1}{2}\right)$$

$$\text{Now } f(-x)=\frac{\sin^{101}(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}}=\frac{-\sin^{101}x}{-\left(\left[\frac{x}{\pi}\right]+\frac{1}{2}\right)}=\frac{\sin^{101}x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}$$

$$= f(x)$$

Hence in all cases $f(-x) = f(x)$

36. (a) : We have $\frac{dp}{dt}(t)=\frac{1}{2}p(t)-200$

$$\Rightarrow\ \frac{dp(t)}{dt}=\frac{p(t)-400}{2}\ \Rightarrow\ \int\frac{dp(t)}{p(t)-400}=\frac{1}{2}\int dt$$

$$\Rightarrow\ \log|p(t)-400|=\frac{1}{2}t+\log c \Rightarrow \log\left|\frac{p(t)-400}{c}\right|=\frac{1}{2}t$$

$$\Rightarrow\ p(t)=ce^{t/2}+400 \qquad \text{...(i)}$$

When $p(0) = 100$, then $c = -300$

$\therefore$ Eqn. (i) becomes $p(t) = -300\ e^{t/2} + 400$

37. (a) : We have, $\sin^{-1} x + \sin^{-1} y = c$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0\ \Rightarrow\ \frac{dy}{dx}=\frac{-\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow\ \sqrt{1-x^2}\frac{dy}{dx}=-\sqrt{1-y^2}$$

Squaring both sides w.r.t. x, we get

$$(1-x^2)\left(\frac{dy}{dx}\right)^2=(1-y^2)$$

Again differentiating w.r.t. x, we get

$$(1-x^2)\left[2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)\right]+\left(\frac{dy}{dx}\right)^2(-2x)=-2y\frac{dy}{dx}$$

$$\Rightarrow\ (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$$

38. (c) : Clearly, $\left[1+\frac{1}{n}, 6-\frac{2}{n}\right]\supset[2, 4]$

$$\therefore\ \bigcap_{n=1}^{\infty}\left[1+\frac{1}{n}, 6-\frac{2}{n}\right]=[2, 4]$$

39. (c) : Given that $\log_{10}(x^2 - 2x - 2) \leq 0$

$\Rightarrow\ x^2 - 2x - 2 \leq 10^0 \quad \Rightarrow\ x^2 - 2x - 3 \leq 0$

$\Rightarrow\ -1 \leq x \leq 3$ ... (i)

For logarithm to be defined, $x^2 - 2x - 2 > 0$

$\Rightarrow\ \ x>1+\sqrt{3}$ and $x<1-\sqrt{3}$ ... (ii)

From (i) and (ii), common values of x are given by

$-1\leq x<1-\sqrt{3}$ or $1+\sqrt{3}<x\leq 3$

40. (a) : Let a and b are two numbers.

$\therefore\ \ a, A_1, A_2, b$ are in A.P.

$\Rightarrow\ A_1 + A_2 = a + b$

a, G_1, G_2, b are in G.P. $\ \Rightarrow\ G_1G_2 = ab$

Again, a, H_1, H_2, b are in G.P.

$$\therefore\ \ \frac{1}{H_1}+\frac{1}{H_2}=\frac{1}{a}+\frac{1}{b}$$

$$\Rightarrow\ \frac{H_1+H_2}{H_1H_2}=\frac{a+b}{ab}=\frac{A_1+A_2}{G_1G_2}$$

$$\Rightarrow\ \frac{A_1+A_2}{H_1+H_2}=\frac{G_1G_2}{H_1H_2}$$

41. (d) : We have $(p + 2q)\ x + (p - 3q)\ y - p + q = 0$

$\Rightarrow p(x + y - 1) + q(2x - 3y + 1) = 0$,

Clearly, it represents a family of lines passing through the intersection of the lines $x + y - 1 = 0$ and $2x - 3y + 1 = 0$.

The coordinates of the point of intersection of these two lines are (2/5, 3/5).

42. (a) : Eccentricity of ellipse $=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$

Foci of ellipse are $=(\pm\sqrt{3}, 0)$

Again for the hyperbola, $\sqrt{1+\frac{b^2}{a^2}}=\frac{2}{\sqrt{3}}\ \Rightarrow\ \frac{b}{a}=\frac{1}{\sqrt{3}}$

Also, $\frac{(\sqrt{3})^2}{a^2}-0=1 \Rightarrow\ a=\sqrt{3}$ which gives $b = 1$

Thus, equation of hyperbola is $\frac{x^2}{3}-\frac{y^2}{1}=1.$

Foci of hyperbola are (±2, 0).

43. (b) : $\because\ \ h(x)=f(x)-(f(x))^2+(f(x))^3$

$$\therefore\ \ h'(x)=f'(x)-2f(x)f'(x)+3(f(x))^2f'(x)$$

$$=3f'(x)\left[(f(x))^2-2f(x)\cdot\frac{1}{3}+\left(\frac{1}{3}\right)^2+\frac{1}{3}-\frac{1}{9}\right]$$

$$=3f'(x)\left[\left(f(x)-\frac{1}{3}\right)^2+\frac{2}{9}\right]$$

It is given that $f'(x) > 0\ \forall\ x$

Thus $h'(x) > 0$ for all real values of x.
Therefore $h(x)$ is an increasing function for all real values of x.

44. (a) : We have, $f(1)=\frac{1+1}{2}=\frac{2}{2}=1$

$$f(2)=\frac{2}{2}=1$$

Thus, $f(1) = f(2)$ but $1 \neq 2$

$\therefore$ $f(n)$ is not one-one.

In order to find that f is onto or not, consider element $n \in N$

If n is odd, then $(2n - 1)$ is also odd and

$$f(2n-1)=\frac{2n-1+1}{2}=\frac{2n}{2}=n$$

If n is even, then $2n$ is also even and

$$f(2n)=\frac{2n}{2}=n$$

Thus, for each $n \in N$, there exists its pre-image in N.

$\therefore$ f is onto.

45. (c) : Let $A \equiv (7, 1, -3)$ and $B \equiv (4, 5, c)$

We have, $AB = 13$

$\Rightarrow$ $AB^2 = 169$

$\Rightarrow$ $(7-4)^2 + (1-5)^2 + (-3-c)^2 = 169$

$\Rightarrow$ $9 + 16 + c^2 + 9 + 6c = 169$

$\Rightarrow$ $c^2 + 6c - 135 = 0$

$\Rightarrow$ $c^2 + 15c - 9c - 135 = 0$

$\Rightarrow$ $(c + 15)(c - 9) = 0$

$\Rightarrow$ $c = -15$ or $c = 9$

46. (a) : $f(x) = \frac{3x+5}{2x-3}$,

$$f_2(x) = f(f(x)) = \frac{3f(x)+5}{2f(x)-3}=\frac{3\left(\frac{3x+5}{2x-3}\right)+5}{2\left(\frac{3x+5}{2x-3}\right)-3}=x$$

$\therefore$ $f_2(x) = f_4(x) = \ldots = f_{2008}(x) = x,$

$$f(x) = f_3(x) = \ldots = f_{2009}(x) = \frac{3x+5}{2x-3}$$

$\therefore$ $f_{2008}(x) + f_{2009}(x) = x+\frac{3x+5}{2x-3}=\frac{2x^2+5}{2x-3}$

47. (c) : We have, $\lim\limits_{x\to 0}\frac{f(x)}{x}=2,$

Given that $f(x)=\min(\sin\sqrt{[m]}x, |x|)$

$$=\lim_{x\to 0}\frac{\min(\sin\sqrt{[m]}x, |x|)}{x}=2$$

$$=\lim_{x\to 0}\frac{\sin\sqrt{[m]}x}{x}=2 \quad [\because \sin kx \le |x| \ \forall\ k, x \in R]$$

$$=\lim_{x\to 0}\frac{\sin\sqrt{[m]}x}{\sqrt{[m]}x}\cdot\sqrt{[m]}=2$$

$\Rightarrow$ $\sqrt{[m]}=2 \ \Rightarrow\ [m]=4$

$\Rightarrow$ $m \in [4, 5)$ *i.e.* $4 \le m < 5$.

48. (a) : For $x > 10$, we have

$|\cos x|<1$ and $1 + x^8 > 10^8 \ \Rightarrow\ \frac{1}{1+x^8}\le 10^{-8}$

$$\therefore \left|\int_{10}^{19}\frac{\cos x\,dx}{1+x^8}\right|\le\int_{10}^{19}\frac{|\cos x|}{1+x^8}\,dx\le\int_{10}^{19}10^{-8}\,dx$$

$= 9 \times 10^{-8} < 10^{-7}$

49. (a) : We have

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 1+2\\0 & 1\end{bmatrix}$$

Again $\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 6\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 1+2+3\\0 & 1\end{bmatrix}$

By induction, $\text{LHS}=\begin{bmatrix}1 & \sum\limits_{k=1}^{n}k\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 378\\0 & 1\end{bmatrix}$

Therefore, $\frac{n(n+1)}{2}=378$

$\Rightarrow n(n + 1) = 27 \times 28$

Hence, $n = 27$.

50. (c) : Let $y = (x + \sec x)(x - \tan x)$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=(x+\sec x)\frac{d}{dx}(x-\tan x)+(x-\tan x)\frac{d}{dx}(x+\sec x)$$

$= (x + \sec x)(1 - \sec^2 x) + (x - \tan x)(1 + \sec x \tan x)$

$\therefore$ $A = 1 - \sec^2 x$ and $B = 1 + \sec x \tan x$

51. (d) **52. (c)** **53. (b)**
54. (c) **55. (a)** **56. (c)**
57. (a) **58. (c)** **59. (d)**
60. (a) **61. (a)** **62. (d)**
63. (a) **64. (a)** **65. (b)**
66. (a, c) **67. (a, d)** **68. (a, b, c, d)**
69. (b, c, d) **70. (a, b, c)** **71. (a, b, c, d)**
72. (b, c) **73. (b, d)** **74. (a, b, d)**
75. (a, b)

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YOU ASK
WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If $a_1, a_2, \ldots, a_n$ are in A.P. and S_n is the sum of first n terms, show that $\sum\limits_{k=0}^{n} {}^nC_k S_k = 2^{n-2}(na_1 + S_n)$.

(Dravyansh, U.P.)

Ans. Let d be the common difference of the A.P., then we have

$$\sum_{k=0}^{n} {}^nC_k S_k = \sum_{k=0}^{n} {}^nC_k \cdot \frac{k}{2}[2a_1 + (k-1)d]$$

$$= \left(a_1 - \frac{d}{2}\right)\sum_{k=0}^{n} k \cdot {}^nC_k + \frac{d}{2}\sum_{k=0}^{n} k^2 \cdot {}^nC_k$$

$$= \left(a_1 - \frac{d}{2}\right)n2^{n-1} + \frac{d}{2}[n2^{n-1} + n(n-1)2^{n-2}]$$

$$= a_1n2^{n-1} + dn(n-1)2^{n-3}$$

$$= a_1n2^{n-1} + n(a_n - a_1)2^{n-3}[\because\ a_n - a_1 = (n-1)d]$$

$$= n2^{n-3}[4a_1 + a_n - a_1] = n2^{n-3}(2a_1 + a_1 + a_n)$$

$$= 2^{n-2}\left[na_1 + \frac{n(a_1 + a_n)}{2}\right] = 2^{n-2}(na_1 + S_n).$$

2. A bag contains m white and 4 black balls. Balls are drawn one by one without replacement till all the black balls are drawn. Find the probability that the process ends at r^{th} draw ($r \geq 4$). *(Prateek Fegade, Hyderabad)*

Ans. If the process ends at the r^{th} draw, then the r^{th} ball must be a black ball and the previous $(r - 1)$ draws must contain 3 black balls and $(r - 4)$ white balls.

Let E = {First $(r - 1)$ draws contains 3 black balls and $(r - 4)$ white balls}

and F = {r^{th} draw is a black ball}

Now, $n(E) = {}^4C_3 \times {}^mC_{r-4}$

Also, total number of ways of drawing $(r - 1)$ balls $= {}^{m+4}C_{r-1}$

$$P(E) = \frac{{}^4C_3 \times {}^mC_{r-4}}{{}^{m+4}C_{r-1}}$$

Hence, P(process of drawing balls ends at r^{th} draw)

$= P(E \cap F) = P(E)\,P(F/E)$

$$= \frac{{}^4C_3 \cdot {}^mC_{r-4}}{{}^{m+4}C_{r-1}} \times \frac{1}{(m+4)-(r-1)}$$

$$= 4 \cdot \frac{m!}{(r-4)!(m-r+4)!} \cdot \frac{(r-1)!(m-r+5)!}{(m+4)!} \cdot \frac{1}{m-r+5}$$

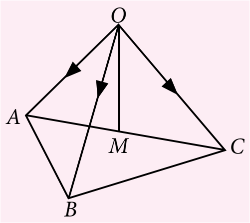
$$= \frac{4(r-1)(r-2)(r-3)}{(m+4)(m+3)(m+2)(m+1)}$$

3. If l is the length of an edge of a regular tetrahedron, then find the distance of any vertex from its opposite face.

(Umang Bhardwaj, Delhi)

Ans. The figure shows a regular tetrahedron $OABC$. Let O be the origin and the position vectors of the vertices A, B, C be $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then, we have

Volume of $OABC = \frac{1}{3}$ area $(\Delta ABC) \times (OM)$



i.e., $\frac{1}{6}\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

$= \frac{1}{3}$ area $(\Delta ABC) \times (OM)$

Now, we have ΔABC

$= \frac{\sqrt{3}}{4} l^2$ [ΔABC is equilateral]

$$\text{and } \left[\vec{a}\,\vec{b}\,\vec{c}\right]^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix}$$

$$= l^6 \begin{vmatrix} 1 & \cos 60° & \cos 60° \\ \cos 60° & 1 & \cos 60° \\ \cos 60° & \cos 60° & 1 \end{vmatrix} = \frac{1}{2} l^6$$

Hence, we have

$$\frac{1}{6} \times \frac{l^3}{\sqrt{2}} = \frac{1}{3}\left(\frac{\sqrt{3}}{4} l^2\right)(OM) \text{ i.e., } OM = \sqrt{\frac{2}{3}}\,l.$$

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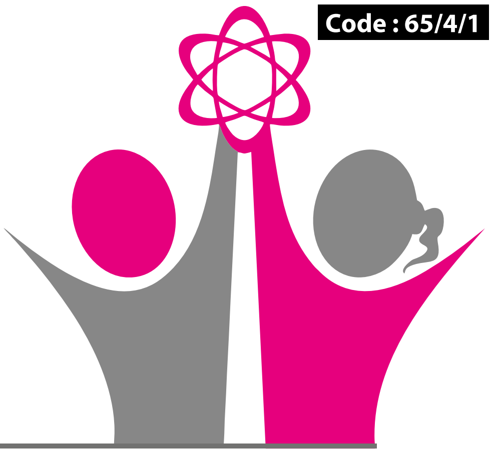
EXAM ALERT 2019

Exam	Date
COMEDK (Engg.)	12th May
MHT-CET	2nd to 13th May
BITSAT	16th to 26th May
AMU (Engg.)	26th May **(Revised)**
WB JEE	26th May
JEE Advanced	27th May **(Revised)**

CBSE BOARD SOLVED PAPER 2019*



Code : 65/4/1

Time Allowed : 3 hours **Maximum Marks : 100**

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Questions **1-4** in Section-A are very short-answer type questions carrying **1** mark each.
(iv) Questions **5-12** in Section-B are short-answer type questions carrying **2** marks each.
(v) Questions **13-23** in Section-C are long-answer-**I** type questions carrying **4** marks each.
(vi) Questions **24-29** in Section-D are long-answer-**II** type questions carrying **6** marks each.

SECTION - A

1. Form the differential equation representing the family of curves $y=\frac{A}{x}+5$, by eliminating the arbitrary constant A.

2. If A is a square matrix of order 3 with $|A| = 9$, then write the value of $|2 \cdot \text{adj } A|$.

3. Find the acute angle between the planes $\vec{r}\cdot(\hat{i}-2\hat{j}-2\hat{k})=1$ and $\vec{r}\cdot(3\hat{i}-6\hat{j}+2\hat{k})=0$.

OR

Find the length of the intercept, cut off by the plane $2x + y - z = 5$ on the x-axis.

4. If $y = \log(\cos e^x)$, then find $\frac{dy}{dx}$.

SECTION - B

5. Find : $\int_{-\frac{\pi}{4}}^{0} \frac{1+\tan x}{1-\tan x}\,dx$

6. Let $*$ be an operation defined as $*: R \times R \to R$ such that $a * b = 2a + b$, $a, b \in R$. Check if $*$ is a binary operation. If yes, find if it is associative too.

7. X and Y are two points with position vectors $3\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ respectively. Write the position vector of a point Z which divides the line segment XY in the ratio 2 : 1 externally.

OR

Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$ be two vectors. Show that the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

8. If A and B are symmetric matrices, such that AB and BA are both defined, then prove that $AB - BA$ is a skew symmetric matrix.

9. 12 cards numbered 1 to 12 (one number on one card), are placed in a box and mixed up thoroughly. Then a card is drawn at random from the box. If it is known that the number on the drawn card is greater than 5, find the probability that the card bears an odd number.

10. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

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OR

In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

11. Solve the following differential equation:

$$\frac{dy}{dx}+y=\cos x-\sin x$$

12. Find : $\int x\cdot\tan^{-1}x\,dx$

OR

Find : $\int\frac{dx}{\sqrt{5-4x-2x^2}}$

SECTION - C

13. Using properties of determinants, find the value of x for which $\begin{vmatrix}4-x & 4+x & 4+x\\ 4+x & 4-x & 4+x\\ 4+x & 4+x & 4-x\end{vmatrix}=0.$

14. Solve the differential equation :

$\frac{dy}{dx}=1+x^2+y^2+x^2y^2$, given that $y = 1$ when $x = 0$.

OR

Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$, given that $y = 1$ when $x = 0$.

15. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x)=\frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1}.

OR

Show that the relation S in the set $A=\{x\in Z:0\le x\le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by $3\}$ is an equivalence relation.

16. Integrate the function $\frac{\cos(x+a)}{\sin(x+b)}$ with respect to x.

17. If $x = \sin t$, $y = \sin pt$, prove that

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0.$$

OR

Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to $\cos^{-1}x^2$.

18. Prove that :

$$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$$

19. If $y = (x)^{\cos x} + (\cos x)^{\sin x}$, then find $\frac{dy}{dx}$.

20. Prove that $\int_0^a f(x)\,dx=\int_0^a f(a-x)dx$ and hence evaluate $\int_0^1 x^2(1-x)^n\,dx$.

21. Find the value of x, for which the four points $A(x, -1, -1)$, $B(4, 5, 1)$, $C(3, 9, 4)$ and $D(-4, 4, 4)$ are coplanar.

22. A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

23. Find the vector equation of the plane determined by the points $A(3, -1, 2)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$. Hence, find the distance of the plane, thus obtained, from the origin.

SECTION - D

24. Using integration, find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$

25. An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver and a car driver are 0.3, 0.05 and 0.02 respectively. One of the insured persons meets with an accident. What is the probability that he is a cyclist?

26. Using elementary row transformations, find the inverse of the matrix $\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$.

OR

Using matrices, solve the following system of linear equations:

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

27. Using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

OR

Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.

28. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. It costs ₹ 50 per kg to produce food I. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C and it costs ₹ 70 per kg to produce food II. Formulate this problem as a LPP to minimise the cost of a mixture that will produce the required diet. Also find the minimum cost.

29. Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line $\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$. Also, find the distance between these two lines.

OR

Find the coordinates of the foot of the perpendicular Q drawn from $P(3, 2, 1)$ to the plane $2x - y + z + 1 = 0$. Also, find the distance PQ and the image of the point P treating this plane as a mirror.

SOLUTIONS

1. Given, $y = \frac{A}{x} + 5$ ...(i)

Differentiating (i) w.r.t. x, we get, $\frac{dy}{dx} = \frac{-A}{x^2}$ ...(ii)

Putting value of A from (ii) into (i), we get

$$y = -x^2 \frac{dy}{dx} \cdot \frac{1}{x} + 5 \quad \Rightarrow \quad y + x\frac{dy}{dx} = 5 \Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{5}{x}$$

which is required differential equation.

2. Given, $|A| = 9$

We know that, $|k \text{ adj } A| = k^n |A|^{n-1}$, where n is the order of the matrix A.

$\therefore \quad |2 \cdot \text{adj } A| = 2^3 (9)^2 = 8 \times 81 = 648.$

3. The given planes are

$\vec{r} \cdot (\hat{i} - 2\hat{j} - 2\hat{k}) = 1$ and $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 0$

If θ is the angle between given planes then

$$\cos\theta = \left| \frac{(\hat{i} - 2\hat{j} - 2\hat{k}) \cdot (3\hat{i} - 6\hat{j} + 2\hat{k})}{|\hat{i} - 2\hat{j} - 2\hat{k}||3\hat{i} - 6\hat{j} + 2\hat{k}|} \right|$$

$$= \left| \frac{(1)(3) + (-2)(-6) + (-2)(2)}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{3^2 + 6^2 + 2^2}} \right| = \left| \frac{3 + 12 - 4}{\sqrt{9}\sqrt{49}} \right| = \left| \frac{11}{3 \times 7} \right| = \frac{11}{21}$$

$\therefore \quad$ Required angle, $\theta = \cos^{-1}\left(\frac{11}{21}\right)$

OR

We have, $2x + y - z = 5 \Rightarrow \frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1,$

which is the equation of plane in intercept form.

$\therefore \quad$ Length of the intercept on x-axis is $\frac{5}{2}$.

4. Given $y = \log(\cos e^x)$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{\cos e^x}(-\sin e^x \cdot e^x) = -e^x \tan e^x$$

5. Let $I = \int_{\frac{-\pi}{4}}^{0} \frac{(1 + \tan x)}{(1 - \tan x)} dx = \int_{\frac{-\pi}{4}}^{0} \frac{\left(1 + \frac{\sin x}{\cos x}\right)}{\left(1 - \frac{\sin x}{\cos x}\right)} dx$

$$= \int_{\frac{-\pi}{4}}^{0} \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Put $\cos x - \sin x = t \Rightarrow -(\sin x + \cos x)\, dx = dt$

When $x = 0, t = 1$, when $x = \frac{-\pi}{4}, t = \sqrt{2}$

$\therefore \quad I = \int_{\sqrt{2}}^{1} -\frac{dt}{t} = \int_{1}^{\sqrt{2}} \frac{dt}{t} = [\log t]_1^{\sqrt{2}} = \log\sqrt{2} - \log 1 = \frac{1}{2}\log 2$

6. For every real number a and b, $2a + b$ is also a real number.

$\therefore \quad *$ is a binary operation.

Associativity : $*$ will be associative if

$(a * b) * c = a * (b * c)$

Now, $(a * b) * c = (2a + b) * c = 2(2a + b) + c$

$= 4a + 2b + c$

Also, $a * (b * c) = a * (2b + c) = 2a + 2b + c$

$\because \quad (a * b) * c \neq a * (b * c)$

$\therefore \quad$ Binary operation is not associative.

7. Position vector which divides the line segment joining points with position vectors $3\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ in the ratio 2 : 1 externally is given by

$$= \frac{2(\vec{a}-3\vec{b})-1(3\vec{a}+\vec{b})}{2-1} = \frac{2\vec{a}-6\vec{b}-3\vec{a}-\vec{b}}{1} = -\vec{a}-7\vec{b}$$

OR

Given, $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$

Now, $\vec{a}+\vec{b}=4\hat{i}+\hat{j}-\hat{k}$

Also, $\vec{a}-\vec{b}=-2\hat{i}+3\hat{j}-5\hat{k}$

Now, $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=(4\hat{i}+\hat{j}-\hat{k})\cdot(-2\hat{i}+3\hat{j}-5\hat{k})$

$= (4)(-2) + (1)(3) + (-1)(-5) = -8 + 3 + 5 = 0$

Hence, $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

8. Given, A and B are symmetric matrices

$\therefore$ $A' = A$ and $B' = B$

Now, $(AB-BA)' = (AB)' - (BA)' = (B'A') - (A'B')$

$= (BA - AB)$ $\qquad [\because A' = A \text{ and } B' = B]$

$= -(AB - BA)$

Thus, $(AB-BA)' = -(AB-BA)$

Hence,$(AB - BA)$ is a skew symmetric matrix.

9. The sample space, S is given by

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Let A be the event that number on the drawn card is odd, and B be the event that number on the drawn card is greater than 5.

$\therefore$ $A = \{1, 3, 5, 7, 9, 11\}$; $B = \{6, 7, 8, 9, 10, 11, 12\}$

and, $A \cap B = \{7, 9, 11\}$

Now, $P(A) = \frac{n(A)}{n(S)} = \frac{6}{12}$, $P(B) = \frac{n(B)}{n(S)} = \frac{7}{12}$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{12}$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{3/12}{7/12} = \frac{3}{7}$

Hence, required probability is $\frac{3}{7}$.

10. Total number of students = 8

The number of ways to select 4 students out of 8 students

$= {}^8C_4 = \frac{8!}{4!\,4!} = 70$

The number of ways to select 2 boys and 2 girls

$= {}^3C_2 \times {}^5C_2 = \frac{3!}{2!1!} \times \frac{5!}{2!3!} = 3 \times 10 = 30$

$\therefore$ Required probability $= \frac{30}{70} = \frac{3}{7}$.

OR

Let X represent the number of correct answers obtained by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer, $p = \frac{1}{3}$

$\therefore$ Probability of getting an incorrect answer, $q = 1 - \frac{1}{3} = \frac{2}{3}$

Required probability $= P\,(X \geq 4)$

$= P(X=4) + P(X=5)$

$$= {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 + {}^5C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^0$$

$$= 5\times\frac{1}{81}\times\frac{2}{3}+1\cdot\frac{1}{243}=\frac{10+1}{243}=\frac{11}{243}$$

11. We have, $\frac{dy}{dx}+y=\cos x-\sin x$, which is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=1$, $Q=\cos x-\sin x$

$\therefore$ I.F. $= e^{\int dx}=e^x$

The solution of given differential equation is

$$ye^x=\int e^x(\cos x-\sin x)dx+C$$

$$\Rightarrow\ ye^x=e^x\cos x+C \Rightarrow y=\cos x+Ce^{-x}$$

12. Let $I=\int x\cdot\tan^{-1}x\,dx$

Integrating by parts, we get

$$I=\tan^{-1}x\int xdx-\int\left\{\frac{d}{dx}(\tan^{-1}x)\int xdx\right\}dx$$

$$=(\tan^{-1}x)\frac{x^2}{2}-\int\frac{1}{(1+x^2)}\frac{x^2}{2}dx$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int dx+\frac{1}{2}\int\frac{1}{1+x^2}dx$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{x}{2}+\frac{1}{2}\tan^{-1}x+C$$

$$=\frac{1}{2}(1+x^2)\tan^{-1}x-\frac{x}{2}+C$$

OR

Let $I=\int\frac{dx}{\sqrt{5-4x-2x^2}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\frac{5}{2}-2x-x^2}}$

$$=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\frac{7}{2}-1-2x-x^2}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2-(x+1)^2}}$$

$$=\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{x+1}{\sqrt{\frac{7}{2}}}\right)+C=\frac{1}{\sqrt{2}}\sin^{-1}\left[\sqrt{\frac{2}{7}}(x+1)\right]+C\cdot$$

13. Given, $\begin{vmatrix}4-x & 4+x & 4+x\\ 4+x & 4-x & 4+x\\ 4+x & 4+x & 4-x\end{vmatrix}=0$

Applying $C_1\to C_1+C_2+C_3$, we get

$$\begin{vmatrix}12+x & 4+x & 4+x\\ 12+x & 4-x & 4+x\\ 12+x & 4+x & 4-x\end{vmatrix}=0$$

Applying $R_2\to R_2-R_1, R_3\to R_3-R_1$, we get

$$\begin{vmatrix}12+x & 4+x & 4+x\\ 0 & -2x & 0\\ 0 & 0 & -2x\end{vmatrix}=0$$

$$\Rightarrow\quad (12+x)(-2x)^2=0 \Rightarrow 4x^2(12+x)=0 \Rightarrow x=0,-12$$

14. We have, $\frac{dy}{dx}=1+x^2+y^2+x^2y^2$

$$\therefore\ \frac{dy}{dx}=1+x^2+y^2(1+x^2)\ =(1+x^2).(1+y^2)$$

$$\Rightarrow\ \frac{dy}{1+y^2}=(1+x^2)dx$$

Integrating both sides, we get

$$\int\frac{dy}{1+y^2}=\int(1+x^2)dx\ \Rightarrow\ \tan^{-1}y=x+\frac{x^3}{3}+C$$

When $x=0, y=1,\ \tan^{-1}1=0+0+C\Rightarrow C=\frac{\pi}{4}$

$$\therefore\ \tan^{-1}y=x+\frac{1}{3}x^3+\frac{\pi}{4}$$

is the required particular solution.

OR

We have, $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ ...(i)

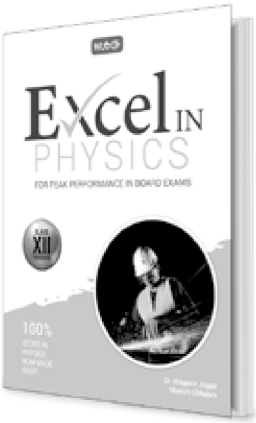
This is a homogeneous linear differential equation.

$\therefore$ Put $y=vx\ \Rightarrow\ \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore$ (i) becomes $v+x\frac{dv}{dx}=\frac{x\cdot vx}{x^2+v^2x^2}$

$$\Rightarrow\ v+x\frac{dv}{dx}=\frac{v}{1+v^2}\quad\Rightarrow\ x\frac{dv}{dx}=\frac{v}{1+v^2}-v$$

$$\Rightarrow\ x\frac{dv}{dx}=\frac{-v^3}{1+v^2}\quad\Rightarrow\ \frac{dx}{x}=-\left(\frac{1+v^2}{v^3}\right)dv$$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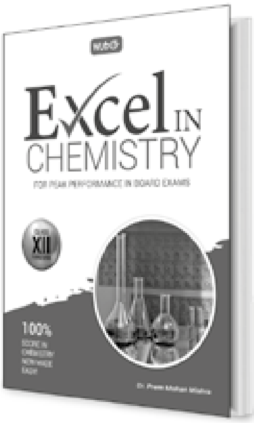
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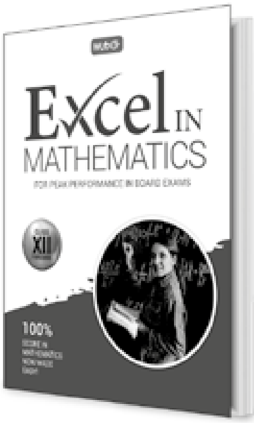
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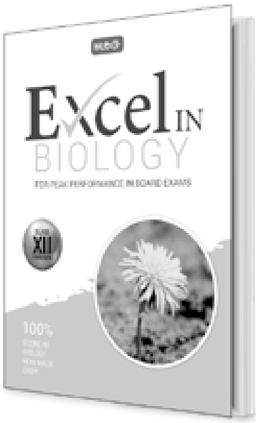
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Integrating both sides, we get

$$\int \frac{dx}{x} = -\int v^{-3} dv - \int \frac{1}{v} dv \Rightarrow \log x = \frac{1}{2v^2} - \log v + C$$

$$\Rightarrow \log x = \frac{x^2}{2y^2} - \log y + \log x + C \Rightarrow \log y = \frac{x^2}{2y^2} + C$$

When $y = 1, x = 0 \Rightarrow \log 1 = 0 + C \Rightarrow C = 0$

$\therefore$ Particular solution is $y = e^{\frac{x^2}{2y^2}}$.

15. Here, $f : A \to B$ is given by $f(x) = \frac{x-1}{x-2}$,

where $A = R - \{2\}$ and $B = R - \{1\}$.

Let $f(x_1) = f(x_2)$, where $x_1, x_2 \in A$ (*i.e.*, $x_1 \neq 2, x_2 \neq 2$)

$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$

$\Rightarrow (x_1 - 1)(x_2 - 2) = (x_1 - 2)(x_2 - 1)$

$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - x_1 - 2x_2 + 2$

$\Rightarrow -2x_1 - x_2 = -x_1 - 2x_2 \Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.

Let $y \in B = R - \{1\}$ *i.e.*, $y \in R$ and $y \neq 1$

such that $f(x) = y$

$\Leftrightarrow \frac{x-1}{x-2} = y \Leftrightarrow (x-2)y = x - 1$

$\Leftrightarrow xy - 2y = x - 1 \Leftrightarrow x(y - 1) = 2y - 1$

$\Leftrightarrow x = \frac{2y-1}{y-1}$ ...(i)

$\therefore f(x) = y$ when $x = \frac{2y-1}{y-1} \in A$ (as $y \neq 1$)

Hence, f is onto.

Thus, f is one–one and onto.

From (i), $f^{-1} : B \to A$ is given by $x = f^{-1}(y)$

i.e., $f^{-1}(y) = \frac{2y-1}{y-1}$.

OR

We have, $A = \{x \in Z : 0 \leq x \leq 12\}$

$\therefore A = \{0, 1, 2, 3, ..., 12\}$

Also, $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by $3\}$

(i) Reflexive : For any $a \in A$,

$|a - a| = 0$, which is divisible by 3

Thus, $(a, a) \in S \;\therefore S$ is reflexive

(ii) Symmetric : For any $a, b \in A$, let $(a, b) \in S$

$\Rightarrow |a - b|$ is divisible by 3

$\Rightarrow |b - a|$ is divisible by 3 $\Rightarrow (b, a) \in S$

i.e. $(a, b) \in S \Rightarrow (b, a) \in S$

$\therefore S$ is symmetric.

(iii) Transitive : For any $a, b, c \in A$

Let $(a, b) \in S$ and $(b, c) \in S$

$\Rightarrow |a - b|$ is divisible by 3 and $|b - c|$ is divisible by 3

$\Rightarrow (a - b) = \pm 3k_1$ and $(b - c) = \pm 3k_2; k_1, k_2 \in N$

$\Rightarrow (a - b) + (b - c) = \pm 3(k_1 + k_2)$

$\Rightarrow (a - c) = \pm 3(k_1 + k_2) ; k_1, k_2 \in N$

$\Rightarrow |a - c|$ is a multiple of 3

$\Rightarrow |a - c|$ is divisible by 3 $\Rightarrow (a, c) \in S$

$\therefore S$ is transitive.

Hence, S is an equivalence relation.

16. $\int \frac{\cos(x+a)}{\sin(x+b)} dx = \int \frac{\cos(x+b+a-b)}{\sin(x+b)} dx$

$$= \int \frac{\cos(x+b)\cos(a-b) - \sin(x+b)\sin(a-b)}{\sin(x+b)} dx$$

$$= \cos(a-b) \int \frac{\cos(x+b)}{\sin(x+b)} dx - \sin(a-b) \int dx$$

$= \cos(a - b) \log \sin(x + b) - x \sin(a - b) + C$

For complete solutions refer to

MTG CBSE Champion Mathematics Class 12.

CBSE releases list of new courses to help students explore further possibilities after Class 12

The Central Board of Secondary Education (CBSE) has released a long list of all possible courses that students can pursue after class 12 under a student-friendly initiative. CBSE has mentioned the names of colleges, their eligibility criteria and other details related to that course.

"As a sequel to the ongoing initiatives, CBSE has prepared a compendium of suggestive courses for students which will help them get information about various course choices, institutes and combinations available in higher education beforehand after class 10 itself," said CBSE chairperson, Anita Karwal in an official statement.

Our country has more than 900 universities and 41,000 colleges, that provide higher education presently and CBSE has tried to cover almost all of the traditional, new age, and popular courses. The purpose of this collection is to raise curiosity among the students to explore further scope, possibilities, avenues, for each of these courses and to look for other options beyond these courses as well.

The list consists of 113 career options including:

- Art Restoration
- Public Relations
- Engineering
- Languages
- Actuarial Sciences
- Corporate Intelligence
- Liberal Studies

The list expands up to 122 pages and students can check the list at the official website : cbse.nic.in.

CBSE warm-up!

CLASS-XII

Synopsis and Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 1

Relations and Functions

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

- (i) All questions are compulsory.
- (ii) This question paper contains **36** questions.
- (iii) Question **1-20** in Section-A are very short-answer-objective type questions carrying **1** mark each.
- (iv) Question **21-26** in Section-B are short-answer type questions carrying **2** marks each.
- (v) Question **27-32** in Section-C are long-answer-**I** type questions carrying **4** marks each.
- (vi) Question **33-36** in Section-D are long-answer-**II** type questions carrying **6** marks each.

SECTION - A

1. The range of the function $f(x)=\frac{x-2}{2-x}$, $x \neq 2$ is

(a) $\mathbb{R}$ (b) $\mathbb{R}-\{1\}$ (c) $\{-1\}$ (d) $\mathbb{R}-\{-1\}$

2. If $f:\mathbb{R}\to\mathbb{R}$, $g:\mathbb{R}\to\mathbb{R}$ and $h:\mathbb{R}\to\mathbb{R}$ is such that $f(x)=x^2$, $g(x)=\tan x$ and $h(x)=\log x$, then the value of $[ho(gof)]\,(x)$, if $x=\frac{\sqrt{\pi}}{2}$ will be

(a) 0 (b) 1 (c) –1 (d) 10

3. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R\beta \Leftrightarrow \alpha \perp \beta, \alpha, \beta \in L$. Then, R is

(a) reflexive only (b) symmetric only
(c) transitive only (d) none of these

4. The range of the function $f(x)=3x^2+7x+10$ is

(a) $[10, \infty)$ (b) $\left[\frac{71}{12}, \infty\right)$
(c) $[1, \infty)$ (d) none of these

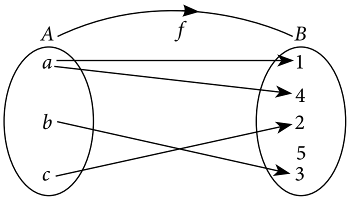
5. If $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=2x+3$ and $g(x)=x^2+7$, then the value of x for which $f(g(x))=25$ is

(a) ±1 (b) ±2 (c) ±3 (d) ±4

6. The relation R is defined on the set of natural numbers as $\{(a, b): a = 2b\}$. Then, R^{-1} is given by

(a) $\{(2, 1), (4, 2), (6, 3),...\}$
(b) $\{(1, 2), (2, 4), (3, 6),...\}$
(c) R^{-1} is not defined (d) none of these

7. The diagram given below shows that



(a) f is a function from A to B
(b) f is a one-one function from A to B
(c) f is an onto function from A to B
(d) f is not a function

8. Let $f(x)=\dfrac{x-1}{x+1}$, then $f(f(x))$ is

(a) $\dfrac{1}{x}$ (b) $-\dfrac{1}{x}$ (c) $\dfrac{1}{x+1}$ (d) $\dfrac{1}{x-1}$

9. If $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\dfrac{2x-7}{4}$ is an invertible function, then find f^{-1}.

(a) $\dfrac{4x+5}{2}$ (b) $\dfrac{4x+7}{2}$

(c) $\dfrac{3x+2}{2}$ (d) $\dfrac{9x+3}{5}$

10. The domain of the function $f=\{(1,3),(3,5),(2,6)\}$ is

(a) 1, 3 and 2 (b) $\{1, 3, 2\}$

(c) $\{3, 5, 6\}$ (d) 3, 5 and 6

11. Prove that the relation R defined on the set N of natural numbers by $xRy \Leftrightarrow 2x^2 - 3xy + y^2 = 0$ *i.e.*, by $R = \{(x, y) : x, y \in N \text{ and } 2x^2 - 3xy + y^2 = 0\}$ is not symmetric.

12. Determine if the function given below is one-to-one. "To each person on earth assign the number which corresponds to his height".

13. Find gof, if $f(x) = |x|$ and $g(x) = |5x - 2|$.

14. Let $R = \{(a, a^3) : a \text{ is a prime number less than } 5\}$ be a relation. Find the range of R.

15. Find the interval in which the function $f(x) = \sqrt{3}\,\sin 2x - \cos 2x + 4$ is one-one.

16. Let the functions f, g, h are defined from $\mathbb{R}$ to $\mathbb{R}$ such that

$f(x) = x^2 - 1$, $g(x)=\sqrt{x^2+1}$ and

$h(x)=\begin{cases}0, & \text{if } x<0\\ x, & \text{if } x>0\end{cases}$, then find $ho(fog)(x)$.

17. Set A has three elements and set B has four elements. Find the number of injections that can be defined from A to B.

18. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and if $R = \{(x, y) : y$ is half of $x; x, y \in A\}$ is a relation on A, then write R as a set of ordered pairs.

19. Let f be an invertible real function. Write $(f^{-1} of)(1) + (f^{-1} of)(2) + ... + (f^{-1} of)(100)$.

20. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$, write $fog\,(7)$.

SECTION - B

21. If $f:\mathbb{R}\to\mathbb{R}$ is given by $f(x) = (3 - x^3)^{1/3}$, then show that $fof = I_{\mathbb{R}}$, where $I_{\mathbb{R}}$ is the identity map on $\mathbb{R}$.

22. Let $A = \{1, 2, 3\}$ and $R = \{(a, b) : a, b \in A, a$ divides b and b divides $a\}$. Show that R is an identity relation on A.

23. Show that the fractional part function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x) = x - [x]$ is neither one-one nor onto.

OR

Show that the signum function $f:\mathbb{R}\to\mathbb{R}$ defined by

$f(x)=\begin{cases}1, & x > 0\\ 0, & x = 0\\ -1, & x < 0\end{cases}$ is neither one-one nor onto.

24. Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class of 0.

25. Let R be a relation defined on the set of natural numbers N as $R = \{(x, y) : x, y \in N, 2x + y = 41\}$. Find the domain and range of R.

26. Let $f(x) = [x]$ and $g(x) = |x|$. Find

$(gof)\left(\dfrac{-5}{3}\right)-(fog)\left(\dfrac{-5}{3}\right)$.

OR

Determine whether the relation R in the set A of human beings in a city at a particular time defined by $R = \{(x, y) : x$ is wife of y ; $x, y \in A\}$ is reflexive and symmetric.

SECTION - C

27. If $f:\mathbb{R}\to\mathbb{R}$ be given by $f(x) = \sin^2 x + \sin^2 (x + \pi/3) + \cos x \cos (x + \pi/3)$ for all $x \in \mathbb{R}$, and $g:\mathbb{R}\to\mathbb{R}$ be a function such that $g\,(5/4) = 1$, then prove that $(gof):\mathbb{R}\to\mathbb{R}$ is a constant function.

28. Show that the relation S in the set
$A = \{x \in Z : 0 \le x \le 12\}$ given by
$S = \{(a, b) : a, b \in Z, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

OR

Show that the relation R defined by $(a, b)\ R\ (c, d)$ iff $a + d = b + c$ on the set $N \times N$ is an equivalence relation.

29. If the function $f : [1, \infty) \to [1, \infty)$ defined by $f(x) = 2^{x(x-1)}$ is invertible, find $f^{-1}(x)$.

30. Show that the relation R on the set $\mathbb{R}$ of all real numbers, defined as $R = \{(a, b) : a \le b^2\}$ is neither reflexive nor symmetric nor transitive.

31. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \rightarrow B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto ? Justify your answer.

OR

Let $f : Z \rightarrow Z$ be defined by $f(n) = 3n$ for all $n \in Z$ and $g : Z \rightarrow Z$ be defined by

$$g(n) = \begin{cases} \frac{n}{3}, \text{ if } n \text{ is a multiple of } 3 \\ 0, \text{ if } n \text{ is not a multiple of } 3 \end{cases} \quad \text{for all } n \in Z.$$

Show that $gof = I_Z$ and $fog \neq I_Z$.

32. Consider $f : \mathbb{R}^+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where $\mathbb{R}^+$ is the set of all non-negative real numbers.

SECTION - D

33. Show that the function $f : N \rightarrow N$ given by, $f(n) = n - (-1)^n$ for all $n \in N$ is a bijection.

OR

Consider $f : \mathbb{R}^+ \rightarrow [-9, \infty[$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{54+5y}-3}{5}\right)$.

34. Show that the relation R, defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides$\}$, is an equivalence relation on A. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

OR

Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\ R(c, d)$ if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

35. Show that the function $f : [-1, 1] \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{x+2}$ is one-one. Find the range of f. Also find the inverse of the function $f : [-1, 1] \rightarrow$ range of f.

36. Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g : A \rightarrow B$ be functions defined by $f(x) = x^2 - x$, $x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1$, $x \in A$. Find $(gof)(x)$ and hence show that $f = g = gof$.

SOLUTIONS

1. (c) : We have, $f(x) = \frac{x-2}{2-x} = -\frac{x-2}{x-2} = -1$ if $x \neq 2$

$\therefore$ Range of $f = \{-1\}$

2. (a) : $[ho(gof)](x) = ho[g(f(x))]$

$= h[g(x^2)] = h[\tan x^2] = h\left[\tan\frac{\pi}{4}\right] = h[1] = \log 1 = 0$

3. (b) : Given, $\alpha R \beta \Leftrightarrow \alpha \perp \beta$

$\therefore \quad \alpha \perp \beta \Leftrightarrow \beta \perp \alpha \Rightarrow \beta R \alpha$

Hence, R is symmetric.

4. (b) : If $y = f(x)$, then $y = 3x^2 + 7x + 10$

or $\quad 3x^2 + 7x + 10 - y = 0$

Since x is real, $(7)^2 - 4 \cdot 3(10 - y) \geq 0$ *i.e.*, $y \geq \frac{71}{12}$

5. (b) : $f(g(x)) = f(x^2 + 7) = 2(x^2 + 7) + 3 = 25$ (given)

$\Rightarrow \quad 2x^2 = 8 \quad \Rightarrow \quad x = \pm 2$

6. (b) : $R = \{(2, 1), (4, 2), (6, 3),...\}$

So, $R^{-1} = \{(1, 2), (2, 4), (3, 6), ...\}$

7. (d) : As $f(a)$ is not unique, thus f is not a function.

8. (b) : $f(f(x)) = f\left(\frac{x-1}{x+1}\right) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = \frac{-2}{2x} = \frac{-1}{x}$

9. (b) : Let $y = f(x) = \frac{2x-7}{4} \quad \Rightarrow \quad x = \frac{4y+7}{2}$

$\therefore \quad f^{-1}(y) = \frac{4y+7}{2} \Rightarrow f^{-1}(x) = \frac{4x+7}{2}$

10. (b) : Domain of f is the set of those points on which f is defined. Here, $f(1) = 3$, $f(3) = 5$ and $f(2) = 6$. So, $D_f = \{1, 2, 3\}$

11. For $x = 1, y = 2$; $2x^2 - 3xy + y^2 = 0$

$\therefore \quad 1R2$ *i.e.*, $(1, 2) \in R$

But $2 \cdot 2^2 - 3 \cdot 2 \cdot 1 + 1^2 = 3 \neq 0$

So, $(2, 1) \notin R$

$\therefore \quad R$ is not symmetric.

12. Since more than one person may have same height, therefore the function defined by $f(x) = y$, where x is a person and y is his height, is not a one-one function.

13. Given $f(x) = |x|$ and $g(x) = |5x - 2|$

Now $gof(x) = g(f(x)) = g(|x|) = |5|x| - 2|$

$[\because \ g(x) = |5x - 2|]$

14. We have, $R = \{(a, a^3) : a$ is prime less than $5\}$

$\Rightarrow \quad R = \{(a, a^3) : a = 2, 3\}$

$\Rightarrow \quad R = \{(2, 8), (3, 27)\}$

$\therefore \quad$ Range of $R = \{8, 27\}$

15. $f(x) = 2\sin\left(2x - \frac{\pi}{6}\right) + 4$

Since, $\sin x$ is one-one in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \quad -\frac{\pi}{2} \le 2x - \frac{\pi}{6} \le \frac{\pi}{2} \Rightarrow x \in \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$

16. $ho(fog)(x) = hof(\sqrt{x^2+1})$

$= h\{(\sqrt{x^2+1})^2 - 1\} = h\{x^2\} = x^2$

17. Since $3 < 4$, injective functions from A to B are defined and the total number of such functions is

${}^4P_3 = \frac{4!}{(4-3)!} = 24.$

18. We have $R = \{(x, y) : y$ is half of $x\,;\, x, y \in A\}$

$\therefore \quad R = \{(2, 1), (4, 2), (6, 3), (8, 4)\}$

19. Since, f is an invertible function, so $(f^{-1}of)(x) = x$

$\Rightarrow \quad (f^{-1}of)(1) + (f^{-1}of)(2) + \ldots + (f^{-1}of)(100)$

$= 1 + 2 + \ldots + 100$

$= 100 \times \frac{(100+1)}{2} = 5050$

20. $fog(x) = f(g(x)) = f(x - 7) = x - 7 + 7 = x$

$\Rightarrow \quad fog(7) = 7$

21. As $f : \mathbb{R} \to \mathbb{R}$, fof exists and $fof : \mathbb{R} \to \mathbb{R}$ is given by

$(fof)(x) = f(f(x)) = f((3 - x^3)^{1/3})$

$= \left(3 - \left(\left(3 - x^3\right)^{1/3}\right)^3\right)^{1/3}$

$= (3 - (3 - x^3))^{1/3} = (x^3)^{1/3} = x$, for all $x \in R$

$\Rightarrow \quad fof = I_{\mathbb{R}}.$

22. Given, $A = \{1, 2, 3\}$

Since, $a \in A$, $b \in A$, a divides b and b divides a

$\Rightarrow \quad a = b$

$\therefore \quad R = \{(a, a), a \in A\} = \{(1, 1), (2, 2), (3, 3)\}.$

Hence, R is the identity relation on A.

23. Since $f\left(\frac{1}{2}\right) = \frac{1}{2} - \left[\frac{1}{2}\right] = \frac{1}{2} - 0 = \frac{1}{2}$

and $f\left(\frac{3}{2}\right) = \frac{3}{2} - \left[\frac{3}{2}\right] = \frac{3}{2} - 1 = \frac{1}{2}$, so the two different elements $\frac{1}{2}, \frac{3}{2} \in R$ (domain of f) have same image.

$\Rightarrow \quad f$ is not one-one.

Also, range of $f = [0, 1)$, which is a proper subset of $\mathbb{R}$ (codomain of f)

$\Rightarrow \quad f$ is not onto.

OR

As $f(x) = 1$ for all $x > 0 \Rightarrow f(1) = 1$ and $f(2) = 1$.

So, the two different elements $1, 2 \in R$ (domain of f) have same image.

$\Rightarrow \quad f$ is not one-one.

Also, range of $f = \{-1, 0, 1\}$, which is a proper subset of $\mathbb{R}$ (codomain of f)

$\Rightarrow \quad f$ is not onto.

24. Here, $R = \{(a, b) \in A \times A : 2$ divides $(a - b)\}$

This is the given equivalence relation, where

$A = \{0, 1, 2, 3, 4, 5\}$

$\therefore \quad [0] = \{0, 2, 4\}.$

25. Here $R = \{(x, y) : x, y \in N, 2x + y = 41\}$

$\therefore \quad$ Domain $R = \{1, 2, 3, \ldots, 19, 20\}$,

Range $R = \{39, 37, 35, \ldots, 7, 5, 3, 1\}$.

26. We have, $f(x) = [x]$ and $g(x) = |x|$

Clearly, domain $(f) = R$ and domain $(g) = R$. Therefore, each of fog, gof has domain R.

$(gof)\left(\frac{-5}{3}\right) - (fog)\left(\frac{-5}{3}\right) = g\left\{f\left(\frac{-5}{3}\right)\right\} - f\left\{g\left(\frac{-5}{3}\right)\right\}$

$= g\left\{\left[\frac{-5}{3}\right]\right\} - f\left\{\left|-\frac{5}{3}\right|\right\}$

$= g(-2) - f\left(\frac{5}{3}\right) = |-2| - \left[\frac{5}{3}\right] = 2 - 1 = 1$

OR

The relation R on the set A of human beings is defined by $R = \{(x, y) : x$ is wife of $y\}$.

Consider any $x \in A$, then x is not the wife of x.

$\Rightarrow \quad (x, x) \notin R \Rightarrow R$ is not reflexive.

If $x, y \in A$ are such that $(x, y) \in R$ *i.e.*, x is wife of y, then y cannot be wife of x.

$\Rightarrow \quad (y, x) \notin R \Rightarrow R$ is not symmetric.

27. $f(x) = \sin^2 x + \sin^2(x + \pi/3) + \cos x \cos(x + \pi/3)$

$= \frac{1}{2}\left[2\sin^2 x + 2\sin^2\left(x + \frac{\pi}{3}\right) + 2\cos x \cos\left(x + \frac{\pi}{3}\right)\right]$

$= \frac{1}{2}\left[1 - \cos 2x + 1 - \cos\left(2x + \frac{2\pi}{3}\right) + \cos\left(2x + \frac{\pi}{3}\right) + \cos\frac{\pi}{3}\right]$

$= \frac{1}{2}\left[\frac{5}{2} - \cos 2x - \cos\left(2x + \frac{2\pi}{3}\right) + \cos\left(2x + \frac{\pi}{3}\right)\right]$

$= \frac{1}{2}\left[\frac{5}{2} - \left\{\cos(2x) + \cos\left(2x + \frac{2\pi}{3}\right)\right\} + \cos\left(2x + \frac{\pi}{3}\right)\right]$

$$=\frac{1}{2}\left[\frac{5}{2}-2\cos\left(2x+\frac{\pi}{3}\right)\cos\frac{\pi}{3}+\cos\left(2x+\frac{\pi}{3}\right)\right]$$

$$=\frac{1}{2}\left[\frac{5}{2}-\cos\left(2x+\frac{\pi}{3}\right)+\cos\left(2x+\frac{\pi}{3}\right)\right]$$

$$=\frac{5}{4}, \text{ for all } x \in R.$$

Therefore, for any $x \in R$, we have

$$(gof)(x) = g(f(x)) = g\left(\frac{5}{4}\right)=1.$$

Thus, $(gof)(x) = 1$ for all $x \in R$.

Hence, $(gof) : \mathbb{R} \to \mathbb{R}$ is a constant function.

28. We have, $A = \{x \in Z : 0 \le x \le 12\}$

$\therefore \quad A = \{0, 1, 2, 3, ..., 12\}$

and $S = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$

(i) Reflexive : For any $a \in A$,

$|a - a| = 0$, which is a multiple of 4.

Thus, $(a, a) \in S$

$\therefore \quad S$ is reflexive.

(ii) Symmetric : For any $a, b \in A$,

Let $(a, b) \in S$

$\Rightarrow \quad |a - b|$ is a multiple of 4.

$\Rightarrow \quad |b - a|$ is a multiple of 4 $\Rightarrow \quad (b, a) \in S$

i.e., $(a, b) \in S \Rightarrow (b, a) \in S$

$\therefore \quad S$ is symmetric.

(iii) Transitive : For any $a, b, c \in A$,

Let $(a, b) \in S$ and $(b, c) \in S$

$\Rightarrow \quad |a - b|$ is a multiple of 4 and $|b - c|$ is a multiple of 4.

$\Rightarrow \quad a - b = \pm 4k_1$ and $b - c = \pm 4k_2$; $k_1, k_2 \in N$

$\Rightarrow \quad (a - b) + (b - c) = \pm 4 (k_1 + k_2)$; $k_1, k_2 \in N$

$\Rightarrow \quad a - c = \pm 4 (k_1 + k_2)$; $k_1, k_2 \in N$

$\Rightarrow \quad |a - c|$ is a multiple of 4 $\Rightarrow (a , c) \in S$

$\therefore \quad S$ is transitive.

Hence, S is an equivalence relation.

The set of elements related to 1 is $\{5, 9\}$.

OR

Consider any $(a, b), (c, d), (e, f) \in N \times N$.

(i) Since $a + b = b + a$ for all $a, b \in N$

(Commutative law of addition in N)

$\Rightarrow \quad (a, b)\ R\ (a, b) \Rightarrow R$ is reflexive.

(ii) Let $(a, b)\ R\ (c, d) \Rightarrow a + d = b + c \Rightarrow d + a = c + b$

$\Rightarrow \quad c + b = d + a \Rightarrow (c, d)\ R\ (a, b)$

Thus, $(a, b)\ R\ (c, d) \Rightarrow (c, d)\ R\ (a, b) \Rightarrow R$ is symmetric.

(iii) Let $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

$\Rightarrow \quad a + d = b + c$ and $c + f = d + e$

$\Rightarrow \quad a + d + c + f = b + c + d + e$

$\Rightarrow \quad a + f = b + e \Rightarrow (a, b)\ R\ (e, f)$.

Thus, $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f) \Rightarrow (a, b)\ R\ (e, f)$

$\Rightarrow \quad R$ is transitive.

Therefore, the relation R is reflexive, symmetric and transitive, and hence it is an equivalence relation.

29. It is given that f is invertible with f^{-1} as its inverse.

$\therefore \quad (fof^{-1})(x) = x$ for all $x \in [1, \infty)$

$\Rightarrow \quad f(f^{-1}(x)) = x$

$\Rightarrow \quad 2^{f^{-1}(x)(f^{-1}(x)-1)} = x$

$\Rightarrow \quad f^{-1}(x)\ \{f^{-1}(x)-1\} = \log_2 x$

$\Rightarrow \quad \{f^{-1}(x)\}^2 - f^{-1}(x) - \log_2 x = 0$

$$\Rightarrow \quad f^{-1}(x)=\frac{1\pm\sqrt{1+4\log_2 x}}{2}$$

$$\Rightarrow \quad f^{-1}(x)=\frac{1+\sqrt{1+4\log_2 x}}{2}$$

$[\because f^{-1}(x) \in [1, \infty) \therefore f^{-1}(x) \ge 1]$

30. We have, $R = \{(a, b) : a \le b^2\}$, where $a, b \in \mathbb{R}$.

Reflexive : Since $\frac{1}{2} \le \left(\frac{1}{2}\right)^2$ is not true. Therefore, $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$.

So, R is not reflexive.

Symmetric : Since, $-1 \le 3^2$ but $3 \nleq (-1)^2$

i.e., $(-1, 3) \in R$ but $(3, -1) \notin R$.

So, R is not symmetric.

Transitive : Since $2 \le (-3)^2$ and $-3 \le 1^2$ but $2 \nleq 1^2$

i.e., $(2, -3) \in R$ and $(-3, 1) \in R$ but $(2, 1) \notin R$.

So, R is not transitive.

31. Given $A = \mathbb{R} - \{3\}$; $B = \mathbb{R} - \{1\}$

and $f : A \to B$ defined as

$$f(x)=\left(\frac{x-2}{x-3}\right) \forall\ x \in A.$$

Here, f is defined $\forall\ x \in A$, as $3 \notin A$.

Also, $f(x) \ne 1$

$[\because f(x) = 1 \Leftrightarrow \frac{x-2}{x-3} = 1 \Leftrightarrow x - 2 = x - 3$

$\Leftrightarrow -2 = -3$, which is absurd]

Let $x_1, x_2 \in A$ be such that $f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3}$$

$\Rightarrow \quad (x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$

$\Rightarrow \; x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow \; -3x_1 - 2x_2 = -2x_1 - 3x_2 \Rightarrow x_1 = x_2$

$\therefore$ f is a one-one function.

Let $y \in B = \mathbb{R} - \{1\} \Rightarrow y \neq 1$.

We want to solve $y = f(x)$ for some $x \in A$

$$\Leftrightarrow \; y = \frac{x-2}{x-3} \; \Leftrightarrow \; xy - 3y = x - 2$$

$$\Leftrightarrow \; x(y-1) = -2 + 3y$$

$$\Leftrightarrow \; x = \frac{3y-2}{y-1} \in A \text{ (as } y \neq 1)$$

$\therefore$ f is also onto.

$\Rightarrow$ $f : A \to B$ is a bijective function.

OR

Since $f : Z \to Z$ and $g : Z \to Z$.

Therefore, $gof : Z \to Z$ and $fog : Z \to Z$.

For $n \in Z$, $(gof)(n) = g(f(n)) = g(3n) = \frac{3n}{3} = n$

$\therefore$ $(gof)(n) = n$, for all $n \in Z$.

Hence $gof = I_Z$.

For $n \in Z$, $fog(n) = f(g(n))$

$$= \begin{cases} f\left(\dfrac{n}{3}\right), \text{ if } n \text{ is a multiple of } 3 \\ f(0), \text{if } n \text{ is not a multiple of } 3 \end{cases}$$

$$= \begin{cases} 3\left(\dfrac{n}{3}\right), \text{ if } n \text{ is a multiple of } 3 \\ 3(0), \text{if } n \text{ is not a multiple of } 3 \end{cases}$$

$$= \begin{cases} n, \text{ if } n \text{ is a multiple of } 3 \\ 0, \text{if } n \text{ is not a multiple of } 3 \end{cases}$$

Thus, $(fog)(n) = n$ only when n is a multiple of 3.

and $(fog)(n) \neq n$, when n is not a multiple of 3.

Therefore, fog is not an identity function on Z.

Hence, $fog \neq I_Z$.

32. $f : \mathbb{R}^+ \to [4, \infty)$ given by $f(x) = x^2 + 4$

(i) Let $x_1, x_2 \in \mathbb{R}^+$ such that $f(x_1) = f(x_2)$

$\Rightarrow \; x_1^2 + 4 = x_2^2 + 4 \Rightarrow x_1^2 = x_2^2$

$\Rightarrow \; x_1 = x_2 \qquad (\because \; x_1, x_2 \in \mathbb{R}^+)$

$\Rightarrow$ f is one-one.

(ii) $y = f(x) \; \forall \; y \in [4, \infty)$

$\Rightarrow \; x^2 + 4 = y \Rightarrow x = \sqrt{y-4}$

Now, x is defined if $y - 4 \geq 0$

$\Rightarrow \sqrt{y-4} \in \mathbb{R}^+$ and

$$f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = (y-4) + 4 = y$$

$\Rightarrow$ f is onto.

$\therefore$ f is one–one and onto.

$\Rightarrow$ f is invertible and f^{-1} exists.

$\therefore$ $f^{-1}(y) = \sqrt{y-4}$.

33. We have, $f(n) = n - (-1)^n$ for all $n \in N$.

$$\Rightarrow \quad f(n) = \begin{cases} n-1, \text{ if } n \text{ is even} \\ n+1, \text{if } n \text{ is odd} \end{cases}$$

One-one : Let n, m be any two even natural numbers, then $f(n) = f(m) \Rightarrow n - 1 = m - 1 \Rightarrow n = m$

If n, m are any two odd natural numbers,

then $f(n) = f(m) \Rightarrow n + 1 = m + 1 \Rightarrow n = m$.

Thus in both the cases, $f(n) = f(m) \Rightarrow n = m$.

If n is even and m is odd, then $n \neq m$. Also $f(n)$ is odd and $f(m)$ is even. So, $f(n) \neq f(m)$.

Thus, $n \neq m \Rightarrow f(n) \neq f(m)$.

So, f is one-one function.

Onto : Let n be an arbitrary natural number.

If n is an odd natural number, then there exists an even natural number $n + 1$ such that

$f(n + 1) = n + 1 - 1 = n$

If n is an even natural number, then there exists an odd natural number $(n - 1)$ such that

$f(n - 1) = n - 1 + 1 = n$

Thus, every n $(\in N)$ has its pre-image in N.

So, $f : N \to N$ is onto.

Hence, $f : N \to N$ is a bijection.

OR

Refer to answer 44, Page no. 12 (MTG CBSE Champion Mathematics Class-12)

34. Given A = Set of all polygons

$R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides$\}$

(i) Reflexive : Let $P \in A$

Clearly, number of sides of P = number of side of P

$\therefore$ $(P, P) \in R, \forall P \in A$

Hence, R is reflexive.

(ii) Symmetric : Let $P_1, P_2 \in A$

Let $(P_1, P_2) \in R \Rightarrow P_1$ and P_2 have same number of sides

$\Rightarrow P_2$ and P_1 have same number of sides

$\Rightarrow (P_2, P_1) \in A$

Hence, R is symmetric.

(iii) Transitive : Let $P_1, P_2, P_3 \in A$

Let $(P_1, P_2) \in R$ and $(P_2, P_3) \in R$

$$\Rightarrow \begin{Bmatrix} \text{Number of sides of } P_1 = \text{number of sides of } P_2 \\ \text{and number of sides of } P_2 = \text{number of sides of } P_3 \end{Bmatrix}$$

$\Rightarrow$ Number of sides of P_1 = number of sides of P_3

$\Rightarrow$ $(P_1, P_3) \in R$

Hence, R is transitive.

Thus R is reflexive, symmetric and transitive and hence R is an equivalence relation on A.

Given T = a right angle triangle with sides 3, 4, 5

$\therefore$ Number of sides of $T = 3$

Hence all polygons having 3 sides will be R-related to T

$\therefore$ All triangles will be R-related to T

Hence, set of all elements in A which are R-related to T = Set of all triangles.

OR

Refer to answer 15, Page no. 7 (MTG CBSE Champion Mathematics, Class-12)

35. The function f is one-one.

Consider any $x_1, x_2 \in [-1, 1]$ such that $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{x_1+2} = \frac{x_2}{x_2+2} \Rightarrow x_1x_2 + 2x_1 = x_1x_2 + 2x_2$$

$\Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.

For the range of f :

$$\text{Let } y = f(x) \Rightarrow y = \frac{x}{x+2} \Rightarrow y = 1 - \frac{2}{x+2}$$

As domain of $f = [-1, 1]$, $-1 \le x \le 1$

$$\Rightarrow 1 \le x + 2 \le 3 \Rightarrow \frac{1}{1} \ge \frac{1}{x+2} \ge \frac{1}{3}$$

$$\Rightarrow -2 \le -\frac{2}{x+2} \le -\frac{2}{3}$$

$$\Rightarrow -1 \le 1 - \frac{2}{x+2} \le \frac{1}{3} \Rightarrow -1 \le y \le \frac{1}{3} \Rightarrow y \in \left[-1, \frac{1}{3}\right].$$

Hence, the range of $f = \left[-1, \frac{1}{3}\right]$.

Further, the function $f : [-1, 1] \to$ range of f is onto. Also f is one-one, therefore, its inverse exists.

To find f^{-1}:

$$\text{Let } f(x) = y \Rightarrow \frac{x}{x+2} = y \Rightarrow xy + 2y = x$$

$$\Rightarrow 2y = x(1-y) \Rightarrow x = \frac{2y}{1-y}.$$

Now $f(x) = y$ and f is invertible

$\Rightarrow f^{-1}(y) = x$

$$\Rightarrow f^{-1}(y) = \frac{2y}{1-y}.$$

Thus, the inverse of the function $f : [-1, 1] \to$ range of f is given by $f^{-1}(y) = \dfrac{2y}{1-y}$.

36. Given $f(x) = x^2 - x$, $x \in A$ *i.e.*, $x = -1, 0, 1, 2$.

$f(-1) = (-1)^2 - (-1) = 1 + 1 = 2$, $f(0) = 0^2 - 0 = 0$,

$f(1) = 1^2 - 1 = 0$, $f(2) = 2^2 - 2 = 2$

$\Rightarrow R_f = \{0, 2\}$

We note that $R_f = \{0, 2\} \subset A = D_f$

$\Rightarrow D_{gof} = D_f = A$.

$\therefore$ gof is defined with domain A and

$$(gof)(-1) = g(f(-1)) = g(2) = 2\left|2 - \frac{1}{2}\right| - 1 = 2 \times \frac{3}{2} - 1 = 2,$$

$$(gof)(0) = g(f(0)) = g(0) = 2\left|0 - \frac{1}{2}\right| - 1 = 2 \times \frac{1}{2} - 1 = 0,$$

$(gof)(1) = g(f(1)) = g(0) = 0$,

$(gof)(2) = g(f(2)) = g(2) = 2$.

Also, $g(x) = 2\left|x - \frac{1}{2}\right| - 1$, $x \in A$ *i.e.*, $x = -1, 0, 1, 2$.

$$g(-1) = 2\left|-1 - \frac{1}{2}\right| - 1 = 2 \times \frac{3}{2} - 1 = 2,$$

$$g(0) = 2\left|0 - \frac{1}{2}\right| - 1 = 2 \times \frac{1}{2} - 1 = 0,$$

$$g(1) = 2\left|1 - \frac{1}{2}\right| - 1 = 2 \times \frac{1}{2} - 1 = 0,$$

$$g(2) = 2\left|2 - \frac{1}{2}\right| - 1 = 2 \times \frac{3}{2} - 1 = 2.$$

We find that $D_f = D_g = D_{gof} = A$ and

$f(x) = g(x) = (gof)(x)$ for all $x \in A$,

$\therefore$ $f = g = gof$.

CBSE warm-up!

CLASS-XI

Synopsis and Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 1 Sets, Relations and Functions

Time Allowed : 3 hours
Maximum Marks : 80

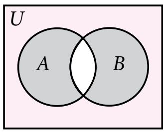
GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) This question paper contains **36** questions.
(iii) Question **1**-**20** in Section-A are very short-answer-objective type questions carrying **1** mark each.
(iv) Question **21**-**26** in Section-B are short-answer type questions carrying **2** marks each.
(v) Question **27**-**32** in Section-C are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **33**-**36** in Section-D are long-answer-**II** type questions carrying **6** marks each.

SECTION - A

1. Let $A = \{(x, y) : y = e^{2x}, \forall\ x \in R\}$ and $B = \{(x, y) : y = e^{-2x}, \forall\ x \in R\}$, then $A \cap B$ is
(a) not a set (b) singleton set
(c) empty set (d) none of these

2. The shaded region in the figure represents
(a) $A \cap B$
(b) $A \cup B$
(c) $B - A$
(d) $(A - B) \cup (B - A)$



3. If $f(x) = e^x$, then $\frac{f(a)}{f(-b)}$ is equal to
(a) $f(a + b)$ (b) $f(a - b)$
(c) $f(-a + b)$ (d) $f(-a - b)$

4. Let $A = \{x : x^2 - 5x + 6 = 0\}$, $B = \{2, 4\}$, $C = \{4, 5\}$, then $A \times (B \cap C)$ is
(a) $\{(2, 4), (3, 4)\}$
(b) $\{(4, 2), (4, 3)\}$
(c) $\{(2, 4), (3, 4), (4, 4)\}$
(d) $\{(2, 2), (3, 3), (4, 4), (5, 5)\}$

5. Let $n(A) = 6$ and $n(B) = p$. Then, the total number of non-empty relations that can be defined from A to B is
(a) 6^p (b) $n^p - 1$
(c) $6p - 1$ (d) $2^{6p} - 1$

6. If $A = \{1, 3, 5, 6\}$, then number of elements in $P\{P(A)\}$ is
(a) 2^{14} (b) 2^{16} (c) 2^{10} (d) 2^8

7. Let $A = \{x \in N : x$ is a multiple of 3$\}$ and $B = \{x \in N : x$ is a multiple of 6$\}$, then $A - B$ is equal to
(a) $\{6, 12, 18, \ldots\}$ (b) $\{3, 6, 9, 12, \ldots\}$
(c) $\{3, 9, 15, 21, \ldots\}$ (d) none of these

8. If $f(x) = 9x - x^2$, $x \in R$, then $f(a + 1) - f(a - 1)$ is equal to
(a) $4(4 - a)$ (b) $2(9 - 2a)$
(c) $4(2 + a)$ (d) $2(4 + a)$

9. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is
(a) $(-\infty, 0)$ (b) $(-\infty, \infty) - \{0\}$
(c) $(-\infty, \infty)$ (d) $(0, \infty)$

10. If A and B are two sets, then $(A \cup B')' \cap (A' \cup B)'$ is
(a) null set (b) universal set
(c) A' (d) B'

11. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 2^x$, then find range of f.

12. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 4, 6\}$. We define xRy if $x < y$. Is R a relation from A to B?

13. Describe the set $\{x : x \in Z \text{ and } |x| \le 2\}$ in roster form.

14. If $f(x) = x + 1$ and $g(x) = 2x - 3$ be two real functions, find the value of $f^2 - 3g$.

15. If $a \in N$ such that $aN = \{ax : x \in N\}$. Describe the set $3N \cap 7N$.

16. Find the domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

17. If $A = \{1, 2, 3, 4\}$ and $x, y \in A$, then form the set of all ordered pairs (x, y) such that x is a divisor of y.

18. If $A \subseteq B$, then find the value of $B' - A'$.

19. If $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 3, 5, 6, 7, 8, 9\}$, then show that $A - B \ne B - A$.

20. Find the range of the function $f(x) = \dfrac{|x-4|}{x-4}$.

SECTION - B

21. State which of the following sets are finite or infinite.
(i) $A = \{x : x \in N \text{ and } x \text{ is odd}\}$
(ii) $B = \{x : x \text{ is a month of a year having less than 31 days}\}$.

22. Find $f(x) + f(1 - x)$, if $f(x) = \dfrac{a^x}{a^x + \sqrt{a}}$.

OR

Let f be defined by $f(x) = x - 4$ and g be defined by

$$g(x) = \begin{cases} \dfrac{x^2-16}{x+4} & , \quad x \ne -4 \\ \lambda & , \quad x = -4 \end{cases}.$$

Find λ such that $f(x) = g(x)$ for all $x \in \mathbb{R}$.

23. If A and B are two non-empty sets having n elements in common, then prove that $A \times B$ and $B \times A$ have n^2 elements in common.

24. If $A = \{-1, 1\}$, then form the set $A \times A \times A$.

OR

If $R = \{(x, y) : x, y \in Z, x^2 + y^2 = 64\}$, then write R in roster form.

mcq's MEMORY CONTEST

1 Who can participate

If you have taken any of the exams given below and possess plenty of grey cells, photographic memory then you are the right candidate for this contest. All you have to do is write down as many questions (with all choices) you can remember, neatly on a paper with name of the exam, your name, address, age, your photograph and mail them to us.

2 The Exams

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Engineering: VITEEE, UPSEE, SRMJEEE, BITSAT, COMED-K.... etc.

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4 And Lastly The Pitfalls

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- Kindly note that each question should be complete.
- Payment will be made only for complete questions.
- Preference will be given to the reader sending the maximum complete and correct questions. Other conditions apply. The decision of the Editor, MTG shall be final and binding.

25. If $f(x)=\dfrac{|x|}{x}$, $x\neq 0$ prove that $|f(\alpha)-f(-\alpha)|=2,\ \alpha\neq 0$.

26. If $A=\{0\}$, $B=\{x:x<5 \text{ and } x>15\}$, $C=\{x:x-5=0\}$, $D=\{x:x^2=25\}$, $E=\{x:x$ is a positive integral root of the equation $x^2-2x-15=0\}$, then find all pairs of equal sets (if any).

SECTION - C

27. Find the domain and the range of the relation R given by $R=\{(x,y):y=x+\dfrac{6}{x}$, where $x,y\in N$ and $x<6\}$.

28. If $A=\{1,3,5,\ldots,17\}$ and $B=\{2,4,6,\ldots,18\}$ and N the set of natural numbers is the universal set, then show that $A'\cup((A\cup B)\cap B')=N$.

29. Let $f=\left\{\left(x,\dfrac{x^2}{1+x^2}\right),x\in\mathbb{R}\right\}$ be a function from $\mathbb{R}$ into $\mathbb{R}$. Determine the range of f.

OR

Let $A=\{-3,-2,-1,4\}$ and $f:A\to Z$ given by $f(x)=x^2+x+2$. Find
(a) the range of f
(b) pre-images of 6 and 4.

30. Let R be a relation from Q to Q defined by $R=\{(a,b):a,b\in Q$ and $a-b\in Z\}$. Show that
(a) $(a,a)\in R$ for all $a\in Q$.
(b) $(a,b)\in R$ implies that $(b,a)\in R$
(c) $(a,b)\in R$ and $(b,c)\in R$ implies that $(a,c)\in R$

31. If $f(x)=\dfrac{1+x}{1-x}$, prove that $\dfrac{f(x)\cdot f(x^2)}{1+[f(x)]^2}=\dfrac{1}{2}$.

32. Two finite sets A and B have m and k elements respectively. If the ratio of total number of subsets of A to total number of subsets of B is 64 : 1 and $n(A)+n(B)=12$, then find the values of m and k.

OR

Let $U=\{1,2,3,4,5,6,7,8,9\}$, $A=\{1,2,3,4\}$, $B=\{2,4,6,8\}$ and $C=\{3,4,5,6\}$.
Verify : $A-(B\cup C)'=(A\cap B)\cup(A\cap C)$

SECTION - D

33. If $f(x)=e^x$ and $g(x)=\log x$, then find the following functions:
(i) $f+g$ (ii) $f-g$ (iii) fg (iv) $\dfrac{f}{g}$
(v) $\dfrac{1}{f}$ (vi) f^2.

OR

Find the domain of the function $f(x)$ given by $f(x)=\log_4\{\log_5(\log_3(18x-x^2-77))\}$.

34. Let A be the set of first ten natural numbers and let R be a relation on A defined by $(x,y)\in R\Leftrightarrow x+2y=10$, *i.e.*, $R=\{(x,y):x\in A,\ y\in A$ and $x+2y=10\}$. Express R as sets of ordered pairs. Also, determine (i) domain of R (ii) range of R.

35. If $U=\{x:x\in N,\ x\le 30\}$, $A=\{x:x$ is prime $<5\}$, $B=\{x:x$ is a perfect square $\le 10\}$ and $C=\{x:x$ is a perfect cube $\le 30\}$, then show that
(i) $(A\cup B)'=A'\cap B'$
(ii) $(A\cap B)'=A'\cup B'$
(iii) $(A\cap B)\cap C=A\cap(B\cap C)$

OR

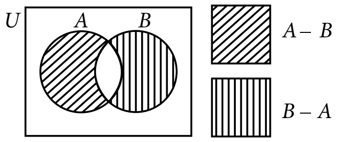
In a group of 50 people, 30 like to play cricket, 25 like to play football and 32 like to play hockey. Assume that each person in the group likes to play atleast one of the three games. If 15 people like to play both cricket and football, 11 like to play football and hockey and 18 like to play cricket and hockey. Find
(i) How many like to play all three games?
(ii) How many like to play only football?
(iii) How many like to play only hockey?
(iv) How many like to play exactly one game?

36. Find the domain and range of the following real valued functions :
(a) $f(x)=\dfrac{1}{\sqrt{x^2-81}}$ (b) $f(x)=\dfrac{2x}{6-x}$

SOLUTIONS

1. (b) : Given $A=\{(x,y):y=e^{2x},\ \forall x\in R\}$
and $B=\{(x,y):y=e^{-2x},\ \forall x\in R\}$
$\Rightarrow\ e^{2x}=\dfrac{1}{e^{2x}}\ \Rightarrow e^{4x}=1\ \Rightarrow\ e^{4x}=e^0$
$\Rightarrow\ x=0$ and $y=1$
As e^{2x} and e^{-2x} both intersect each other at (0, 1), so $A\cap B$ is a singleton set.

2. (d) :



$\Rightarrow$ Shaded part represents $(A-B)\cup(B-A)$.

3. (a) : $\dfrac{f(a)}{f(-b)}=\dfrac{e^a}{e^{-b}}=e^ae^b=e^{a+b}=f(a+b)$

4. **(a) :** $A = \{x : x^2 - 5x + 6 = 0\}$

$x^2 - 5x + 6 = 0 \Rightarrow (x - 3)(x - 2) = 0 \Rightarrow x = 3, 2$

$\therefore\ A = \{2, 3\}, B = \{2, 4\}, C = \{4, 5\}$

Now, $B \cap C = \{4\}$

$\Rightarrow\ A \times (B \cap C) = \{2, 3\} \times \{4\} = \{(2, 4), (3, 4)\}$.

5. **(d) :** Given $n(A) = 6$ and $n(B) = p$

$\therefore$ Total number of relations from A to $B = 2^{6p}$

$\therefore$ Total number of non-empty relations from A to $B = 2^{6p} - 1$

6. **(b) :** Given, $A = \{1, 3, 5, 6\}$

$\therefore\ n\{P(A)\} = 2^4 = 16 \ \Rightarrow\ n[P\{P(A)\}] = 2^{16}$

7. **(c) :** $A = \{3, 6, 9, 12, ...\}$, $B = \{6, 12, 18, 24, ...\}$

$\therefore\ A - B = \{3, 9, 15, 21, ...\}$.

8. **(b) :** $f(a + 1) - f(a - 1)$

$= [9(a + 1) - (a + 1)^2] - [9(a - 1) - (a - 1)^2]$

$= 18 - 4a = 2(9 - 2a)$.

9. **(a) :** We have, $f(x) = \dfrac{1}{\sqrt{|x| - x}}$

Now, $|x| - x > 0 \ \Rightarrow\ |x| > x$

Thus, x must be negative $\ \Rightarrow\ x \in (-\infty, 0)$.

10. **(a) :** Consider $(A \cup B')' \cap (A' \cup B)'$

$= (A' \cap B) \cap (A \cap B') = (B - A) \cap (A - B) = \phi$

11. Given $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 2^x$.

Since $D_f = \mathbb{R}$ and for all $x \in \mathbb{R}$, $2^x > 0$

$\therefore$ Range of $f = (0, \infty)$.

12. We have, xRy if $x < y$

$\therefore\ R = \{(1, 2), (1, 4), (1, 6), (2, 4), (2, 6), (3, 4), (3, 6), (4, 6), (5, 6)\}$

Here, $R \subseteq A \times B$

$\therefore$ R is a relation from A to B.

13. Here x is an integer satisfying $|x| \le 2$.

$\Rightarrow\ |x| = 0, 1, 2 \ \Rightarrow\ x = 0, \pm 1, \pm 2$

So, x can take values $-2, -1, 0, 1, 2$.

$\therefore\ \{x : x \in Z \text{ and } |x| \le 2\} = \{-2, -1, 0, 1, 2\}$.

14. $(f^2 - 3g)(x) = (f^2)(x) - (3g)(x) = (f(x))^2 - 3g(x)$

$= (x + 1)^2 - 3(2x - 3) = x^2 + 2x + 1 - 6x + 9$

$= x^2 - 4x + 10$, for all $x \in R$.

15. We have, $aN = \{ax : x \in N\}$

$\therefore\ 3N = \{3x : x \in N\} = \{3, 6, 9, 12, \ldots\}$

and $7N = \{7x : x \in N\} = \{7, 14, 21, 28, \ldots\}$

Hence, $3N \cap 7N = \{21, 42, \ldots\} = \{21x : x \in N\} = 21N$.

16. The values of x for which $f(x)$ and $g(x)$ are equal are given by $f(x) = g(x)$

$\Rightarrow\ 2x^2 - 1 = 1 - 3x \ \Rightarrow\ 2x^2 + 3x - 2 = 0$

$\Rightarrow\ (x + 2)(2x - 1) = 0 \ \Rightarrow\ x = -2, 1/2$.

Thus, $f(x)$ and $g(x)$ are equal on the set $\{-2, 1/2\}$.

17. Given $A = \{1, 2, 3, 4\}$ and $x, y \in A$.

The set of all ordered pairs (x, y) such that x is a divisor of $y = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)\}$.

18. Let $x \in B' \Rightarrow x \notin B \Rightarrow x \notin A \Rightarrow x \in A'$

$\Rightarrow\ B' \subseteq A' \Rightarrow B' - A' = \phi$.

19. Here $A - B = \{2, 4\}$ and $B - A = \{9\}$

$\therefore\ A - B \ne B - A$.

20. We have, $f(x) = \dfrac{|x - 4|}{x - 4}$

Clearly, $f(x)$ is defined for all $x \in \mathbb{R}$ except at $x = 4$.

$$\text{Now, } f(x) = \begin{cases} \dfrac{x - 4}{x - 4} = 1 & , \ \text{if } x > 4 \\[2mm] \dfrac{-(x - 4)}{x - 4} = -1 & , \ \text{if } x < 4 \end{cases}$$

Hence, Range$(f) = \{-1, 1\}$.

21. (i) $A = \{x : x \in N \text{ and } x \text{ is odd}\} = \{1, 3, 5, 7, 9, 11, \ldots\}$.

Since odd numbers are infinite in numbers, so A is an infinite set.

(ii) $B = \{x : x$ is a month of a year having less than 31 days$\}$, *i.e.*, {February, April, June, September, November}.

$\therefore$ B is a finite set.

22. Given, $f(x) = \dfrac{a^x}{a^x + \sqrt{a}}$ ...(i)

$$\therefore \quad f(1-x) = \frac{a^{1-x}}{a^{1-x}+\sqrt{a}} = \frac{a \cdot a^{-x}}{a \cdot a^{-x}+\sqrt{a}}$$

$$= \frac{a}{a+\sqrt{a} \cdot a^x} = \frac{\sqrt{a}}{\sqrt{a}+a^x} \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$f(x) + f(1-x) = \frac{a^x}{a^x+\sqrt{a}} + \frac{\sqrt{a}}{\sqrt{a}+a^x} = \frac{a^x+\sqrt{a}}{a^x+\sqrt{a}} = 1$$

OR

We have, $f(x) = g(x)$ for all $x \in R$

$\Rightarrow$ $f(-4) = g(-4)$

$\Rightarrow$ $-4 - 4 = \lambda$

$\Rightarrow$ $\lambda = -8$.

23. Let $C = A \cap B$.

$\therefore$ $C \times C = (A \cap B) \times (A \cap B) = (A \cap B) \times (B \cap A)$
$= (A \times B) \cap (B \times A)$

Since $A \cap B$ has n elements, so C has n elements.

Hence $C \times C$ has n^2 elements.

$\therefore$ $(A \times B) \cap (B \times A)$ has n^2 elements.

Hence $A \times B$ and $B \times A$ has n^2 elements in common.

24. Here $A = \{-1, 1\}$

$\therefore$ $A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$.

OR

Here $R = \{(x, y) : x, y \in Z, x^2 + y^2 = 64\}$.

When $x = 0, y = 8, -8$;

When $x = 8, y = 0$; when $x = -8, y = 0$.

For all other values of $x \in Z$, we do not get $y \in Z$.

$\therefore$ $R = \{(0, 8), (0, -8), (8, 0), (-8, 0)\}$.

25. $f(\alpha) - f(-\alpha) = \dfrac{|\alpha|}{\alpha} - \dfrac{|-\alpha|}{-\alpha} = \dfrac{|\alpha|}{\alpha} + \dfrac{|-\alpha|}{\alpha}$

$$= \frac{|\alpha|}{\alpha} + \frac{|\alpha|}{\alpha} = 2\frac{|\alpha|}{\alpha} = \begin{cases} 2, & \alpha > 0 \\ -2, & \alpha < 0 \end{cases}$$

Thus, $f(\alpha) - f(-\alpha) = \pm 2$

$\therefore$ $|f(\alpha) - f(-\alpha)| = |\pm 2| = 2$.

26. The given sets are:

$A = \{0\}, B = \phi, C = \{5\}, D = \{5, -5\}, E = \{5\}$

$(\because\ x^2 - 2x - 15 = 0 \Rightarrow (x-5)(x+3) = 0 \Rightarrow x = 5, -3)$

Thus, the only pair of equal sets is C and E.

27. Given $y = x + \dfrac{6}{x}$, $x, y \in N$ and $x < 6$.

When $x = 1$, $y = 1 + \dfrac{6}{1} = 7$ and $7 \in N$, so $(1, 7) \in R$,

When $x = 2$, $y = 2 + \dfrac{6}{2} = 5$ and $5 \in N$, so $(2, 5) \in R$,

When $x = 3$, $y = 3 + \dfrac{6}{3} = 5$ and $5 \in N$, so $(3, 5) \in R$,

When $x = 4$, $y = 4 + \dfrac{6}{4} \notin N$, and

When $x = 5$, $y = 5 + \dfrac{6}{5} \notin N$

$\Rightarrow$ $R = \{(1, 7), (2, 5), (3, 5)\}$.

$\therefore$ Domain of $R = \{1, 2, 3\}$ and range of $R = \{5, 7\}$.

28. Given $A = \{1, 3, 5, \ldots, 17\}$ and $B = \{2, 4, 6, \ldots, 18\}$,

$\therefore$ $A \cap B = \phi$ ...(i)

Now, $A' \cup ((A \cup B) \cap B') = A' \cup ((A \cap B') \cup (B \cap B'))$ (Distributive law)

$= A' \cup ((A \cap B') \cup \phi)$ $(\because\ B \cap B' = \phi)$

$= A' \cup (A \cap B')$ $(\because\ A \cup \phi = A)$

$= (A' \cup A) \cap (A' \cup B')$ (Distributive law)

$= N \cap (A \cap B)'$ (De-Morgan's law)

$= N \cap \phi'$ [Using (i)]

$= N \cap N$

$= N$

29. Let $f = \left\{\left(x, \dfrac{x^2}{x^2+1}\right), x \in \mathbb{R}\right\}$

We know that $f(x) = \dfrac{x^2}{x^2+1}$ is defined for all $x \in \mathbb{R}$

as $x^2 + 1 \neq 0$ for any $x \in \mathbb{R}$.

So, domain of $f(x)$ is real number.

Let $f(x) = y$

$$\Rightarrow \quad \frac{x^2}{x^2+1} = y \Rightarrow x = \pm\sqrt{\frac{y}{1-y}}$$

$\therefore$ x will be defined when $\dfrac{y}{1-y} \geq 0$

i.e., $0 \leq y < 1 \Rightarrow y \in [0, 1)$

Hence, range of $f = [0, 1)$

OR

Refer to answer 87, page no. 40 (MTG CBSE Champion Mathematics Class 11)

30. We have, $R = \{(a, b) : a, b \in Q$ and $a - b \in Z\}$

(a) Let $a \in Q$. Then $a - a = 0 \in Z$.

$\therefore$ $(a, a) \in R\ \forall\ a \in Q$.

(b) $(a, b) \in R \Rightarrow (a - b) \in Z$

$\Rightarrow$ $-(a - b) \in Z\ (\because (-1) \in Z) \Rightarrow b - a \in Z$

$\Rightarrow\ (b, a) \in R$

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$

(c) Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow\ a - b \in Z$ and $b - c \in Z \Rightarrow \{(a - b) + (b - c)\} \in Z$

$\Rightarrow\ (a - c) \in Z \Rightarrow (a, c) \in R$

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

31. Given $f(x)=\frac{1+x}{1-x} \Rightarrow f(x^2)=\frac{1+x^2}{1-x^2}$

Now, $\frac{f(x)\cdot f(x^2)}{1+[f(x)]^2}=\frac{\left(\frac{1+x}{1-x}\right)\left(\frac{1+x^2}{1-x^2}\right)}{1+\left(\frac{1+x}{1-x}\right)^2}$

$$=\frac{\frac{1+x^2}{(1-x)^2}}{\frac{(1-x)^2+(1+x)^2}{(1-x)^2}}=\frac{1+x^2}{2(1+x^2)}=\frac{1}{2}.$$

32. Given, $n(A) = m$ and $n(B) = k$

$\Rightarrow\ n(P(A)) = 2^m$ and $n(P(B)) = 2^k$

Ratio of $n(P(A))$ to $n(P(B)) = 64 : 1$ [Given]

$\therefore\ \frac{2^m}{2^k}=\frac{64}{1} \Rightarrow 2^{m-k} = 64$

$\Rightarrow\ 2^{m-k} = 2^6 \Rightarrow m - k = 6$ ...(i)

Also, $n(A) + n(B) = 12 \Rightarrow m + k = 12$ ...(ii)

Solving (i) and (ii) simultaneously, we get $m = 9$ and $k = 3$.

OR

We have, $B \cup C = \{2, 4, 6, 8\} \cup \{3, 4, 5, 6\}$
$= \{2, 3, 4, 5, 6, 8\}$ ...(i)

$A \cap B = \{1, 2, 3, 4\} \cap \{2, 4, 6, 8\} = \{2, 4\}$ ...(ii)

$A \cap C = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\} = \{3, 4\}$ ...(iii)

L.H.S. $= A - (B \cup C)'$

$= \{1, 2, 3, 4\} - \{2, 3, 4, 5, 6, 8\}'$ (From (i))

$= \{1, 2, 3, 4\} - \{1, 7, 9\}$

$= \{2, 3, 4\}$ ...(iv)

R.H.S. $= (A \cap B) \cup (A \cap C)$

$= \{2, 4\} \cup \{3, 4\}$ (From (ii) and (iii))

$= \{2, 3, 4\}$ ...(v)

From (iv) and (v), we get L.H.S. = R.H.S.

33. Given $f(x) = e^x$ and $g(x) = \log x$

$D_f = \mathbb{R}$ and $D_g = \mathbb{R}^+$, where $\mathbb{R}^+$ is the set of positive reals.

Let $D = D_f \cap D_g = \mathbb{R} \cap \mathbb{R}^+ = \mathbb{R}^+$

(i) $(f + g)(x) = f(x) + g(x) = e^x + \log x$, with domain $\mathbb{R}^+$.

(ii) $(f - g)(x) = f(x) - g(x) = e^x - \log x$, with domain $\mathbb{R}^+$.

(iii) $(fg)(x) = f(x)\,g(x) = e^x \log x$, with domain $\mathbb{R}^+$.

(iv) $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{e^x}{\log x}$, with domain D_1, where $D_1 = \mathbb{R}^+ - \{1\}$.

(v) $\left(\frac{1}{f}\right)(x)=\frac{1}{f(x)}=\frac{1}{e^x}$, with domain $\mathbb{R}$.

(vi) $(f^2)(x) = (f(x))^2 = (e^x)^2 = e^{2x}$, with domain $D_f = \mathbb{R}$.

OR

We have, $f(x) = \log_4\{\log_5(\log_3(18x - x^2 - 77))\}$

Since $\log_a x$ is defined for all $x > 0$.

$\therefore\ f(x)$ is defined if

$\log_5\{\log_3(18x - x^2 - 77)\} > 0$ and $18x - x^2 - 77 > 0$

$\Rightarrow\ \log_3(18x - x^2 - 77) > 5^0$ and $x^2 - 18x + 77 < 0$

$\Rightarrow\ \log_3(18x - x^2 - 77) > 1$ and $(x - 11)(x - 7) < 0$

$\Rightarrow\ 18x - x^2 - 77 > 3^1$ and $7 < x < 11$

$\Rightarrow\ 18x - x^2 - 80 > 0$ and $7 < x < 11$

$\Rightarrow\ x^2 - 18x + 80 < 0$ and $7 < x < 11$

$\Rightarrow\ (x - 10)(x - 8) < 0$ and $7 < x < 11$

$\Rightarrow\ 8 < x < 10$ and $7 < x < 11 \Rightarrow 8 < x < 10$

$\Rightarrow\ x \in (8, 10)$.

Hence, the domain of $f(x)$ is $(8, 10)$.

34. We have, $(x, y) \in R$

$\Leftrightarrow\ x + 2y = 10 \Leftrightarrow y = \frac{10-x}{2}$, $x, y \in A$,

where $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Now, $x = 1 \Rightarrow y = \frac{10-1}{2}=\frac{9}{2}\notin A$.

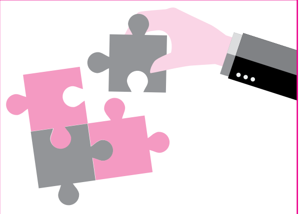
This shows that 1 is not related to any element in A. Similarly, 3, 5, 7, 9 and 10 are not related to any element of A under the defined relation.

For $x = 2$, $y = \frac{10-2}{2} = 4 \in A$. $\therefore\ (2, 4) \in R$

For $x = 4$, $y = \frac{10-4}{2} = 3 \in A$. $\therefore\ (4, 3) \in R$

PUZZLE CORNER

ANSWER · APRIL 2019



8+ 3	5	3× 1	12+ 2	6	4
7+ 5	72× 6	3	1	120× 4	2÷ 2
2	3	4	6	5	1
12× 1	2	6	12+ 4	3	5
15+ 4	1	3– 5	2– 3	9+ 2	6
6	4	2	5	1	3 3

For $x = 6, y = \dfrac{10-6}{2} = 2 \in A.$ $\therefore (6, 2) \in R$

For $x = 8, y = \dfrac{10-8}{2} = 1 \in A.$ $\therefore (8, 1) \in R$

Thus, $R = \{(2, 4), (4, 3), (6, 2), (8, 1)\}$
Clearly, $\text{Dom}(R) = \{2, 4, 6, 8\}$
and, Range $(R) = \{4, 3, 2, 1\}$

35. Given, $U = \{1, 2, 3, \ldots, 30\}$, $A = \{2, 3\}$, $B = \{1, 4, 9\}$ and $C = \{1, 8, 27\}$.

(i) $A \cup B = \{1, 2, 3, 4, 9\}$

$\Rightarrow$ $(A \cup B)' = \{5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$

Now, $A' = \{1, 4, 5, 6, \ldots, 30\}$ and

$B' = \{2, 3, 5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$

$\Rightarrow$ $A' \cap B' = \{5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$.

$\therefore$ $(A \cup B)' = A' \cap B'$

(ii) $A \cap B = \{2, 3\} \cap \{1, 4, 9\} = \phi \Rightarrow (A \cap B)' = U$

$A' \cup B' = \{1, 4, 5, 6, \ldots, 30\} \cup \{2, 3, 5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$

$= \{1, 2, 3, 4, \ldots, 30\} = U$

$\therefore$ $(A \cap B)' = A' \cup B'$

(iii) $A \cap B = \{2, 3\} \cap \{1, 4, 9\} = \phi$

$\Rightarrow$ $(A \cap B) \cap C = \phi \cap C = \phi \cap \{1, 8, 27\} = \phi$

Now, $B \cap C = \{1, 4, 9\} \cap \{1, 8, 27\} = \{1\}$

$\Rightarrow$ $A \cap (B \cap C) = \{2, 3\} \cap \{1\} = \phi$

$\therefore$ $(A \cap B) \cap C = A \cap (B \cap C)$

OR

Let C, F and H be the set of persons who like to play cricket, football and hockey respectively.

$n(C) = 30$, $n(F) = 25$, $n(H) = 32$, $n(C \cup F \cup H) = 50$
$n(C \cap F) = 15$, $n(F \cap H) = 11$, $n(C \cap H) = 18$

(i) Number of persons who like to play all three games
$= n(C \cap F \cap H) = n(C \cup F \cup H) - n(C) - n(F) - n(H) + n(C \cap F) + n(F \cap H) + n(C \cap H)$
$= 50 - 30 - 25 - 32 + 15 + 11 + 18 = 7$

(ii) Number of persons who like to play only football
$= n(F) - n(F \cap C) - n(F \cap H) + n(C \cap F \cap H)$
$= 25 - 15 - 11 + 7 = 6$

(iii) Number of persons who like to play only hockey
$= n(H) - n(C \cap H) - n(F \cap H) + n(C \cap F \cap H)$
$= 32 - 18 - 11 + 7 = 10$

(iv) Number of persons who like to play exactly one game $= n(C \cup F \cup H) - n(C \cap F) - n(C \cap H) - n(F \cap H) + 2n(C \cap F \cap H)$

$= 50 - 15 - 18 - 11 + 2(7) = 20$

36. (a) Here, $f(x) = \dfrac{1}{\sqrt{x^2 - 81}}$

$f(x)$ is defined for $x^2 - 81 > 0$

$\Rightarrow$ $x^2 > 81 \Rightarrow x < -9, x > 9$

$\therefore$ Domain of f is $(-\infty, -9) \cup (9, \infty)$

Let $y = f(x)$

$$\Rightarrow y = \frac{1}{\sqrt{x^2 - 81}} \Rightarrow y^2 = \frac{1}{x^2 - 81}$$

$$\Rightarrow x^2 - 81 = \frac{1}{y^2} \Rightarrow x = \pm\sqrt{81 + \frac{1}{y^2}}$$

$\therefore$ Range of $f = \mathbb{R} - \{0\}$

(b) Here, $f(x) = \dfrac{2x}{6 - x}$

$f(x)$ is not defined for $6 - x = 0 \Rightarrow x = 6$

$\therefore$ Domain of $f = \mathbb{R} - \{6\}$

Let $y = f(x) \Rightarrow y = \dfrac{2x}{6 - x} \Rightarrow 6y - xy = 2x$

$\Rightarrow 6y = 2x + xy \Rightarrow x(2 + y) = 6y \Rightarrow x = \dfrac{6y}{2 + y}$

$\Rightarrow$ Range of f is $\mathbb{R} - \{-2\}$.

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

			4			9									2					1
	8	3											9				8			
	9	4	2		7	6								2			9			4
			9			8	7						5		9					7
								5						8	5		6		2	
7	6				3			2								3	2		4	
	2	5			8	1			5				3			7				
			6	4			8				1	7			4			3		
		8		1		7							1	9			3			6
									6											
								6			2		5	8						
							9		3	1			6							
	7				6	4		2								4		7		2
	9		7		5		3		9	5					9		5			
1				3						6	3								8	
		2		4	1			5					5			6				3
		8										4		9						1
			5			2	9					3	2						4	
4		6						7						2			8	6		
					2	5		6						4		9				7
					3			9					3				6			5

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

MATHS MUSING

SOLUTION SET-196

1. (a) : $S = \frac{1}{19!} + \frac{1}{3!17!} + \frac{1}{5!15!} + \ldots$ to 10 terms

$\Rightarrow \quad S = \frac{1}{20!}\left\{\frac{20!}{1!19!} + \frac{20!}{3!17!} + \frac{20!}{5!15!} + \ldots \text{ to 10 terms}\right\}$

$\Rightarrow \quad S = \frac{1}{20!}\left\{{}^{20}C_1 + {}^{20}C_3 + {}^{20}C_5 + \ldots + {}^{20}C_{19}\right\},$

$\Rightarrow \quad S = \frac{1}{20!}\left\{2^{20-1}\right\} = \frac{2^{19}}{20!}$

2. (d) : We have, $\begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow \quad a + 8b + 7c = 0, 9a + 2b + 3c = 0, 7a + 7b + 7c = 0$

$\Rightarrow \quad a = 1, b = 6, c = -7$

Clearly, $P(a, b, c)$ lies on the plane $2x + y + z = 1$

$\therefore \quad 7a + b + c = 7 + 6 - 7 = 6$

3. (c) : Let a and b be two given quantities.

Then, $G = \sqrt{ab}$, $A_1 = \frac{2a+b}{3}$ and $A_2 = \frac{a+2b}{3}$

$\therefore \quad 2A_1 - A_2 = a$ and $2A_2 - A_1 = b$

$\Rightarrow \quad (2A_1 - A_2)(2A_2 - A_1) = ab = G^2$

4. (a) : The equation of a conic passing through the intersection of the given conics is

$(a_1x^2 + 2h_1xy + b_1y^2 - c_1) + \lambda(a_2x^2 + 2h_2xy + b_2y^2 - c_2) = 0$

This equation will represent a circle, if

Coeff. of x^2 = Coeff. of y^2 and Coeff. of $xy = 0$

$\Rightarrow \quad (a_1 + \lambda a_2) = (b_1 + \lambda b_2)$ ...(i)

and $2h_1 + 2\lambda\, h_2 = 0$ ...(ii)

Eliminating λ from (i) and (ii), we get

$(a_1 - b_1) = \frac{h_1}{h_2}(a_2 - b_2) \Rightarrow (a_1 - b_1)\, h_2 = (a_2 - b_2)\, h_1$

5. (c) : $x^2 - 2x \sec\theta + 1 = 0$ ...(i)

$\Rightarrow \quad x^2 - x\{(\sec\theta + \tan\theta) + (\sec\theta - \tan\theta)\}$
$+ (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 0$

$\Rightarrow \quad x = \sec\theta + \tan\theta, x = \sec\theta - \tan\theta$

$\therefore \quad \alpha_1 = \sec\theta - \tan\theta$ and $\beta_1 = \sec\theta + \tan\theta$ $(\because \alpha_1 > \beta_1)$

Now, $x^2 + 2x\tan\theta - 1 = 0$ ...(ii)

$\Rightarrow \quad x^2 - \{(\sec\theta - \tan\theta) + (-\tan\theta - \sec\theta)\}\, x$
$+ (\sec\theta - \tan\theta)(-\tan\theta - \sec\theta) = 0$

$\Rightarrow \quad x = \sec\theta - \tan\theta, x = -\tan\theta - \sec\theta$

$\therefore \quad \alpha_2 = \sec\theta - \tan\theta$ and $\beta_2 = -\tan\theta - \sec\theta$ $(\because \alpha_2 > \beta_2)$

Hence, $\alpha_1 + \beta_2 = \sec\theta - \tan\theta - \tan\theta - \sec\theta = -2\tan\theta$.

6. (b) : We have, $f(x) = 1 + 2x + 3x^2 + 4x^3$

$\Rightarrow \quad f'(x) = 2 + 6x + 12x^2 = 2\,(6x^2 + 3x + 1) > 0$ for all $x \in R$

$\Rightarrow \quad f(x)$ is strictly increasing on R ...(i)

Now, $f(-3/4) < 0$ and $f(-1/2) > 0$

$\Rightarrow \quad f(x)$ has a real root between $-3/4$ and $-1/2$ ...(ii)

From (i) and (ii), $s \in \left(-\frac{3}{4}, -\frac{1}{2}\right)$

$\therefore \quad t \in (1/2, 3/4)$

Now, $f''(x) = 6 + 24x$, Put $f''(x) = 0 \Rightarrow x = -1/4$

Clearly, $f'(x)$ is increasing on $(-1/4, \infty)$ and decreasing on $(-\infty, -1/4)$.

$\therefore \quad f'(x)$ is decreasing on $(-t, -1/4)$ and increasing on $(-1/4, t)$.

(7-8) : 7. (a) 8. (d)

$$F(x) = \begin{cases} f(x) & , 0 \le x^2 < 1 \\ \frac{f(x)+g(x)}{2} & , x^2 = 1 \\ g(x) & , x^2 > 1 \end{cases} = \begin{cases} g(x) & , x < -1 \\ \frac{f(-1)+g(-1)}{2} & , x = -1 \\ f(x) & , -1 < x < 1 \\ \frac{f(1)+g(1)}{2} & , x = 1 \\ g(x) & , x > 1 \end{cases}$$

For continuity at $x = -1$, we have $f(-1) = g(-1)$

$\Rightarrow \quad 1 - a + 3 = b - 1 \Rightarrow a + b = 5$ ...(i)

For continuity at $x = 1$, $f(1) = g(1)$

$\Rightarrow \quad 1 + a + 3 = 1 + b \Rightarrow a - b = -3$ ...(ii)

Solving (i) and (ii), we get $a = 1$ and $b = 4$

Now, $f(x) = g(x) \Rightarrow x^2 + x + 3 = x + 4 \Rightarrow x = \pm 1$

9. (4) : Roots are real if discriminant ≥ 0

i.e., if $n^2 - 4\left(\frac{n}{2}\right) \ge 0$

i.e., if $n^2 - 2n \ge 0$ ...(i)

We find that $n = 2, 3, 4$ and 5 satisfy the inequality (i). So, four out of 5 choices are favourable. Hence, required probability = 4/5.

10. (c) : P → 1, Q → 2, R → 4, S → 3

Equation of the normal is $y = mx - 2m - m^3$.

Since it passes through $(3, 0)$

$\therefore \quad 3m - 2m - m^3 = 0$

$\Rightarrow \quad m = 0, \pm 1$. Centroid of $\Delta PQR = (2/3, 0)$

ΔPQR is isosceles.

If (x, y) is the circumcentre, then

$x^2 + y^2 = (x - 1)^2 + (y - 2)^2 = (x - 1)^2 + (y + 2)^2$

$\Rightarrow \quad x = \frac{5}{2}, y = 0 \quad \therefore \quad$ Radius = 5/2

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 2.$$

◆ ◆

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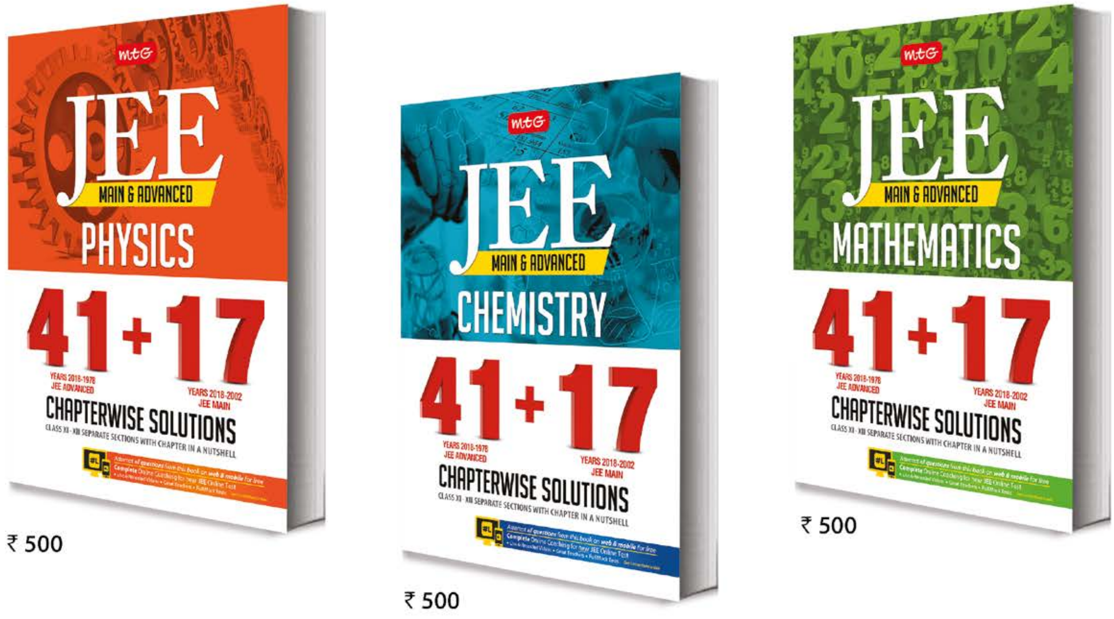
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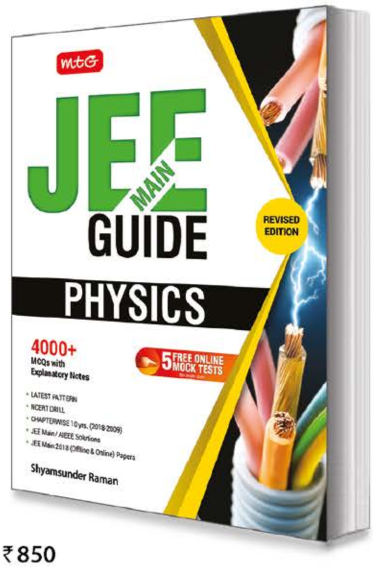

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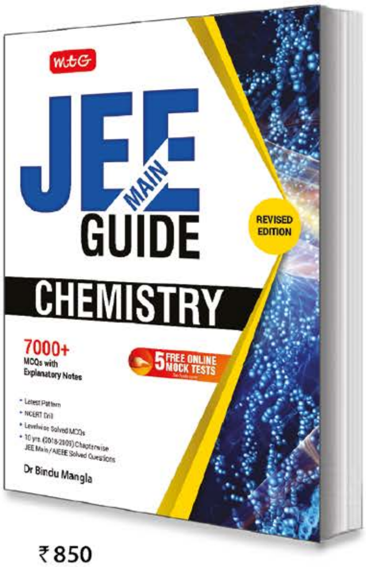

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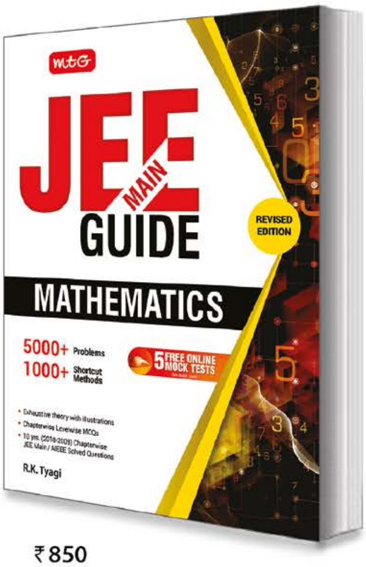

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