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GOOD NEWS FOR OUR READERS

We had earlier shared our intention to increase the price of this magazine and its sister magazines due to steep all round increase in costs.

We have since been flooded with calls & mails requesting us not to increase the costs to benefit the student community at large and especially students from the economically weaker section.

We are happy to share that our editorial board has accepted the request in an endeavour to ensure that the largest circulating monthly magazines continue to be available to students at very reasonable costs.

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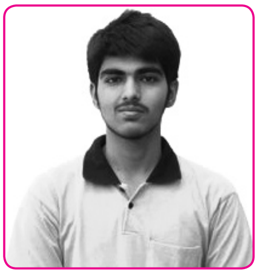
Cracking the JEE Advanced EXAM



Bhavesh Dhingra

JEE Advanced 2016 Topper (AIR 2) Interview : How Bhavesh Dhingra scored 312 out of 372?

Bhavesh Dhingra, AIR 2 from Yamuna Nagar, Haryana believes that in order to achieve success, you need not follow a fixed routine or long term plans. His passion for computers shaped his dream to be a Computer Science Engineer. He is aiming to take admission in IIT Bombay at his preferred branch.

Bhavesh, who is also the topper from IIT Roorkee zone, had scored 92.4% in Class XII. He focused on clarifying concepts first before taking practice tests and advices the same for the aspirants preparing to crack JEE Main and Advanced. He credits his faculty members and family for his success as he feels that without their support and motivation, he wouldn't have been in the top 3 among the 2,00,000 candidates across India.

Congratulations for your achievement in JEE Advanced 2016! How did you celebrate?

Bhavesh Dhingra: Thank you. I celebrated with my parents and family members. Later I joined my friends and mentors at my coaching institute for sharing my success with them.

Were you confident about bagging the top 2 position?

Bhavesh Dhingra: Initially I was not confident about being among the top 3. Later when I analyzed my performance and tallied my chosen answer options with the answer keys, my confidence started building up about getting a high score and rank.

Please share your subject wise and overall score in JEE Advanced 2016. How was your performance in JEE Main?

Bhavesh Dhingra: I have scored 312 out of 372 in JEE Advanced. In Physics my score is 118, 100 in Chemistry and 94 in Mathematics. I have scored 285 in JEE Main.

Who all are there in your family? How supportive were they during your preparation phase?

Bhavesh Dhingra: My parents and two elder sisters comprise my family. My father is a retired bank manager and my mother is a homemaker. My elder sisters are also engineers. The support and motivation they have provided me is beyond my expression. They have provided me best possible facilities in terms of education and taken care of my comfort throughout. They also motivated me and boosted my confidence whenever I felt low.

Why did you decide to pursue higher education in engineering? How did you start with the preparation?

Bhavesh Dhingra: I was fascinated about computers since my childhood and I had decided to be a Computer Science Engineer. I started preparing seriously from Class XI. I had joined coaching and started preparation by gaining conceptual clarity on the subjects.

What was your preparation strategy for JEE?

Bhavesh Dhingra: My strategy included clarifying concepts and then practicing. I focused more on learning the concepts first before taking up practice or solving test papers as it would not have been fruitful. I did not follow any fixed routine and my day to day study timings and pattern was quite flexible. On an average I studied for 6 hours in a day. On other days, when I felt the need to stretch, I studied for around 8 hours.

Tell us about your areas of strength and weaknesses.

Bhavesh Dhingra: Physics is my favourite subject. I find Maths and Chemistry tough. There were areas of strengths and weaknesses in each subject but I worked hard on the weak areas and practiced every possible question types from the strong areas to make them even stronger.

What was your coaching institute's role in your success? Do you think it would be possible for a meritorious student to crack a competitive test like JEE without coaching?

Bhavesh Dhingra: My coaching institute, Allen is a major factor behind my success. They have provided me the right support in terms of study and practice materials, constant performance analysis and motivation throughout. I think the guidance and exposure the coaching institute mentors provide is very crucial for one's improvement.

So what next? Where do you plan to see yourself in the coming 4 years?

Bhavesh Dhingra: I plan to take admission in the Computer Science Engineering in my dream institute, IIT Bombay.

You have had quite a hectic preparation phase so far. Did you get much time to be in touch with your friends or participate in social media activities?

Bhavesh Dhingra: Yes indeed, I could manage to spare some time to spend with my friends. I love sports and played table tennis, cricket and badminton with my friends regularly, apart from discussing about studies in school and coaching classes.

I was less active in Facebook and more on WhatsApp. I used to interact with my classmates in WhatsApp groups on JEE preparation.

What are your hobbies? How often could you pursue them?

Bhavesh Dhingra: Apart from sports, my hobbies include listening to music. I listened to music every day to relax my mind. There is no fixed genre and it depends on my mood. I also enjoy watching movies. The latest movie I watched was Captain America: Civil War.

"Aspirants do not stress yourselves by studying for long hours. Even if you are studying for a short duration, ensure that you give your 100%. Prioritize on clarifying your concepts first before practicing as unless you know the theory well, practice won't be of much help."

What is your message for the JEE aspirants appearing for the exam next year?

Bhavesh Dhingra: My message for the aspirants would be to not stress themselves by studying for long hours. Even if you are studying for a short duration, ensure that you give your 100%. Prioritize on clarifying your concepts first before practicing as unless you know the theory well, practice won't be of much help.

Courtesy : careers360.com

JEE Advanced 2016
Zone wise List

Zone	Number of Candidates within the		Total Qualified
	Top 10	Top 100	
IIT Bombay	3	37	8,810
IIT Delhi	1	18	5,941
IIT Guwahati	0	1	2,468
IIT Kanpur	0	4	4,443
IIT Kharagpur	0	3	4,560
IIT Madras	5	30	6,702
IIT Roorkee	1	7	3,642

NEET | JEE ESSENTIALS

Class XI

Maximize your chance of success, and high rank in NEET, JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

Unit 1 UNITS AND MEASUREMENTS

PHYSICAL QUANTITIES

- All the quantities in terms of which laws of physics can be described and which can be measured are known as physical quantities.
- **Types of Physical Quantities**

  There are two types of physical quantities :
  - **Fundamental quantities :** Those physical quantities which do not depend upon any other quantity are known as fundamental quantities or base quantities. There are seven fundamental or base quantities in SI system. They are Length, Mass, Time, Electric current, Thermodynamic temperature, Amount of substance and Luminous intensity.
  - **Derived quantities :** Those physical quantities which are derived from the fundamental quantities are known as derived quantities.

    $$e.g.\ \text{Speed} = \frac{\text{distance}}{\text{time}}$$

UNITS

- The chosen reference standard of measurement in multiples of which, physical quantity is expressed is called the unit of that quantity.
- **System of Units**
  - The given table shows different system of units:

Type of Physical quantity	Physical Quantity	System		
		CGS (Originated in France)	MKS (Originated in France)	FPS (Originated in Britain)
Funda-mental	Length	cm	m	m
	Mass	g	kg	lb
	Time	s	s	s
Derived	Force	dyne	newton(N)	poundal
	Work	erg	joule (J)	ft-poundal
	Power	erg/s	watt (W)	ft-poundal/s

  - **International system (SI) of units :** This system is modification over the MKS system and so it is also known as Rationalised MKS system. Besides the three base units of MKS system, four fundamental and two supplementary units are also included in this system.
- **Classification of Units**
  - **Fundamental or base units :** Units of fundamental quantities.

Base quantity	SI Units	
	Name	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s

Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

- **Derived units :** Units of derived quantities or the units that can be expressed in terms of the base units.
- **Supplementary units :** radian (rad) for plane angle and steradian (sr) for solid angle.
- **Practical units :** Units defined for practical purposes. *e.g.* light year (ly) is a practical unit of distance.

REPRESENTATION OF ERRORS

- Let a physical quantity a be measured n times. Let the measured values be a_1, a_2, a_3 ..... a_n. To eliminate random error, their arithmetic mean is taken as the true value.

$$\bar{a} = \frac{a_1 + a_2 + ... a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

- **Absolute error :** The magnitude of the difference between the true value and the measured value is called absolute error. Such errors are given by
$\Delta a_1 = \bar{a} - a_1$; $\Delta a_2 = \bar{a} - a_2$; $\Delta a_3 = \bar{a} - a_3$; ....$\Delta a_n = \bar{a} - a_n$
- **Mean absolute error :** The arithmetic mean of the positive magnitudes of all the absolute errors is called mean absolute error.

$$\Delta\bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

- **Relative error :** It is the ratio of the mean absolute error to the mean value.
- **Percentage error :** The relative error expressed in percent is called the percentage error.

$$\text{Percentage error} = \frac{\Delta\bar{a}}{a} \times 100\%$$

- **Propagation of errors in mathematical operations**

Operation	Formula Z	Absolute error ΔZ	Relative error $\Delta Z/Z$	Percentage error $\Delta Z/Z \times 100\%$
Sum	$A + B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A + B}$	$\frac{\Delta A + \Delta B}{A + B} \times 100\%$
Difference	$A - B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A - B}$	$\frac{\Delta A + \Delta B}{A - B} \times 100\%$
Multiplication	$A \times B$	$A\Delta B + B\Delta A$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100\%$
Division	$\frac{A}{B}$	$\frac{B\Delta A + A\Delta B}{B^2}$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100\%$
Power	A^n	$nA^{n-1}\Delta A$	$n\frac{\Delta A}{A}$	$n\frac{\Delta A}{A} \times 100\%$
Root	$A^{1/n}$	$\frac{1}{n}A^{\frac{1}{n}-1}\Delta A$	$\frac{1}{n}\frac{\Delta A}{A}$	$\frac{1}{n}\frac{\Delta A}{A} \times 100\%$

DIMENSIONAL ANALYSIS

- Dimensions of a physical quantity are the powers or exponents to which the base quantities are raised to represent that quantity.
- **Derivation of dimensional formula :** The dimensional formulae of various physical quantities can be obtained by defining them in terms of simple quantities whose dimensions we already know.
e.g. Volume = Length × Breadth × Height
$= [L] \times [L] \times [L] = [L]^3$

- **Applications of Dimensional Analysis**
  - **To check the dimensional consistency of equations :** It is based on principle of homogeneity of dimensions which states that the equation is dimensionally correct if the dimensions of the various terms on either side of the equation are the same.
  - **To deduce relation among the physical quantities:** If we know the dependence of the physical quantity on the other physical quantities, we can

derive a relation among the physical quantities by using the principle of homogeneity of dimensions.

For example, let viscosity (η) of a gas depends upon the mass (m), the effective diameter (d) and the mean speed (v) of the molecules. We have to find η as a function of these given variables.

Then, $\eta \propto m^a d^b v^c$ or $n = km^a d^b v^c$ ...(i)

where k, a, b and c are dimensionless constants. Writing the dimensional formulae of the quantities on both sides of eqn.(i)

$[ML^{-1}T^{-1}] = [M]^a[L]^b[LT^{-1}]^c = [M^aL^{b+c}T^{-c}]$

Comparing the dimensions on both sides

$a = 1, b + c = -1, -c = -1$

$\Rightarrow \quad a = 1, c = 1, b = -1$

$$\therefore \quad \eta = km^1d^{-2}v^1 = k\frac{mv}{d^2}$$

- **To convert one system of units into another system of units :** For this, we use the relation

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

where M_1, L_1, T_1 are fundamental units on one system; M_2, L_2, T_2 are fundamental units on the other system, a, b, c are the dimensions of the quantity in mass, length and time, n_1 is numerical value in one system and n_2 is its numerical value in the other system.

Note : This formula is valid only for absolute units and not for gravitational units.

SIGNIFICANT FIGURES

- The number of digits in the measured value which include certain digits plus one uncertain (doubtful) digit are called significant figures.

Rules for Counting the Significant Figures

- **Rule I** - All non-zero digits are significant.
- **Rule II** - All zeros occurring between the non-zero digits are significant, *i.e.* 230089 contains six significant figures.
- **Rule III** - All zeros to the left of non-zero digit in a number with or without decimal point are not significant, *i.e.* 0.0023 contains two significant figures.
- **Rule IV** - All zeros to the right of non-zero digits in a number without decimal point are not significant, *e.g.* 23000 contains two significant figures.
- **Rule V** - The zeros to the right of non-zero digits (trailing zeros) in a number with a decimal point are significant. *e.g.* 0.2300 contains four significant figures.
- **Remember :** Forget zeros on left but do not forget zeros on the right. Also please note that in scientific notation 23000 should be written as 23.000×10^3. Thus 23 thousands is written as 23×10^3 and it contains two significant figures. Similarly, 230 hundreds is written as 230×10^2 and it contains three significant digits. Thus power (or exponent) of 10 is irrelevant in finding the significant figures in scientific notation. However, all the trailing zeros appearing in the base number in it are significant. The change of units only changes the order of exponent but not the number of significant figures. *e.g.* 1.40 m = 1.40×10^2 cm, both have three significant figures.

Rules for Rounding off

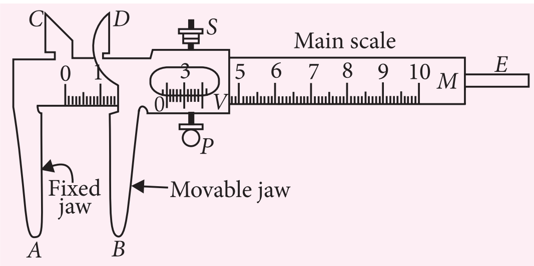
- **Rule I** - If the digit to be dropped is less than 5, then the preceding digit is left unchanged.
  e.g. 8.22 is rounded off to 8.2
- **Rule II** - If the digit to be dropped is more than 5, then the preceding digit is raised by one.
  e.g. 6.87 is rounded off to 6.9
- **Rule III** - If the digit to be dropped is 5 followed by digit other than zero, then the preceding digit is raised by one.
  e.g. 7.851 is rounded off to 7.9
- **Rule IV** - If the digit to be dropped is 5 or 5 followed by zero, then preceding digit is left unchanged, if it is even.
  e.g. 5.250 rounded off to 5.2
- **Rule V** - If the digit to be dropped is 5 or 5 followed by zero, then the preceding digit is raised by one, if it is odd.
  e.g. 3.750 is rounded off to 3.8

- In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.
  e.g. 1.2 + 1.74 + 1.348 = 4.288
  The final result should be rounded off to 4.3.
- In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.
  e.g. $1.2 \times 1.74 \times 1.348 = 2.814624$
  The final result should be 2.8.

☞ MEASURING INSTRUMENTS

- **Vernier Callipers**

➤ It is a device used to measure accurately upto $\left(\dfrac{1}{10}\right)^{\text{th}}$ of a millimetre. The vernier callipers is as shown in the figure.



➤ **Vernier constant :** It is the difference between values of one main scale division and one vernier scale division of vernier callipers. Let n vernier scale divisions (VSD) coincide with $(n-1)$ main scale divisions (MSD)

$\therefore$ n VSD = $(n-1)$ MSD

$$1\ \text{VSD} = \left(\frac{n-1}{n}\right)\text{MSD}$$

Vernier constant, VC = 1 MSD – 1 VSD

$$= 1\ \text{MSD} - \left(\frac{n-1}{n}\right)\text{MSD} = \frac{1}{n}\text{MSD}$$

$$\text{VC} = \frac{\text{Value of one main scale division}}{\text{Total number of divisions on vernier scale}}$$

➤ **Reading of Vernier callipers :** Place the body between the jaws and the zero of vernier scale lies ahead of N^{th} division of main scale. Then

Main scale reading (MSR) = N

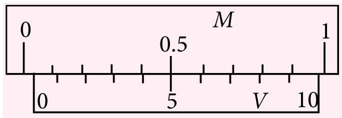
If n^{th} division of vernier scale coincides with any division of main scale, then

Vernier scale reading (VSR) = $n \times$ (VC)

Total reading = MSR + VSR = $N + n \times$ (VC)

➤ **Zero error :** When the jaws A and B touch each other and if the zero of the vernier scale does not coincide with the zero of the main scale, then the instrument has error called zero error. Zero error is always algebraically subtracted from the observed reading.

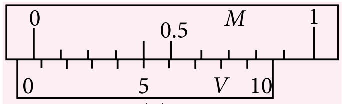
(i) **Positive zero error :** Zero error is said to be positive if the zero of the vernier scale lies on the right of the zero of the main scale as shown in figure (i).



(i)

Here, zero error = 0.00 cm + 5 × VC

(ii) **Negative zero error :** Zero error is said to be negative if the zero of the vernier scale lies on the left of the main scale as shown in figure (ii).

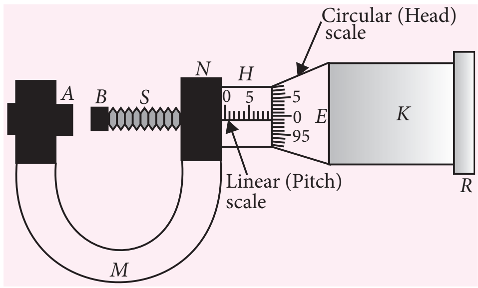


(ii)

Here, zero error = 0.00 cm – (10 – 5) × VC

- **Screw Gauge**

➤ It works on the principle of micrometer screw. A screw gauge is as shown in figure.



➤ **Pitch :** It is defined as the linear distance moved by the screw forward or backward when one complete rotation is given to the circular cap.

Pitch of the screw

$$= \frac{\text{Distance moved on linear scale}}{\text{Number of rotations}}$$

➤ **Least count of the screw gauge**

$$= \frac{\text{Pitch of the screw}}{\text{Total number of divisions on the circular scale}}$$

➤ **Reading of a Screw Gauge :** Place a wire between A and B, the edge of the cap lies ahead of N^{th} division of linear scale. Then

Linear scale reading (LSR) = N

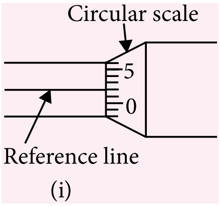
If n^{th} division of circular scale lies over reference line, then

Circular scale reading (CSR) = $n \times$ (LC)

Total reading = LSR + CSR = $N + n \times$ (LC)

➤ **Zero error :** When the two studs A and B of the screw gauge are brought in contact and if the zero of the circular scale does not coincide with the reference line then the screw gauge has an error. This error is called zero error.

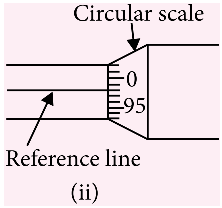
(i) Positive zero error : Zero error is said to be positive if the zero of the circular scale lies below the reference line as shown in figure (i).



(i)

Here , zero error = + 2 × LC

(ii) Negative zero error : Zero error is said to be negative if the zero of the circular scale lies above the reference line as shown in figure (ii).



(ii)

Here, zero error = (98 – 100) × LC = – 2 × LC

SPEED PRACTICE

1. A train goes from station A to D via stations B and C. The measured distances between various stations are : A to B = 648 km, B to C = 64.8 km, C to D = 6.48 km. Total distance covered by the train in proper significant figures is
(a) 719.2 km (b) 719.3 km
(c) 719 km (d) 719.28 km

2. The power of a motor is 200 W. If the unit of length is halved, that of mass is doubled and that of time is also doubled, the power of the motor in the new system is
(a) 3200 W (b) 12.5 W
(c) 12.5 new units (d) 3200 new units

3. Which one of the following statements is false ?
(a) SI unit of all types of energy is joule.
(b) If error in calculating volume of a sphere is 9%, error in measuring its radius is 3%.
(c) Two quantities can be divided only when they have same dimensions.
(d) Two quantities can be subtracted only when they have same dimensions.

4. The time period of a body under S.H.M. is represented by : $T = P^a D^b S^c$ where P is pressure, D is density and S is surface tension, then values of a, b and c are
(a) $-\frac{3}{2},\frac{1}{2},1$ (b) $-1, -2, 3$
(c) $\frac{1}{2},-\frac{3}{2},-\frac{1}{2}$ (d) $1, 2, \frac{1}{3}$

5. In a vernier callipers, N divisions of vernier scale coincide with $(N–1)$ divisions of main scale in which one division represents 1 mm. The least count of the instrument in cm should be
(a) N (b) $N – 1$
(c) $\frac{1}{10N}$ (d) $\frac{1}{N-1}$

6. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are
(a) 5, 1, 5 (b) 5, 5, 2
(c) 4, 4, 2 (d) 5, 1, 2

7. The dimensions of mobility are
(a) $[M^{-1} L^0 T^2 A^1]$ (b) $[M^1 L^0 T^{-2} A^1]$
(c) $[M^0 L^1 T^{-2} A^{-1}]$ (d) $[M^0 L^{-1} T^2 A^{-1}]$

8. If electronic charge e, electron mass m, speed of light in vacuum c and Planck's constant h are taken as fundamental quantities, the permeability of vacuum μ_0 can be expressed in units of
(a) $\left(\frac{hc}{me^2}\right)$ (b) $\left(\frac{h}{me^2}\right)$
(c) $\left(\frac{h}{ce^2}\right)$ (d) $\left(\frac{mc^2}{he^2}\right)$ **[JEE Main 2015]**

9. The least count of a stop watch is 1/5 second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. The minimum percentage error in the measurement of time period will be
(a) 0.1% (b) 8%
(c) 1.8% (d) 0.8%

10. In the expression for torque $\tau = a \times L + b \times I/\omega$
L represents angular momentum, I is moment of inertia and ω is angular velocity. The dimensions of $a \times b$ are :

(a) $[M^0 L^0 T^{-4}]$ (b) $[M^0 L^0 T^{-2}]$
(c) $[M^0 L^0 T^{-1}]$ (d) $[M^0 L^0 T^{-3}]$

11. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be
(a) 92 ± 2 s (b) 92 ± 5.0 s
(c) 92 ± 1.8 s (d) 92 ± 3 s
[JEE Main 2016]

12. If the capacitance of a nanocapacitor is measured in terms of a unit u made by combining the electronic charge e, Bohr radius a_0, Planck's constant h and speed of light c then
(a) $u = \dfrac{e^2 c}{h a_0}$ (b) $u = \dfrac{e^2 h}{c a_0}$
(c) $u = \dfrac{e^2 a_0}{hc}$ (d) $u = \dfrac{hc}{e^2 a_0}$
[JEE Main 2015]

13. A beaker contains a fluid of density ρ kg m^{-3}, specific heat S J kg^{-1} °C^{-1} and viscosity η. The beaker is filled up to height h. To estimate the rate of heat transfer per unit area $(\dot{Q}/A)$ by convection when beaker is put on a hot plate, a student proposes that it should depend on η, $\left(\dfrac{S\Delta\theta}{h}\right)$ and $\left(\dfrac{1}{\rho g}\right)$ when $\Delta\theta$ (in °C) is the difference in the temperature between the bottom and top of the fluid. In that situation the correct option for $(\dot{Q}/A)$ is
(a) $\eta \dfrac{S\Delta\theta}{h}$ (b) $\eta\left(\dfrac{S\Delta\theta}{h}\right)\left(\dfrac{1}{\rho g}\right)$
(c) $\dfrac{S\Delta\theta}{\eta h}$ (d) $\left(\dfrac{S\Delta\theta}{\eta h}\right)\left(\dfrac{1}{\rho g}\right)$
[JEE Main 2015]

14. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it ?
(a) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.
(b) A meter scale.
(c) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.
(d) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.
[JEE Main 2014]

15. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then
(a) $[\varepsilon_0] = [M^{-1} L^2 T^{-1} A]$
(b) $[\varepsilon_0] = [M^{-1} L^{-3} T^2 A]$
(c) $[\varepsilon_0] = [M^{-1} L^{-3} T^4 A^2]$
(d) $[\varepsilon_0] = [M^{-1} L^2 T^{-1} A^{-2}]$ **[JEE Main 2013]**

16. Which of the following will not have the dimensions of potential energy ?
(a) Torque × angular displacement
(b) Rotational inertia × (angular frequency)^2
(c) Position vector × rate of change of momentum
(d) Displacement × momentum

17. In a tangent galvanometer, current i is proportional to tan θ, where θ is angular deflection. The relative error in i will be least when θ is equal to
(a) 30° (b) 45°
(c) 60° (d) 90°

18. If dimensions of critical velocity v_c of a liquid flowing through a tube are expressed as $[\eta^x \rho^y r^z]$ where η, ρ and r are the coefficient of viscosity of liquid, density of liquid and radius of the tube respectively, then the values of x, y and z are given by
(a) −1, −1, −1 (b) 1, 1, 1
(c) 1, −1, −1 (d) −1, −1, 1
[AIPMT 2015]

19. If force (F), velocity (V) and time (T) are taken as fundamental units, then the dimensions of mass are
(a) $[FVT^{-1}]$ (b) $[FVT^{-2}]$
(c) $[FV^{-1}T^{-1}]$ (d) $[FV^{-1}T]$
[AIPMT 2014]

20. In an experiment four quantities a, b, c and d are measured with percentage error 1%, 2%, 3% and 4% respectively. Quantity P is calculated as follows

% error in P is $P = \dfrac{a^3 b^2}{cd}$
(a) 7% (b) 4%
(c) 14% (d) 10% **[NEET 2013]**

SOLUTIONS

1. **(c) :** Here, $AB = 648$ km, $BC = 64.8$ km, $CD = 6.48$ km
Total distance = 719.28 km
Rounding off to no decimal place,
$\therefore$ total distance = 719 km.

2. **(d):** $\because$ Power = $[ML^2T^{-3}]$
$a = 1, b = 2, c = -3$
$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$
$$\text{or } n_2 = n_1\left(\frac{M_1}{M_2}\right)^1\left(\frac{L_1}{L_2}\right)^2\left(\frac{T_1}{T_2}\right)^{-3} = 200\left(\frac{1}{2}\right)^1(2)^2\left(\frac{1}{2}\right)^{-3}$$
= 3200 new units.

3. **(c) :** Two quantities can be divided even when they do not have the same dimensions.

4. **(a) :** $T = P^a D^b S^c$
$[M^0 L^0 T^1] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [MT^{-2}]^c$
$= [M^{a+b+c} L^{-a-3b} T^{-2a-2c}]$
Applying principle of homogeneity of dimensions,
$a + b + c = 0$; $-a - 3b = 0$; $-2a - 2c = 1$
On solving, we get $a = -3/2$, $b = 1/2$, $c = 1$

5. **(c) :** N V.S.D. = $(N - 1)$ M.S.D.
$$1\text{ V.S.D.} = \left(\frac{N-1}{N}\right)\text{M.S.D.}$$
V.C. = Least count = 1 M.S.D. – 1 V.S.D.
$$\text{V.C.} = \left(1 - \frac{N-1}{N}\right)\text{M.S.D.}$$
$$= \frac{1}{N}\text{M.S.D.} = \frac{0.1}{N}\text{cm} = \frac{1}{10N}\text{cm}$$

6. **(d)**

7. **(a) :** $$\text{Mobility} = \frac{\text{Drift velocity}(v)}{\text{Electric field }(E)} = \frac{v}{F/q} = \frac{qv}{F}$$
$$= \frac{[AT][LT^{-1}]}{[MLT^{-2}]} = [M^{-1}L^0T^2A^1]$$

8. **(c) :** $[e] = [AT]$, $[m] = [M]$, $[c] = [LT^{-1}]$
$[h] = [ML^2T^{-1}]$, $[\mu_0] = [MLA^{-2}T^{-2}]$
If $\mu_0 = ke^a m^b c^c h^d$
$[MLA^{-2}T^{-2}] = [AT]^a[M]^b[LT^{-1}]^c[ML^2T^{-1}]^d$
By equating powers, we get
$a = -2$, $b + d = 1$
$c + 2d = 1$, $a - c - d = -2$
Solving these eqns, we get
$a = -2, b = 0, c = -1, d = 1$
$$\therefore [\mu_0] = \left[\frac{h}{ce^2}\right]$$

9. **(d):** Error in measuring 25 s = $\frac{1}{5}$ s = 0.2 s
$$\therefore \text{Percentage error} = \frac{0.2}{25}\times 100 = 0.8\%$$

10. **(a):** Given, $\tau = a \times L + b \times I/\omega$
$$\therefore [a] = \frac{[\tau]}{[L]} = \frac{[I\alpha]}{[I\omega]} = \frac{[T^{-2}]}{[T^{-1}]} = [T^{-1}]$$
$$[b] = \frac{[\tau]}{[I/\omega]} = \frac{[I\alpha]}{[I/\omega]} = [\alpha\omega] = [T^{-2}][T^{-1}] = [T^{-3}]$$
$\therefore$ $[a \times b] = [T^{-1}][T^{-3}] = [M^0L^0T^{-4}]$

11. **(a):** Here, $t_1 = 90$ s, $t_2 = 91$ s, $t_3 = 95$ s, $t_4 = 92$ s
L.C. = 1 s
$$\text{Mean of the measurements, } \bar{t} = \frac{\sum_i t_i}{N}$$
$$\bar{t} = \frac{90+91+95+92}{4} = 92\text{ s}$$
$$\text{Mean deviation} = \frac{\sum_i |\bar{t} - t_i|}{N} = \frac{2+1+3+0}{4} = 1.5\text{ s}$$
Since the least count of the instrument is 1 s, so reported mean time = (92 ± 2) s.

12. **(c) :** Here, capacitance $C = ke^x a_0^y h^z c^a$
$[C] = [M^{-1}L^{-2}A^2T^4]$
$[e] = [AT]$, $[a_0] = [L]$
$[c] = [L^1T^{-1}]$, $[h] = [M^1L^2T^{-1}]$
$\therefore$ $[M^1L^{-2}A^2T^4] = [AT]^x[L]^y[M^1L^2T^{-1}]^z[L^1T^{-1}]^a$
Comparing both sides
$x = 2$; $z = -1$; $y + 2z + a = -2$; $x - z - a = 4$
On solving these eqns, we get
$x = 2, y = 1, z = -1, a = -1$
Also, $[C] = u$
$$\text{So } u = \frac{e^2 a_0}{hc}$$

13. **(a):** Let $\left(\frac{\dot{Q}}{A}\right)$ is derived quantity which is derived from three fundamental quantities $\eta, \left(\frac{S\Delta\theta}{h}\right)$ and $\left(\frac{1}{\rho g}\right)$
By using principle of homogeneity of dimensions
$$\left[\frac{\dot{Q}}{A}\right] = [\eta]^x\left[\frac{S\Delta\theta}{h}\right]^y\left[\frac{1}{\rho g}\right]^z$$
$$\left[\frac{\dot{Q}}{A}\right] = [M^1T^{-3}];\ [\eta] = [M^1L^{-1}T^{-1}]$$
$$\left[\frac{S\Delta\theta}{h}\right] = [L^1T^{-2}];\ \left[\frac{1}{\rho g}\right] = [M^{-1}L^2T^2]$$

$\therefore \quad [M^1L^0T^{-3}] = [M^1L^{-1}T^{-1}]^x[M^0L^1T^{-2}]^y[M^{-1}L^2T^2]^z$

On comparing both sides

$x + 0 - z = 1, - x + y + 2z = 0$ and $-x - 2y + 2z = -3$

On solving these eqns, we get, $x = 1, y = 1$ $z = 0$

so, $\dfrac{\dot{Q}}{A} = \eta \dfrac{S\Delta\theta}{h}$

14. (c) : Measured value of the length of rod = 3.50 cm

So, least count of the measuring instrument must be 0.01 cm = 0.1 mm

For, vernier scale, 10 MSD = 1 cm = 10 mm

$\Rightarrow$ 1 MSD = 1 mm

Also, 9 MSD = 10 VSD

L.C. = 1 MSD – 1 VSD = (1 – 0.9) mm = 0.1 mm

15. (c) : According to Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$\therefore \quad \varepsilon_0 = \frac{1}{4\pi}\frac{q_1q_2}{Fr^2} = \frac{[AT][AT]}{[MLT^{-2}][L]^2} = [M^{-1}L^{-3}T^4A^2]$$

16. (d): Displacement × momentum

= [L] × [MLT^{-1}] = [ML^{2}T^{-1}], which are not the dimensions of potential energy.

17. (b): In a tangent galvanometer,

$i \propto \tan\theta$

$\therefore \quad di \propto \sec^2\theta\, d\theta$

$$\frac{di}{i} = \frac{\sec^2\theta d\theta}{\tan\theta} = \frac{d\theta}{\sin\theta\cos\theta} = \frac{2d\theta}{\sin 2\theta}$$

$$\left(\frac{di}{i}\right)_{\min} = 2\,d\theta, \text{ for } \sin 2\theta = 1 = \sin 90°$$

$$\theta = \frac{90°}{2} = 45°$$

18. (c) : $[v_c] = [\eta^x\rho^y r^z]$ (given) ... (i)

Writing the dimensions of various quantities in eqn. (i), we get

$[M^0LT^{-1}] = [ML^{-1}T^{-1}]^x[ML^{-3}T^0]^y[M^0LT^0]^z$

$= [M^{x+y}L^{-x-3y+z}T^{-x}]$

Applying the principle of homogeneity of dimensions, we get

$x + y = 0; -x - 3y + z = 1; \ -x = -1$

On solving, we get, $x = 1, y = -1, z = -1$

19. (d): Let mass $m \propto F^a V^b T^c$

or $\qquad m = kF^a V^b T^c$ ...(i)

where k is a dimensionless constant and a, b and c are the exponents.

Writing dimensions on both sides, we get

$[ML^0T^0] = [MLT^{-2}]^a [LT^{-1}]^b [T]^c$

$[ML^0T^0] = [M^aL^{a+b}T^{-2a-b+c}]$

Applying the principle of homogeneity of dimensions, we get

$a = 1$ ... (ii)

$a + b = 0$ ... (iii)

$-2a - b + c = 0$ ... (iv)

Solving eqns. (ii), (iii) and (iv), we get

$a = 1, b = -1, c = 1$

From eqn. (i), [m] = [FV^{-1}T]

20. (c) : As $P = \dfrac{a^3b^2}{cd}$

% error in P is

$$\frac{\Delta P}{P}\times 100\% = \left[3\left(\frac{\Delta a}{a}\right) + 2\left(\frac{\Delta b}{b}\right) + \frac{\Delta c}{c} + \frac{\Delta d}{d}\right]\times 100\%$$

$= [3 \times 1\% + 2 \times 2\% + 3\% + 4\%] = 14\%$

SOLUTION OF JUNE 2016 CROSSWORD

1 P	H	O	T	O	C	O	N	2 D	U	C	T	I	V	I	T	Y					
A								O				3 G					4 G				
R		5 F				6 G		7 S	K	Y	W	A	V	E	8 S		R				
S		E				I		E				L			T		A				
E		R				L						9 A	D	V	E	C	T	I	O	10 N	
C		R				B				11 L	U	X			T		I			E	
		I				E						Y			H		N			U	
		T		12 W		R							13 M		O		G			T	
		14 E	L	A	S	T	A	N	C	E			I		S			15 B		R	
				T									C		C			U		O	
16 F				17 E	F	F	U	S	I	O	N		R		O		18 P	O	U	N	D
U				R									O		P			Y			
L		19 C		20 S	O	L	A	R	E	C	L	I	P	S	E		21 L	A	G		22 E
L		O		P									H					N			L
23 M	I	C	R	O	C	H	I	P			24 A	C	O	U	S	T	I	C	S		E
O		H		U									N					Y			C
O		L		T			25 C		26 M	A	27 G	N	E	T	I	T	28 E				T
N		E					R				R						P				R
		A					E				I			29 D	R	A	I	N			O
						30 T	E	R	M	I	N	A	L				T				N
					31 P		P				L						A				V
	32 G	L	I	D	E					33 L	E	V	E	R			X				O
					T						N				34 C	R	Y	S	T	A	L
				35 H	A	R	D	N	E	S	S										T

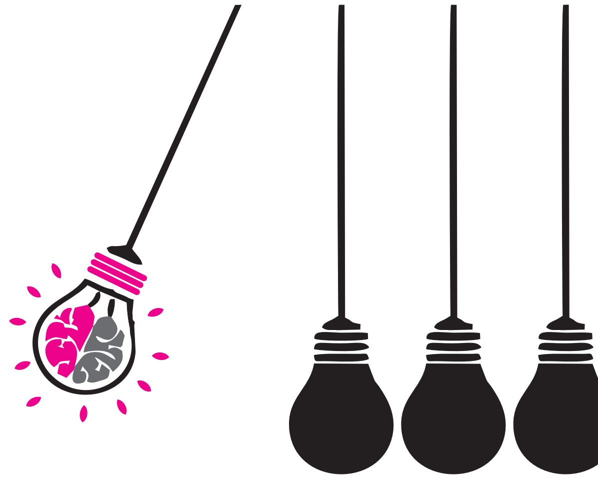
Winners (June 2016)

1. Debasrija Mondal (West Bengal)
2. Saket Jain (New Delhi)

Solution Senders (May 2016)

1. Charit Ghosh (West Bengal)
2. Divyanshu Makhija (Punjab)
3. Nikhil Paradkar (Gwalior)

CORE CONCEPT



GRAVITATION

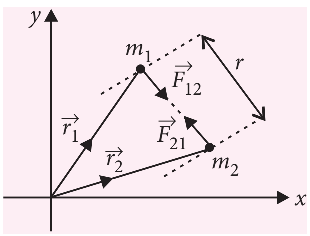
Now-a-days almost every kid knows about the basic of gravity, earth pulls every object towards itself. Ask them why? They would not be able to answer. Ask them, whether they knew that every other object which is being pulled by earth is also pulling earth with same magnitude of force.

In this section ahead we will see in details about how much force of gravitation exists between two objects in consideration.

Newton's law of gravitation states that for two point masses m_1 and m_2 the gravitational force of attraction between them is proportional to

(i) product of masses

(ii) inverse of the square of the distance between them.



$\vec{F}_{12}$ and $\vec{F}_{21}$ indicate the force acting on m_1 due to m_2 and on m_2 due to m_1 respectively.

These are central forces which means that it always acts along the line joining the two particles under consideration.

Magnitude wise F_{12} and F_{21} are identical but just opposite to each other.

Let $F_{12} = F_{21} = F$

$$F \propto \frac{m_1 m_2}{r^2} \quad \Rightarrow \quad F = G\left(\frac{m_1 m_2}{r^2}\right)$$

where $G = 6.67 \times 10^{-11}$ N m^{2}kg^{-2}

$$\approx \frac{20}{3} \times 10^{-11} \text{ N m}^2\text{kg}^{-2}$$

is universal gravitation constant.

(A constant is said to be universal if its value does not change with location, hence gravitational force between two objects does not get affected by the medium present between them).

This equation of force only gives us the magnitude of the force.

In vector form,

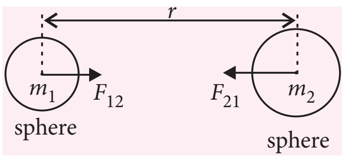
$$\vec{F}_{12} = F_{12}\,\hat{r}_{21} = \left(\frac{Gm_1m_2}{r_{21}^2}\right)\left(\frac{\vec{r}_2 - \vec{r}_1}{r_{21}}\right)$$

$$\vec{F}_{21} = F_{21}\,\hat{r}_{12} = \left(\frac{Gm_1m_2}{r_{12}^2}\right)\left(\frac{\vec{r}_1 - \vec{r}_2}{r_{12}}\right)$$

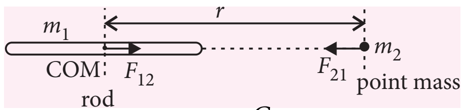
where $r_{12} = r_{21} = r$

The law of gravitation even though is given for point masses but it is found to be equally applicable for radially symmetric spherical mass distribution where we can assume the entire mass to be concentrated at the centre of mass of the sphere. But other than sphere, no other mass can be assumed to be concentrated at its centre of mass (COM).

For calculation of gravitational force



$$F_{12} = F_{21} = \frac{Gm_1m_2}{r^2}$$



$$F_{12} = F_{21} \neq \frac{Gm_1m_2}{r^2}$$

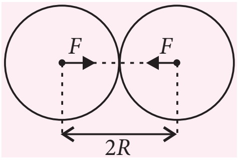
If $\dfrac{Gm_1m_2}{r^2}$ isn't the force between the rod and point mass, then how much is it?

We will see this later, as we progress through the chapter. First we see few applications of what we already learnt.

E1. Two uniform solid spheres of radius R and mass m each are kept in contact and are found to exert force F on each other. If the radius of sphere is doubled, find the force of attraction between them.

Contributed By: Bishwajit Barnwal, Aakash Institute, Kolkata

Sol.: $F = \dfrac{Gm^2}{(2R)^2}$



Now, if R is changed, m also changes.

Applying, $m = \rho V = \rho \cdot \dfrac{4}{3}\pi R^3$

$$F \propto \frac{m^2}{R^2} \propto \frac{R^6}{R^2} \propto R^4$$

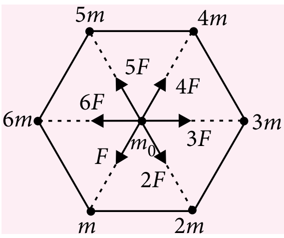
or $F \propto R^4$, $\therefore$ we learn that $F \propto \dfrac{1}{R^2}$ is applicable only if masses are fixed.

$$\therefore \quad \frac{F'}{F} = \left(\frac{2R}{R}\right)^4 = 16$$

$\therefore$ $F' = 16F$

E2. Six point masses m, $2m$, $3m$, ..., $6m$ are placed fixed at the vertices of a regular hexagon of side length a. Another point mass m_0 is placed at the centroid. Find the net force experienced by it.

Sol.:

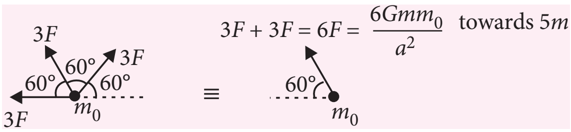


Let us assume,

$$\frac{Gmm_0}{a^2} = F$$

$\therefore$ The force exerted by the masses m, $2m$, ..., $6m$ are F, $2F$, ..., $6F$ as shown.

The vector sum of the forces can be seen as below.



$3F + 3F = 6F = \dfrac{6Gmm_0}{a^2}$ towards $5m$

E3. Three identical point masses m each are located at the vertices of an equilateral triangle of side length a. They are found to perform uniform circular motion (UCM) under their mutual gravitational attraction. Find the speed of each.

Sol.: In such question of UCM under mutual attractive force remember these -

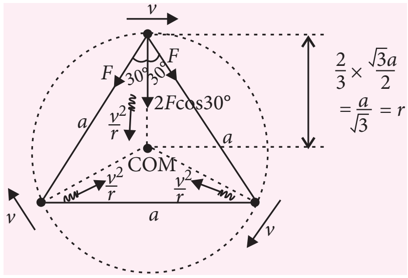
(i) linear momentum of the system is zero

(ii) COM doesn't shift ($\because F_{ext} = 0$ and $u_{COM} = 0$)

(iii) Angular momentum remains conserved ($\because \tau = 0$ due to central force)

(iv) The net force on any mass should be directed towards the centre of mass, since it would be the centripetal force.

(v) The distance of the individual masses from the COM would be the radius of revolution of the masses.



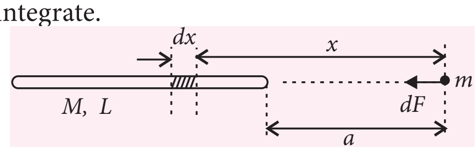
Considering any one mass,

$$F_{net} = 2F\cos 30° = \frac{mv^2}{r}$$

$$\Rightarrow \quad 2\left(\frac{Gm^2}{a^2}\right)\frac{\sqrt{3}}{2} = \frac{mv^2}{(a/\sqrt{3})} \Rightarrow \quad v = \sqrt{\frac{Gm}{a}}$$

These were some examples involving point or spherical masses.

What if the masses were not spherical? For example force between uniform rod and point mass? In such case we choose a small element on the rod and calculate force between chosen element and point mass and then integrate.



$$dF = \frac{Gm\,dM}{x^2} = \frac{Gm\left(\frac{M}{L}\right)dx}{x^2}$$

$$\therefore \quad F = \frac{GmM}{L}\int_a^{a+L}\frac{dx}{x^2} = \frac{GmM}{L}\left[-\frac{1}{x}\right]_a^{a+L} = \frac{GmM}{a(a+L)}$$

Till here we saw few examples of how much force has been acting.

Now, wait ... did we think, how the force acted?

Let me answer.

Whenever two objects exerts force on each other without physical contact, we say that the objects fall (lie) in the field of each other. For example charges interact through electrostatic field, masses through gravitational field, bar magnets through magnetic field.

So, gravitational field can be seen as the region surrounding a particular mass in which its influence can be felt by others. So basically if a mass experiences a gravitational force, it has to lie in the gravitational field of other masses.

Mathematically, gravitational field at any point is defined as the gravitational force experienced per unit test mass kept at that point.

Let us denote it by $\vec{g}$.

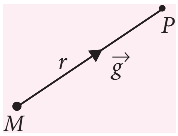
$$\therefore \quad \vec{g} = \frac{\vec{F}_{\text{gravitational}}}{m_0}$$

where $\vec{F}_{\text{gravitational}}$ is the gravitational force experienced by mass m_0 kept at the point.

So, if the gravitational field at any point is known, say $\vec{g}$, if we keep any mass m there, the force experienced by it will be $m\vec{g}$. Simple!

Let us try to calculate gravitational field of some standard configuration.

Field due to a point mass

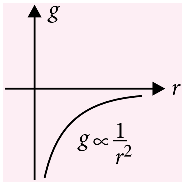


$$\vec{g} = \frac{\vec{F}}{m_0}$$

$$\therefore \quad g = \frac{Gm_0M}{r^2} \cdot \frac{1}{m_0} = \frac{GM}{r^2} \quad \therefore \quad g \propto \frac{1}{r^2}$$

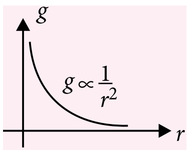
In vector form,

$$\vec{g} = \left(\frac{GM}{r^2}\right)(-\hat{r}) = -\frac{GM}{r^3}(\vec{r})$$



Note that, $\vec{g}$ is opposite to $\vec{r}$, hence for positive r, g has been plotted as negative.

If only magnitude is plotted then,

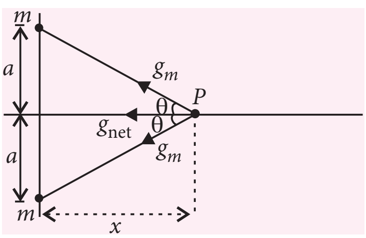


If you notice the expression, $g = \dfrac{GM}{r^2}$, this expression is same as if a mass of 1 kg was kept and force on this 1 kg was being calculated as below:

$$F = \frac{GM \cdot 1}{r^2} = \frac{GM}{r^2}$$

$\therefore$ To calculate g, next time we imagine a mass of 1 kg kept at the point and the calculated gravitational force is the magnitude of the $\vec{g}$.

Field due to two identical point masses at its bisector line

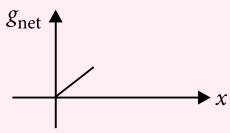
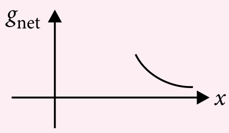


At point P, distant x from the line joining the masses, if g_m is the magnitude of field strength due to one of them then,

$$g_{\text{net}} = 2g_m \cos\theta$$

$$\Rightarrow \quad g_{\text{net}} = 2 \cdot \frac{Gm}{(a^2 + x^2)} \cdot \frac{x}{\sqrt{a^2 + x^2}} = \frac{G(2m)x}{(x^2 + a^2)^{3/2}}$$

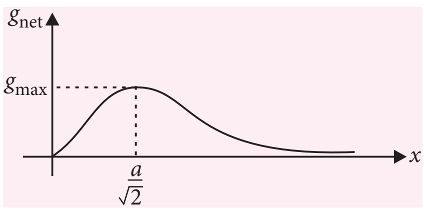
Now, let us try to plot the variation of magnitude of g_{net} with respect to x.

For $x << a$	For $x >> a$
$(x^2 + a^2)^{3/2} \approx a^3$	$(x^2 + a^2)^{3/2} \approx x^3$
$\therefore \; g_{\text{net}} = \dfrac{G(2m)x}{a^3}$ $= \left(\dfrac{G(2m)}{a^3}\right)x$	$\therefore \; g_{\text{net}} \approx \dfrac{G(2m)x}{x^3}$ $= \dfrac{G(2m)}{x^2}$ $\therefore$ It appears as if both the masses are placed at one point.
$\therefore \; g_{\text{net}} \propto x$ (linear)	$\therefore \; g_{\text{net}} \propto \dfrac{1}{x^2}$
 (1)	 (2)

Merging the two graphs, (1) and (2), we understand that the graph, *i.e.*, gravitational field strength attained maximum some where in between $x = 0$ and $x = \infty$, which can easily be found out from maxima/minima concept.

$$\therefore \quad \text{At maxima, } \frac{dg_{\text{net}}}{dx} = 0 \;\Rightarrow\; \frac{d}{dx}\left(\frac{x}{(x^2 + a^2)^{3/2}}\right) = 0$$

$$\Rightarrow \quad x = \pm\left(\frac{a}{\sqrt{2}}\right) \qquad \text{(on solving)}$$



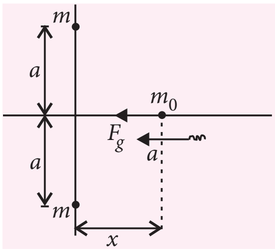
Note that for small x, *i.e.*, $x << a$, $g_{\text{net}} \propto x$ which indicates that if a mass m_0 is kept there, it will experience a restoring force proportional to x, the displacement from mean position which indicates that m_0 will start executing SHM.

$$F_g = m_0 g_{net} = m_0\left(\frac{G(2m)}{a^3}\right)x$$

$$\Rightarrow \quad m_0 a = \left(\frac{G\,2mm_0}{a^3}\right)x$$

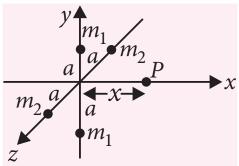
$$\Rightarrow \quad a = \left(\frac{2Gm}{a^3}\right)x \;\Rightarrow\; \text{SHM}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{a^3}{2Gm}}$$



The expression of the field for two identical point masses can be extended to find field due to a thin ring using symmetry.

For the pair of masses m_1 and m_2 field is in the same direction at point P, hence the field can be added like scalar.



$$\therefore \quad g_{net} = g_1 + g_2$$

$$= \frac{G(2m_1)x}{(x^2+a^2)^{3/2}} + \frac{G(2m_2)x}{(x^2+a^2)^{3/2}}$$

$$= \frac{G(2(m_1+m_2))x}{(x^2+a^2)^{3/2}} = \frac{Gm_{total}x}{(x^2+a^2)^{3/2}}$$

$\therefore$ For a ring if we consider two point mass elements dm each on diametric ends, the field strength due to it would be

$$dg = \frac{G(2dm)x}{(x^2+a^2)^{3/2}} \quad \therefore \quad g_{net} = \int dg = \frac{Gx}{(x^2+a^2)^{3/2}}\int 2dm$$

where $\int 2dm$ = sum of all such dm elemental masses = m = mass of ring

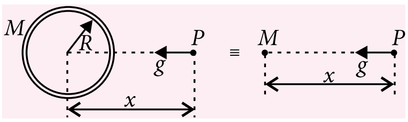
$$\therefore \quad g_{net} = \frac{Gmx}{(x^2+a^2)^{3/2}}$$

where a would be the radius of ring now.

Field due to a thin uniform spherical shell

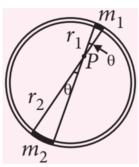
Shell theorem:

1. For points external to the sphere, the entire mass can be assumed to be concentrated at centre.



$$\therefore \quad g_{net} = \frac{GM}{x^2}$$

2. For point internal to the shell, there is no gravitational field. Let us try to prove this. Consider a point P, non-symmetrically considered inside the shell.



If θ is small, m_1 and m_2 will behave as point masses and where m_1 and m_2 will be proportional to respective areas A_1 and A_2.

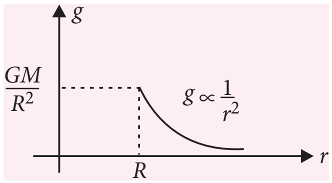
$$\therefore \quad g_p = g_1 - g_2 = \frac{Gm_1}{r_1^2} - \frac{Gm_2}{r_2^2}$$

$$= G\left(\frac{M}{4\pi R^2}\right)\left(\frac{A_1}{r_1^2} - \frac{A_2}{r_2^2}\right)$$

But $\frac{A_1}{r_1^2} = \frac{A_2}{r_2^2}$ = solid angle subtended by two small elemental curved surfaces A_1 and A_2 at point P.

$\therefore \quad g_p = 0$

$\therefore$ If the strength of g is plotted for thin spherical shell for radial distance, r, it would look as below:



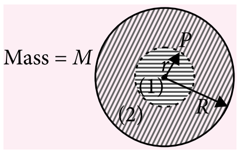
Field due to a uniform solid sphere

For external points, again the entire sphere can be assumed to be concentrated at centre.

But to find field at internal point, we can use what we learnt just above.

Field at P is due to two parts marked (1) and (2).

For parts (1), the point P is just above the surface of the sphere, hence mass of part (1) can be assumed to be concentrated at centre.



For part (2), its an internal point of large number of thin layered concentric shells, hence field due to it is zero.

$$g_p = g_{p_1} + g_{p_2} = \frac{Gm_1}{r^2} = \frac{G\cdot\frac{M}{R^3}\cdot r^3}{r^2} = \left(\frac{GM}{R^3}\right)r$$

$$\Rightarrow \quad g = \left(\frac{GM}{R^2}\right)\left(\frac{r}{R}\right) = g_s\left(\frac{r}{R}\right)$$

where $g_s = \frac{GM}{R^2}$ = field strength at the surface

$\therefore$ Field at internal point is a fraction of the field at surface and the fraction is radius ratio.

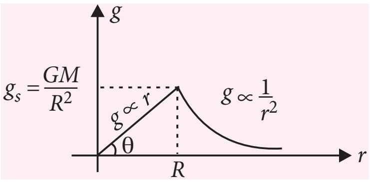
$$\text{Also, } g = G\left(\frac{M}{R^3}\right)r \; = G\left(\frac{M}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi\right)r = \frac{4}{3}\pi G\rho r$$

$\Rightarrow \quad g \propto \rho r$

$\therefore$ For internal points ($r < R$),

$$g = \left(\frac{GM}{R^3}\right) r = g_s\left(\frac{r}{R}\right) = \frac{4}{3}\pi G\rho r$$

Hence the plot of g looks as below:



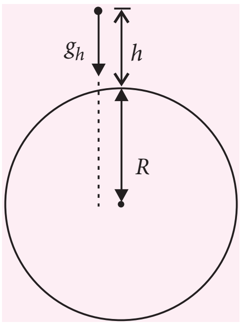
$$\tan\theta = \left(\frac{GM}{R^3}\right)$$

The gravitational field strength when calculated for earth is said to be acceleration due to gravity of earth. Earth itself is almost a solid sphere. Hence the field expressions will be same as above.

For external points, we say g above the surface of earth, whereas for internal points we say at a depth below the surface of earth.

At a height h above earth's surface

$$\therefore \quad g_h = \frac{GM}{(R+h)^2} = \frac{GM}{R^2}\cdot\left(\frac{R}{R+h}\right)^2$$



$$\Rightarrow \quad g_h = g_s\left(\frac{R}{R+h}\right)^2$$

where g_s is the acceleration due to gravity near the surface of earth, *i.e.*,

$$g_s = \frac{GM}{R^2}$$

Special case

If $h << R$, *i.e.* $h < 5\%$ of $R = 6400$ km, *i.e.*, $h < 320$ km, then

$$g_h = g_s\left(1+\frac{h}{R}\right)^{-2}$$

$$\approx g_s\left(1-\frac{2h}{R}\right); \text{since } (1+x)^n \approx 1+nx, \text{ if } |x| << 1$$

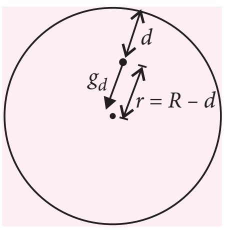
$\therefore \quad g_h = g_s\left(1-\frac{2h}{R}\right)$; this result can be used only as an approximate result, that too if $h < 320$ km.

At a depth d below the surface of earth

$$\therefore \quad g_d = g_s\left(\frac{r}{R}\right)$$

$$= g_s\left(\frac{R-d}{R}\right)$$

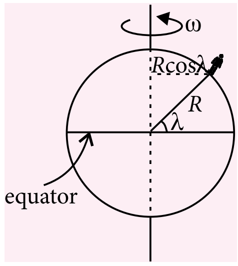
$$g_d = g_s\left(1-\frac{d}{R}\right)$$



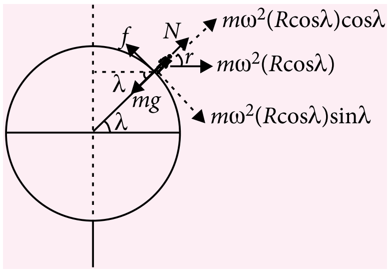
This result has no approximations, hence can be used in all cases.

Effect of rotation of earth on g experienced by objects kept on surface

Let a person be standing at a place where the latitude angle is known to be λ. Due to rotation of earth, he is also revolving in a circle of radius $R\cos\lambda$ with same angular velocity.



$\therefore$ With respect to earth, a non-inertial frame, a centrifugal force (pseudo force) needs to be shown apart from other visible forces as shown in the free body diagram.



$\therefore$ Along the normal,

$N + m\omega^2 R\cos^2\lambda = mg$

$\Rightarrow \quad N = m(g - \omega^2 R\cos^2\lambda)$

$\therefore \quad g_{eff} = g - \omega^2 R\cos^2\lambda$

where g_{eff} can be seen as the effective acceleration due to gravity experienced by the person. The equation clearly shows that the effect of rotation will be felt maximum at equator and no rotational effect at all at the poles.

Suppose a hypothetical situation is given where they say that a person standing on equator starts feeling weightless. What could have been possible?

If the angular speed of earth is increased the component $\omega^2 R$ will also increase and if it equals g then $g_{eff} = 0$.

$\therefore \quad g_{eff} = 0 \quad \Rightarrow \quad \omega^2 R = g$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{R}} \qquad \therefore \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$

Putting the values, $R = 6400$ km, $g = 9.8$ m s^{-2}, we get

$T \approx 84$ minutes ≈ 1.4 hrs

$\therefore$ The length of the entire day will be shrinked to 1.4 hrs instead of 24 hrs.

$\therefore$ It has to spin at a rate $\frac{24}{1.4} \approx 17$ times faster than its present rate.

◆◆

The NEET Solution

By endorsing NEET, Supreme Court comes to the aid of aspiring medical students

Devi Shetty

I was on the Board of Governors of the Medical Council of India (MCI) in 2010. We conceptualised a single national eligibility exam that addressed the pain of lakhs of aspiring medical students who criss-cross India to write over 100 entrance tests. We proposed a fair and simple test, modelled on examination systems like SAT, GRE, TOEFL. Our proposal was eventually adopted as NEET but sadly, there was no political will to implement it then.

In April this year, Judges of the Supreme Court earned the blessings of lakhs of underprivileged students when they asked the CBSE to implement NEET in 2016, despite opposition from states and individuals across party lines. To understand this opposition, you have to understand that as many as 80 parliamentarians are reported to have stakes in medical colleges. Let's try to understand facts behind NEET and dispel some myths.

The total number of MBBS seats, according to the MCI website, is 49,990. Out of this 25,330 are in government medical colleges and 24,660 are in private colleges. Historically, government medical colleges admission happens through AIPMT or CET conducted by various state governments in a transparent manner. Now CET will be replaced by NEET with no major controversy. The controversy over NEET relates to the 24,660 medical seats in private colleges.

Will private colleges lose their "management quota" seats under NEET? No, private medical colleges and minority institutions will continue to have the same number of medical seats as before NEET. However, admission to those seats will be based on NEET ranking, maintaining the objectives of minority institutions and spirit of private enterprise.

Neither will NEET in English adversely affect children from rural India. Most state governments, including states like Karnataka, always conducted CET in English. Fifteen per cent of medical seats across the country are now allotted by the central government through AIPMT conducted by CBSE, which is primarily in English. This year roughly 12 states, including Delhi, MP, Haryana and Odisha, have decided to admit all their students to government medical colleges from AIPMT only.

Historically, for 24,660 MBBS seats in private colleges more than 100 entrance tests are conducted only in English. So there is no change in the language of entrance tests to all the private medical colleges.

Neither are students from government schools handicapped from excelling in NEET. For the 25,330 medical seats in government colleges admission through CET across the country is conducted to assess knowledge of physics, chemistry and biology. Irrespective of the syllabus, the human heart can be taught in only one way. For 24,660 private medical seats, over 100 entrance tests are done not based on any specific syllabus. Prestigious colleges like CMC Vellore and St John's Medical College have their own style of conducting entrance tests with integrity, not based on any syllabus.

> To understand the opposition to NEET, you have to understand that as many as 80 parliamentarians are reported to have stakes in medical colleges

Historically, children from English medium schools always had greater chances of getting admission to medical colleges, IIT and IAS. NEET cannot change that reality. In 2013, when a vernacular NEET was conducted not a single vernacular language candidate could get admission.

A vital piece of information is concealed from the general public in the whole NEET debate: MBBS is taught only in English. According to MCI regulations, only students who have passed the 12th standard English paper are eligible to join medical college irrespective of their medium of education. Historically, students could not join medical college in India without proficiency in English.

As a student from a Kannada medium school, I really struggled in PUC and first year of MBBS. When English medium students learnt the constitution of the human body, I was struggling to learn English to learn about the human body. But that did not prevent me from getting trained in some of England's best hospitals.

Neither will bright students from progressive states take away seats from the state quota of weaker states. Each state government will create a separate merit list for their students based on NEET ranking. NEET is just a tool to grade students and make the selection fair, without taking away from a state's privilege to protect the interest of its students.

The same applies to minority institutions. They retain the privilege of offering seats to the students of their community, NEET will only provide a rank based on which minority institutions can create their merit list in a fair manner.

The time given by the Supreme Court to students to prepare for NEET is not too short either. Most students aspiring to join medical colleges prepare for CET or AIPMT. NEET is not different from existing exams conducted by the government. Time given for preparing for the exam is the same for everyone, standardising conditions for everybody.

I would like to request MCI to convert medical education into an apprenticeship programme like in the West, emphasising skill building rather than physical infrastructure. This will allow conversion of 500 district hospitals in backward parts of the country into medical colleges, with modest investment. In my experience doctors with fire in the belly and magic in their fingers, who have changed the face of medicine, come from deprived backgrounds. NEET is our last chance to give those underprivileged students a fair chance at a medical education, and build a corps of dedicated doctors that will ensure a healthy future for our country.

The writer is a cardiac surgeon and Chairman and Founder, Narayana Health.

Courtesy : The Times of India

CLASS XI Series 1

ACE YOUR WAY CBSE

Units and Measurements
Motion in a Straight Line

Time Allowed : 3 hours
Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION-A

1. Does a quantity have different dimensions in different systems of units?

2. If the size of a nucleus ($\approx 10^{-15}$m) is scaled up to the tip of a sharp pin ($\approx 10^{-5}$m), what roughly is the size of an atom?

3. In the expression, surface area = $4\pi r^2$, the factor 4 is an exact number. How many number of significant figures are there in the factor 4?

4. A body covered a distance of l metre along a semicircular path. Calculate the magnitude of displacement of the body and the ratio of distance to displacement.

5. Can earth be regarded as a point object when it is describing its yearly journey around the sun?

SECTION-B

6. Two railway tracks are parallel to west-east direction. Along one track, A moves with a speed of 54 km h^{-1} from west to east while along the second track, train B moves with a speed of 90 km h^{-1} from east to west. Find (in m s^{-1}) the (i) relative velocity of B with respect to A and (ii) relative velocity of earth with respect to B.

7. Find the value of 60 joule per minute on a system which has 100 g, 100 cm and 1 minute as the fundamental units.

OR

The equation of a wave is given by :

$$y = r\sin\omega\left(\frac{p}{v} - q\pi\right)$$

where the symbols have their usual meanings. What are the dimensions of p and q?

8. The position of a particle moving along a straight line is given by $x = 2 - 5t + 6t^2$.
Find the acceleration of the particle at $t = 2$ s.

9. Show dimensionally that the relation : $t = 2\pi\left(\sqrt{\frac{l}{g}}\right)$ is incorrect, where l is length, t is time period of a simple pendulum and g is acceleration due to gravity. Find correct form of the relation, dimensionally.

10. The displacement x of the body in motion is given by $x = A\sin(\omega t + \theta)$. Determine the time at which the displacement is maximum.

SECTION-C

11. (a) Explain the principle of homogeneity of dimensions.
(b) Pressure is defined as momentum per unit volume. Is it true?

12. (a) Give two drawbacks of dimensional analysis.
(b) What is the importance of dimensional analysis inspite of its drawbacks?

13. If a, b, c be the distances moved by a particle moving with a constant acceleration during the l^{th}, m^{th} and n^{th} second of its motion respectively, show that $a(m - n) + b(n - l) + c(l - m) = 0$.

OR

The driver of a car travelling at a velocity v suddenly sees a broad wall in front of him at a distance r. Is it better to brake or to turn sharply? Explain.

14. The velocity of a train increases at a constant rate α from 0 to v and then remains constant for some time interval and then finally decreases to zero at a constant rate β. If the total distance covered by the particle be x, then show that the total time taken will be
$$t = \frac{x}{v} + \frac{v}{2}\left[\frac{1}{\alpha} + \frac{1}{\beta}\right].$$

15. (a) State the rules for rounding off a measurement.
(b) Add 7.21, 12.141 and 0.0028 and express the result to an appropriate number of significant figures.

16. Which of the following is the most precise device for measuring length and why?
(a) A vernier calliper with 20 divisions on the sliding scale, coinciding with 19 main scale divisions where smallest division on the main scale is 1 mm.
(b) A screw gauge of pitch 1 mm and 100 division on the circular scale.
(c) An optical instrument that can measure length to within a wavelength of visible light.

17. An object has uniformly accelerated motion. The object always slows down before the time, when its velocity becomes zero. Prove this statement graphically, when (a) both u and a are positive (*b*) u is negative and a is positive (*c*) u is positive and a is negative and (*d*) both u and a are negative.

18. (a) What is the necessity of selecting some units as fundamental units ?
(b) For the estimation of Young's modulus of a wire, $Y = \frac{4Mg}{\pi d^2}\cdot\frac{L}{l}$.
For the specimen of a wire, following observation were recorded : $L = 2.890$, $M = 3.00$, $d = 0.082$, $l = 0.087$. Calculate the maximum percentage error in the value of Y and mention which physical quantity causes maximum error.

19. A balloon is ascending at the rate of 14 m s^{-1} at a height of 98 m above the ground when a packet is dropped from the balloon. After how much time and with what velocity does it reach the ground?

20. A raindrop starts falling from clouds at a considerable height above the surface of the Earth. During the fall, the raindrop experiences retardation due to air resistance. The retardation is directly proportional to the instantaneous speed of the drop. Obtain an expression for (a) the distance travelled by the drop in time t and (b) the terminal speed.

21. (a) What are positive and negative accelerations ? Give examples.
(b) The position of an object moving along x-axis is given by $x = a + bt^2$, where $a = 8.5$ m, $b = 2.5$ m s^{-2} and t is measured in seconds. What is its velocity at $t = 0$ *s* and $t = 2$ *s* ? What is the average velocity between $t = 2$ *s* and $t = 4$ *s* ?

22. (a) How can one determine (i) the distance (ii) the displacement covered by a uniformly accelerated body from its velocity-time graph?
(b) Draw position-time graphs for two objects having zero relative velocity.

SECTION-D

23. Read the given paragraph and answer the following questions :
As is known, the result of an experiment is calculated by performing mathematical operations (like addition, subtraction, multiplication, division, etc.) on several measurements, which have different degrees of accuracy. It has been established that
(a) When $x = a + b$; $\Delta x = \pm(\Delta a + \Delta b)$
(b) When $x = a - b$; $\Delta x = \pm(\Delta a + \Delta b)$
(c) When $x = a \times b$; $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

(d) When $x = \frac{a}{b}; \frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

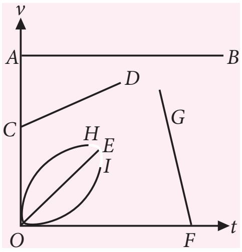
(i) Why is absolute error in $x = (a - b)$, sum of the absolute errors in a and b ?

(ii) Why is fractional error in $x = \frac{a}{b}$; sum of fractional errors in a and b ?

(iii) What do you learn from this?

SECTION-E

24. Figure shows $v - t$ graph for various situations. What does each graph indicate?



OR

The motion of a particle along a straight line is described by the equation : $x = 6 + 4t^2 - t^4$ where x is in metres and t is the positive time in seconds.

(i) Find the position, velocity and acceleration at $t = 2$ s.

(ii) During what time interval is the velocity positive ?

(iii) During what time interval is x positive?

(iv) What is the maximum possible velocity attained by the particle?

25. How is random error eliminated? What do you mean by (i) absolute error (ii) mean absolute error (iii) relative error and (iv) percentage error ?

OR

State the rules for counting the number of significant figures in a measured quantity.

26. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between $t = 0$ to 12 s.

OR

A man is standing on top of a building 100 m high. He throws two balls vertically, one at $t = 0$ and other after a time interval (less than 2 seconds). The later ball is thrown at a velocity of half the first. The vertical gap between first and second ball is +15 m at $t = 2$ s. The gap is found to remain constant. Calculate the velocity with which the balls were thrown and the exact time interval between their throw.

SOLUTIONS

1. No. A quantity has same dimensions in different systems of units.

2. Magnification $= \frac{\text{size of tip of pin}}{\text{size of nucleus}} = \frac{10^{-5}}{10^{-15}} = 10^{10}$

As actual size of atom $\approx 10^{-10}$ m and it is magnified 10^{10} times,

$\therefore$ Apparent size of atom $= 10^{-10} \times 10^{10} = 10^0 = 1$ m

3. In the expression, surface area $= 4\pi r^2$, the factor 4 is an exact number which has infinite number of significant figures. It can be written as 4 or 4.0 or 4.00 and so on.

4. Let r be the radius of semicircular path. Here, $l = (2\pi\, r/2)$ or $r = l/\pi$.

Magnitude of displacement = diameter = $2r = 2\, l/\pi$

$\therefore \quad \frac{\text{Distance}}{\text{Displacement}} = \frac{l}{2l/\pi} = \frac{\pi}{2}.$

5. Yes, because the diameter of the earth is very small as compared to the radius of the orbital path of earth around the sun.

6. Considering the positive direction of x-axis from west to east, we have

v_{AE} = Relative velocity of A with respect to earth = $+54$ km h^{-1} = $+15$ m s^{-1}

v_{BE} = Relative velocity of B with respect to earth = -90 km h^{-1} = -25 m s^{-1}

(i) Relative velocity of B with respect to A is

$v_{BA} = v_{BE} + v_{EA} = v_{BE} - v_{AE} = (-25) - (+15)$

$= -40$ m s^{-1}

(ii) Relative velocity of earth with respect to B is

$v_{EB} = v_{EE} + v_{EB} = v_{EE} - v_{BE} = 0 - (-25) = 25$ m s^{-1}

7. 60 J min^{-1} = 1 J s^{-1} = 1 watt

Now watt is the SI unit of power.

SI units	Units of new system
$n_1 = 1$	$n_2 = ?$
$M_1 = 1$ kg	$M_2 = 100$ g
$L_1 = 1$ m	$L_2 = 100$ cm
$T_1 = 1$ s	$T_2 = 1$ min

[Power] = [ML^{2}T^{-3}] $\therefore \quad a = 1;\; b = 2;\; c = -3$

Now, $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

$$=1\left[\frac{1\text{ kg}}{100\text{ g}}\right]^1\left[\frac{1\text{ m}}{100\text{ cm}}\right]^2\left[\frac{1\text{ s}}{1\text{ min}}\right]^{-3}$$

$$=1\left[\frac{1000\text{ g}}{100\text{ g}}\right]^1\left[\frac{100\text{ cm}}{100\text{ cm}}\right]^2\left[\frac{1\text{ s}}{60\text{ s}}\right]^{-3}=1[10]^1[1]^2\left[\frac{1}{60}\right]^{-3}$$

$= 10 \times 60 \times 60 \times 60 = 2.16 \times 10^6$ new units

OR

Since, $\omega\left(\frac{p}{v}-q\pi\right)$ is an angle, it is a dimensionless quantity. It means that $\omega p/v$ as well as $\omega q\pi$ is dimensionless.

$\therefore\ \frac{\omega p}{v}=[M^0L^0T^0]=1$ or $[p]=\frac{[v]}{[\omega]}=\frac{[LT^{-1}]}{[T^{-1}]}=[L]$

i.e., p has the dimensions of distance.

Also, $\omega q\pi = [M^0L^0T^0] = 1\ \therefore\ [q]=\left[\frac{1}{\omega}\right]=[T]$

i.e., q has the dimensions of time.

8. Given, $x = 2 - 5t + 6t^2$

$$v=\frac{dx}{dt}=\frac{d}{dt}(2-5t+6t^2)=-5+12t$$

$$\therefore\ a=\frac{dv}{dt}=\frac{d}{dt}(-5+12t)=12$$

Acceleration of the particle is constant and independent of time.

9. $[\text{RHS}]=\left[2\pi\left(\frac{l}{g}\right)\right]=\frac{[L]}{[LT^{-2}]}=[T^2]\neq[T]=[\text{LHS}]$

$\therefore$ Formula is incorrect.

Let $t = k\,l^a g^b$

$[M^0L^0T^1] = L^a[LT^{-2}]^b = [L^{a+b}T^{-2b}]$

Using principle of homogeneity of dimensions,

$a+b=0, -2b=1 \Rightarrow b=-\frac{1}{2}\ \therefore\ a=-b=\frac{1}{2}$

From (i), $t=k\,l^{1/2}g^{-1/2}=k\sqrt{\frac{l}{g}}$

10. The value of displacement x will be maximum, when the value of $\sin(\omega t+\theta)$ is maximum. It will be so if $\sin(\omega t+\theta) = 1 = \sin \pi/2$

or $\omega t+\theta=\frac{\pi}{2}$ or $\omega t=\frac{\pi}{2}-\theta$

$$\therefore\ t=\left(\frac{\pi}{2\omega}-\frac{\theta}{\omega}\right)$$

11. (a) According to this principle, a physical relation is dimensionally correct if the dimensions of fundamental quantities (mass, length and time etc.) are the same in each and every term on either side of the equation. This principle is based on the fact that only quantities of the same kind (or dimensions) can be added or subtracted. For example, consider the equation, $A = B + C$. Here the quantities A, B and C must have the same dimensions.

(b) No, because this is not dimensionally true.

$$\frac{\text{Momentum}}{\text{Volume}}=\frac{\text{Mass}\times\text{Velocity}}{\text{Volume}}=\frac{[MLT^{-1}]}{[L^3]}=[ML^{-2}T^{-1}]$$

$$\text{Pressure}=\frac{\text{Force}}{\text{Area}}=\frac{[MLT^{-2}]}{[L^2]}=[ML^{-1}T^{-2}]$$

12. (a) Two salient drawbacks of dimensional analysis are :

(i) The method does not tell us where the equation is wrong.

(ii) If the dimensions had been the same on each side of the equation, we would know only that it might be correct, for the method does not provide a check on numerical factors.

(b) In many physical situations, it is very difficult to obtain the formula of a physical quantity. It is because the mathematical analysis involved is to difficult. In such situations, dimensional analysis can be a powerful tool.

13. Let u be the initial velocity of the particle and A be its uniform acceleration. Using the relation

$D_n=u+\frac{A}{2}(2n-1)$, we have

$$a=u+\frac{A}{2}(2l-1) \quad \text{...(i)}$$

$$b=u+\frac{A}{2}(2m-1) \quad \text{...(ii)}$$

and $c=u+\frac{A}{2}(2n-1)$ ...(iii)

Subtracting (iii) from (ii)

$$b-c=\frac{A}{2}(2m-2n)=A(m-n)$$

$\therefore\ a(m-n) = a(b-c)/A = (ab-ac)/A$ ...(iv)

Subtracting (i) from (iii)

$$c-a=\frac{A}{2}(2n-2l)=A(n-l)$$

$\therefore\ b(n-l) = b(c-a)/A = (bc-ab)/A$ ...(v)

Subtracting (ii) from (i)

$$a-b=\frac{A}{2}(2l-2m)=A(l-m)$$

or $\quad c\,(l-m)=c\,(a-b)/A=(ac-bc)/A \quad$ ...(vi)

Adding (iv), (v) and (vi), we get

$a(m-n)+b(n-l)+c\,(l-m)$

$$=\frac{ab-ac}{A}+\frac{bc-ab}{A}+\frac{ac-bc}{A}=0$$

OR

It is better to apply brake than to turn sharply for the reason discussed here.

Let m be the mass of the car. When the driver applies the brakes, let the car stop at distance x. Then retardation, $a=v^2/2x$

Retarding force, $F=ma=\frac{mv^2}{2x}$ or $x=\frac{mv^2}{2F}$.

There will be no collision if $x\le r$

or $\frac{mv^2}{2F}\le r$ or $F\ge\frac{mv^2}{2r}$ ...(i)

If the driver takes a sharp turn of radius x, then centripetal force on car is,

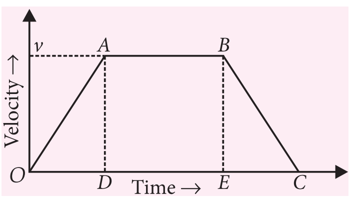
$$F'=\frac{mv^2}{x} \quad \text{or} \quad x=\frac{mv^2}{F'}$$

To avoid collision, $x\le r$ so $\frac{mv^2}{F'}\le r$

or $F'\ge\frac{mv^2}{r}$ ...(ii)

From (i) and (ii) we note that, to avoid collision braking force required is half the centripetal force. Therefore, braking is better.

14. The given motion can be represented by the velocity-time graph $OABC$ as shown in figure. The portion OA represents the motion with constant accleration α, the straight line AB parallel to time-axis represents the motion with uniform velocity v and the line BC represents the motion with constant deceleration β.



$$\alpha=\text{slope of } OA=\frac{AD}{OD}=\frac{v}{OD}$$

$$\therefore \quad OD=\frac{v}{\alpha}$$

$$\beta=\text{slope of } BC=\frac{BE}{EC}=\frac{v}{EC}$$

$$\therefore \quad EC=\frac{v}{\beta}$$

Now, total distance covered

= Area under the graph $OABC$

or $\quad x=\frac{1}{2}[AB+OC]\times AD$

$$=\frac{1}{2}[DE+OD+DE+EC]\times AD \qquad [\because AB=DE]$$

$$=\frac{1}{2}\left[2DE+\frac{v}{\alpha}+\frac{v}{\beta}\right]\times v$$

or $\quad \frac{2x}{v}=2\,DE+\frac{v}{\alpha}+\frac{v}{\beta}$

or $\quad DE=\frac{1}{2}\left[\frac{2x}{v}-\frac{v}{\alpha}-\frac{v}{\beta}\right]$

Now, total time taken is given by

$$t=OD+DE+EC$$

$$=\frac{v}{\alpha}+\frac{1}{2}\left[\frac{2x}{v}-\frac{v}{\alpha}-\frac{v}{\beta}\right]+\frac{v}{\beta}$$

or $\quad t=\frac{x}{v}+\frac{v}{2}\left[\frac{1}{\alpha}+\frac{1}{\beta}\right].$

15. (a) Rules for rounding off a measurement :

(i) If the digit to be dropped is smaller than 5, then the preceding digit is left unchanged.

(ii) If the digit to be dropped is greater than 5, then the preceding digit is increased by 1.

(iii) If the digit to be dropped is 5, followed by non-zero digit , then the preceding digit is increased by 1.

(iv) If the digit to be dropped is 5, then the preceding digit is left unchanged if it is even.

(v) If the digit to be dropped is 5, then the preceding digit is increased by 1 if it is odd.

(b)

	7.21
	+ 12.141
	+ 0.0028
Sum	= 19.3538

Corrected sum = 19.35

[Rounded off upto 3rd decimal place]

Here 7.21 has minimum number of decimal places (two), so result is rounded off upto second place of decimal point.

16. The device that has minimum least count will be more precise for measuring length.

(a) Least count of vernier callipers

$$= 1\text{ MSD} - 1\text{ VSD} = 1\text{ MSD} - \frac{19}{20}\text{MSD} = \frac{1}{20}\text{MSD}$$

$$= \frac{1}{20}\times 1\text{mm} = \frac{1}{200}\text{cm} = 0.005\text{ cm.}$$

(b) Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{No. of division on circular scale}}$$

$$= \frac{1.0\text{mm}}{100} = \frac{1}{1000}\text{cm} = 0.001\text{ cm}$$

(c) Least count of optical instrument

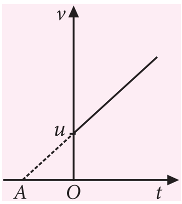
= Wavelength of visible light

= 5000 Å = 5000×10^{-8} cm

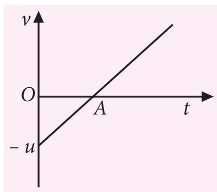
= 0.00005 cm

Hence the most precise device for measuring length is the given optical instrument.

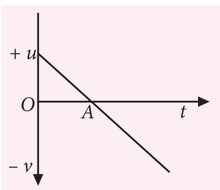
17. (a) When both u and a are positive : In such a case, the v-t graph will be as shown in figure. At the time corresponding to point A, the velocity becomes zero. It can be seen that before this time, the velocity is negative but its magnitude decreases with time till it becomes zero at A.



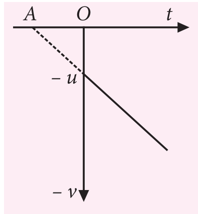
(b) When u is negative and a is positive : In this case, graph will be as shown in figure. At the time corresponding to point A, the velocity becomes zero. It can be seen that before this time the velocity is negative but its magnitude decreases with time till it becomes zero at A.



(c) When u is positive and a is negative : In such a case, graph between v and t will be as shown in figure. Again at A, velocity is zero. The velocity decreases before the time corresponding to point A.



(d) When both u and a are negative: In this case, v - t graph will be as shown in figure. If we produce graph backwards, it meets the time-axis at point A. Before this time velocity is positive and decreases till it becomes zero at point A.



18. (a) The number of physical units required to be measured is very large. If a separate unit is defined for each of them, then it will become very difficult to remember all of them as they will be quite unrelated to each other.

(b) Given, $Y = \frac{4Mg}{\pi d^2}\cdot\frac{L}{l}$

As 4, g and π are constants so the percentage error in Y will be

$$\frac{\Delta Y}{Y}\times 100\,\% = \left(\frac{\Delta M}{M} + \frac{\Delta L}{L} + 2\frac{\Delta d}{d} + \frac{\Delta l}{l}\right)\times 100\%$$

It is known from the given data that

$\Delta M = 0.01$, $\Delta L = 0.001$, $\Delta d = 0.001$, $\Delta l = 0.001$

$$\therefore \frac{\Delta Y}{Y}\times 100\% = \left(\frac{0.01}{3.00} + \frac{0.001}{2.890} + \frac{2\times 0.001}{0.082} + \frac{0.001}{0.087}\right)\times 100\%$$

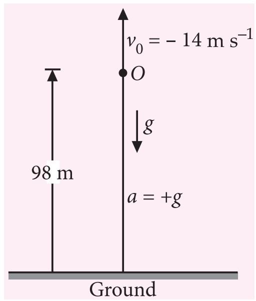
= (0.0033 + 0.0003 + 0.0244 + 0.0115) × 100%

= 0.0395 × 100% = 3.95%

Diameter of the wire causes maximum error in the value of Y.

19. Let a point O, which is 98 m above the ground be taken as the origin, and the vertically downward direction as positive as shown in figure.

When the packet is dropped from the balloon, at O, it has the same velocity (v_0) as that of the balloon, *i.e.*, 14 m s^{-1} in the vertically upward direction. In order to reach the ground, it must fall through a height of 98 m.



From $h = v_0 t + \frac{1}{2}gt^2$,

$$98 = -14t + \frac{1}{2}\times 9.8t^2$$

or $4.9t^2 - 14t - 98 = 0$

or $49t^2 - 140t - 980 = 0$

or $7t^2 - 20t - 140 = 0$

or $t = \left[\frac{20 \pm \sqrt{400+3920}}{14}\right]$ s

$= \left[\frac{20 \pm 65.7}{14}\right]$ s

Thus, either $t = \left(\frac{20+65.7}{14}\right)\text{s} = 6.1\text{ s}$

or $\quad t = \left(\frac{20-65.7}{14}\right)\text{s} = -3.3\text{ s}$

Neglecting negative time, $t = 6.1$ s

Let v be the velocity with which the packet hits the ground.

Thus, $v = v_0 + gt = [-14 + (9.8)(6.1)]\text{ m s}^{-1}$

or $\quad v = [-14 + 60]\text{ m s}^{-1} = 46\text{ m s}^{-1}$

20. Instantaneous retardation due to air resistance = αv where α is a constant of proportionality

Clearly, net instantaneous acceleration,

$a = g - \alpha v$

or $\quad \frac{dv}{dt} = (g - \alpha v)$ or $\frac{dv}{(g-\alpha v)} = dt$

Integrating, $\int_0^v \frac{dv}{g-\alpha v} = \int_0^t dt$

or $\quad \left|\frac{\ln(g-\alpha v)}{-\alpha}\right|_0^v = |t|_0^t = t$

or $\quad [\ln(g - \alpha v) - \ln g] = -\alpha t$

or $\quad \ln\left(\frac{g-\alpha v}{g}\right) = -\alpha t$

or $\quad \frac{g-\alpha v}{g} = e^{-\alpha t}$

or $\quad v = \frac{g}{\alpha}(1 - e^{-\alpha t}) \qquad$ ...(i)

or $\quad \frac{ds}{dt} = \frac{g}{\alpha}(1-e^{-\alpha t})$

or $\quad ds = \frac{g}{\alpha}(1-e^{-\alpha t})dt$

Integrating, $\int_0^s ds = \frac{g}{\alpha}\int_0^t (1-e^{-\alpha t})dt$

$$= \frac{g}{\alpha}\int_0^t dt - \frac{g}{\alpha}\int_0^t e^{-\alpha t}dt$$

or $\quad |s|_0^s = \frac{g}{\alpha}|t|_0^t - \frac{g}{\alpha}\left|\frac{e^{-\alpha t}}{-\alpha}\right|_0^t$

or $\quad s = \frac{g}{\alpha}t + \frac{g}{\alpha^2}e^{-\alpha t} - \frac{g}{\alpha^2}$

i.e., $\quad s = \frac{g}{\alpha^2}(e^{-\alpha t} - 1) + \frac{g}{\alpha}t$

(b) From eqn. (i), it is clear that v increases with t and attains its highest value, called the terminal speed, when $t = \infty$ or $e^{-\alpha t} = 0$. Clearly, terminal speed = g/α

21. (a) Positive acceleration: If the velocity of an object increases with time, its acceleration is positive. When a bus leaves a bus-stop, its acceleration is positive.

Negative acceleration: If the velocity of an object decreases with time, its acceleration is negative. Negative acceleration is also called retardation or deceleration. When a bus slows down on approaching a bus-stop, its acceleration is negative.

(b) Given $\quad x = a + bt^2$

Instantaneous velocity,

$v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 0 + b \times 2t = 2bt$

At $\quad t = 0, v = 0$

At $\quad t = 2$ s, $v = 2\times2.5 \times 2 = 10\text{ m s}^{-1}$

At $\quad t = 2$ s, $x = a + 4b$

At $\quad t = 4$ s, $x = a + 16b$

Average velocity

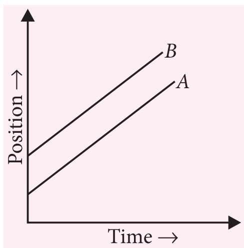
$= \frac{x_2 - x_1}{t_2 - t_1} = \frac{(a+16b)-(a+4b)}{4-2} = 6b$

$= 6 \times 2.5 = 15\text{ m s}^{-1}$

22. (a) (i) The distance travelled by a body in a given interval of time is equal to total area of velocity time graph, without considering sign. It means, even if the body is moving with negative velocity, the area of velocity time graph is to be taken positive for the measurement of distance travelled by the body.

(ii) Displacement of a body in a given interval of time is equal to total area of velocity-time graph, during the given interval of time, which is to be added with proper sign.

(b) As the relative velocity is zero, the two bodies A and B have equal velocities. Hence their position-time graph are parallel straight lines, equally inclined to the time-axis as shown in figure.



23. (i) We have made errors twice ; in measuring a and in measuring b. Therefore, absolute error in $x = (a - b)$ has to be sum of the two errors.

(ii) Same argument as in (i) applies to fractional error in $x = a/b$.

(iii) From this, we learn that errors committed any number of times just add. No mathematical operation can reduce the net overall error. So be warned. Multiple errors are going to cost you dearly.

24. Acceleration = slope of $v - t$ graph

(i) Straight line AB indicates that the acceleration of the moving body is zero. Clearly, the body is moving with constant velocity.

(ii) Straight line CD indicates that the body has constant positive acceleration with initial velocity OC. In this case, the velocity of the body is increasing.

(iii) Straight line OE indicates that the body has positive constant acceleration with zero initial velocity.

(iv) Curve OI shows the increasing acceleration. Here the slope of the graph increases with time.

(v) Curve OH indicates decreasing acceleration. Here the slope of the graph decreases with time.

(vi) The straight line FG indicates that the body is moving with constant negative acceleration. Here the slope of the graph is negative. It means the velocity of the body is decreasing at a constant rate.

OR

(i) $x = 6 + 4t^2 - t^4$

Velocity, $v = \dfrac{dx}{dt} = \dfrac{d}{dt}(6 + 4t^2 - t^4) = 8t - 4t^3$

Acceleration, $a = \dfrac{dv}{dt} = \dfrac{d}{dt}(8t - 4t^3) = 8 - 12t^2$

At $t = 2$ s, $x = 6 + 4(2)^2 - (2)^4 = 6$ m

$v = 8 \times 2 - 4(2)^3 = -16$ m s^{-1}

$a = 8 - 12(2)^2 = -40$ m s^{-2}

(ii) The velocity v is positive if $8t - 4t^3 \geq 0$. That is if $4t(2 - t^2) > 0$

i.e., $0 < t < \sqrt{2}$ s or $0 < t < 1.41$ s

(iii) The position x is positive if $6 + 4t^2 - t^4 \geq 0$.

$t^2 \leq 2 \pm 3.16$ s. Since t is positive, t lies between 0 and $\sqrt{5.16}$ or 0 and 2.27 s.

(iv) The velocity is maximum for t given by

$\dfrac{dv}{dt} = 0$ or $8 - 12t^2 = 0$ or $t = \sqrt{\dfrac{2}{3}} = 0.816$ s

$\therefore$ Maximum velocity, $v_{max} = (8t - 4t^3)_{t = 0.816\text{ s}}$

$= [8 \times 0.816 - 4 \times (0.816)^3] = 4.355$ m s^{-1}

25. Elimination of error : Normal or Gaussian law of random errors shows that the probability of occurrence of positive and negative errors is same, so random error can be minimised by repeating measurements a large number of times. Then the arithmetic mean of all measurements can be taken as the true value of the measured quantity.

If $a_1, a_2, a_3, ...a_n$ be the n measured values of a physical quantity, then its true value, $\bar{a}$ is given by the arithmetic mean,

$$\bar{a} \text{ or } a_{mean} = \frac{a_1 + a_2 + a_3 + ... + a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i.$$

(i) Absolute error: The magnitude of the difference between the true value of the quantity measured and the individual measured value is called absolute error. If we take arithmetic mean $\bar{a}$ as the true value, then the absolute errors in the individual measured values will be

$\Delta a_1 = \bar{a} - a_1$, $\Delta a_2 = \bar{a} - a_2$, $\Delta a_3 = \bar{a} - a_3$, ...

$\Delta a_n = \bar{a} - a_n$

(ii) Mean or final absolute error : The arithmetic mean of the positive magnitudes of all the absolute errors is called mean absolute error. It is given by

$$\frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

Thus the final result of the measurement of a physical quantity can be expressed as $a = \bar{a} \pm \Delta\bar{a}$. Clearly, any measured value of a will be such that $\bar{a} - \Delta\bar{a} \leq a \leq \bar{a} + \Delta\bar{a}$

(iii) Relative error : The ratio of the mean absolute error to the true value of the measured quantity is called relative error.

Relative error, $\delta a = \dfrac{\Delta\bar{a}}{a}$

(iv) Percentage error : The relative error expressed in percent is called percentage error.

Percentage error $= \dfrac{\Delta\bar{a}}{\bar{a}} \times 100\%$

OR

Rules for determining the number of significant figures :

(i) All non-zero digits are significant. So 13.75 has four significant figures.

(ii) All zeros between two non-zero digits are significant. Thus 100.05 km has five significant figures.

(iii) All zeros to the right of a non-zero digit but to the left of an understood decimal point are not significant. For example, 86400 has three significant figures.

(iv) All zeros to the right of a non-zero digit but to the left of a decimal point are significant. For example, 648700. has six significant figures.

(v) All zeros to the right of a decimal point are significant. So 161 cm, 161.0 cm and 161.00 cm have three, four and five significant figures respectively.
(vi) All zeros to the right of a decimal point but to the left of a non-zero digit are not significant. So 0.161 cm and 0.0161 cm, both have three significant figures. Moreover, zero conventionally placed to the left of the decimal point is not significant.
(vii) The number of significant figures does not depend on the system of units. So 16.4 cm, 0.164 m and 0.000164 km, all have three significant figures.

26. Taking vertical downward motion of ball from a height 90 m, we have

$u = 0,\ a = 10\text{ m s}^{-2};\ S = 90\text{ m},\ t = ?\ v = ?$

$$t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2\times 90}{10}} = 3\sqrt{2}\text{ s} = 4.24\text{ s}$$

$$v = \sqrt{2aS} = \sqrt{2\times 10\times 90} = 30\sqrt{2}\text{ m s}^{-1}$$

Rebound velocity of ball,

$$u' = \frac{9}{10}v = \frac{9}{10}\times 30\sqrt{2} = 27\sqrt{2}\text{ m s}^{-1}$$

Time to reach the highest point is,

$$t' = \frac{u'}{a} = \frac{27\sqrt{2}}{10} = 2.7\sqrt{2} = 3.81\text{ s}$$

Total time $= t + t' = 4.24 + 3.81 = 8.05$ s

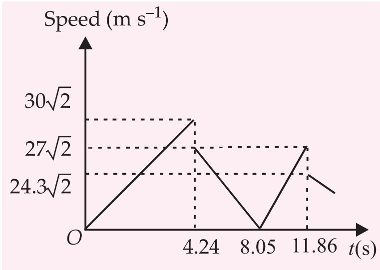
The ball will take further 3.81 s to fall back to floor, where its velocity before striking the floor $= 27\sqrt{2}\text{ m s}^{-1}$

Velocity of ball after striking the floor

$$= \frac{9}{10}\times 27\sqrt{2} = 24.3\sqrt{2}\text{ m s}^{-1}$$

Total time elapsed before upward motion of ball = 8.05 + 3.81 = 11.86 s

Thus the speed-time graph of this motion will be as shown in figure.



OR

Let the speed of the two balls (1 and 2) be u_1 and u_2, where $u_1 = 2u$, $u_2 = u$.

If y_1 and y_2 are the distances covered by the balls 1 and 2 respectively, before coming to rest, then

$$y_1 = \frac{u_1^2}{2g} = \frac{4u^2}{2g}\text{ and } y_2 = \frac{u_2^2}{2g} = \frac{u^2}{2g}$$

Since $y_1 - y_2 = 15$ m, $\frac{4u^2}{2g} - \frac{u^2}{2g} = 15\text{ m}$

or $\frac{3u^2}{2g} = 15\text{ m}$ or $u^2 = \sqrt{5\text{m}\times(2\times 10\text{ m s}^{-2})}$

or $u = 10\text{ m s}^{-1}$

Clearly, $u_1 = 20\text{ m s}^{-1}$ and $u_2 = 10\text{ m s}^{-1}$

As $y_1 = \frac{u_1^2}{2g} = \frac{(20\text{ m s}^{-1})^2}{2\times 10\text{ m s}^{-2}} = 20\text{ m},$

$y_2 = y_1 - 15\text{ m} = 5\text{ m}$

If t_2 is the time taken by the ball 2 to cover a distance of 5 m, then from $y_2 = u_2 t - \frac{1}{2}gt_2^2,$

$$5 = 10t_2 - 5t_2^2$$

or $t_2^2 - 2t_2 + 1 = 0$, or $t_2 = 1$ s

Since t_1 (time taken by ball 1 to cover a distance of 20 m) is 2 s, time interval between the two throws $= t_1 - t_2 = 2\text{ s} - 1\text{ s} = 1\text{ s}$

CENTURY OF PERFECT SCORE IN 1 SUBJECT

Gurgaon : More than 100 students from the city scored a perfect 100 in at least one subject in the Class XII board examinations, results of which were announced on 21st May.

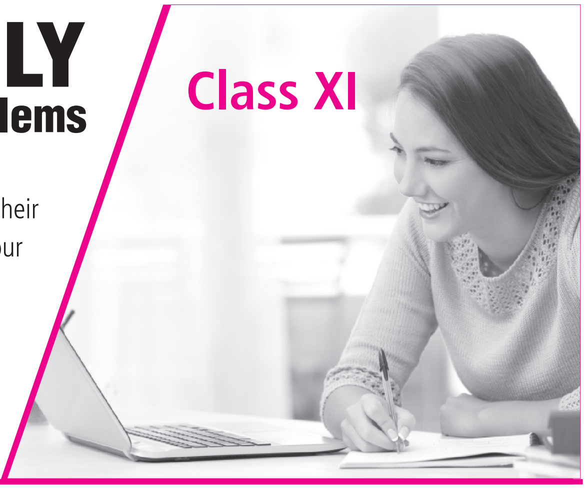
And once again, girls have outshone boys, according to schools. "Most 100% scorers in any subject are girls," said Shikha Deswal, a teacher in the city. "I think girls are more attentive and have better concentration in the classroom," she said.

Most of the centum scorers said studying NCERT books and solving previous years' question papers was practice enough.

Courtesy : The Times of India

MPP-1 MONTHLY Practice Problems

Class XI



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Units and Measurements

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS / PMTs

Only One Option Correct Type

1. Which of the following is the most accurate measurement ?
(a) 20×10^{-3} m (b) 200×10^{-4} m
(c) 2×10^{-2} m (d) 0.02 m

2. When a current of (2.5 ± 0.5) A flows through a wire, it developes a potential difference of (20 ± 1)V. The resistance of wire is
(a) $(8 \pm 2)\ \Omega$ (b) $(8 \pm 1.6)\ \Omega$
(c) $(8 \pm 1.5)\ \Omega$ (d) $(8 \pm 3)\ \Omega$

3. $\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left(\frac{x}{a} - 1\right)$. The value of n is
(a) 0 (b) -1
(c) 1 (d) None of these

4. Crane is British unit of volume (one crane = 170.4742 L). Convert crane into SI units.
(a) 0.170474 m^3 (b) 17.0474 m^3
(c) 0.00170474 m^3 (d) 1704.74 m^3

5. The radius of the proton is about 10^{-15}m. The radius of the observable universe is 10^{26} m. Identify the distance which is half-way between, these two extremes on a logarithmic scale.
(a) 10^{21} m (b) 10^6 m (c) 10^{-6} m (d) 10^0 m

6. The pitch of a screw guage is 1 mm and there are 100 divisions on the circular scale. While measuring diameter of a thick wire, the pitch scale reads 1 mm and 63rd division on the circular scale coincides with the reference. The length of the wire is 5.6 cm. Then
(a) The least count of screw guage is 0.001 mm
(b) The volume of the wire is 0.117 cm^3
(c) The diameter of the wire is 1.63 m
(d) The cross-section area of the wire is 0.0209 cm^3

7. There are atomic clocks capable of measuring time with an accuracy of 1 part in 10^{11}. If two such clocks are operated with precision, then after running for 5000 yr, these will record a difference of
(a) nearly 2 s (b) 1 day
(c) 10^{11} s (d) 1 yr

8. SI unit of intensity of wave is
(a) J $m^{-2}s^{-2}$ (b) J $m^{-1}s^{-2}$
(c) W m^{-2} (d) J m^{-2}

9. The equation of stationary wave is $y = A\sin kx \cos \omega t$, where, y and x are in metre and t in second. Choose the correct option.
(a) The dimensions of A and k are same
(b) The dimensions of A, k and ω are same
(c) The dimensions of k and ω are same
(d) The dimensions of (kx) and (ωt) are same

10. A container contains 35 kg water. 0.2 kg water leaks from the container. Find the amount of water in container.
(a) 34.8 kg (b) 35 kg
(c) 34.80 kg (d) 35.0 kg

11. In the relation $p = \frac{\alpha}{\beta} e^{-\frac{\alpha Z}{k\theta}}$, where p is pressure, Z is distance, k is Boltzmann constant and θ is the temperature. The dimensional formula of β will be
(a) $[M^0L^2T^0]$ (b) $[ML^2T]$
(c) $[ML^0T^{-1}]$ (d) $[M^0L^2T^{-1}]$

12. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. The thickness of hair is
(a) 0.035 mm (b) 0.04 mm
(c) 0.35 mm (d) 0.40 mm

Assertion & Reason Type

Directions : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion
(b) If both assertion and reason are true but reason is not the correct explanation of assertion
(c) If assertion is true but reason is false
(d) If both assertion and reason are false.

13. **Assertion :** A dimensionally correct equation may or may not be correct.

Reason : A dimensionally incorrect equation must be wrong.

14. **Assertion :** If two quantities have same dimensions, then they must represent same physical quantities.

Reason : Work and torque have different dimensions.

15. **Assertion :** Power of a engine depends on mass, angular speed, torque and angular momentum, then the formula of power is not derived with the help of dimensional method.

Reason : In mechanics, if a particular quantity depends on more than three quantities, then we can not derive the formula of the quantity by the help of dimensional method.

JEE MAIN / JEE ADVANCED / PETs

Only One Option Correct Type

16. A gas bubble from an explosion under water oscillates with a period T proportional to $P^a d^b E^c$, where P is the static pressure, d is the density and E is the total energy of the explosion. The values of a, b and c are
(a) $a = 0, b = 1, c = 2$ (b) $a = 1, b = 2, c = 3$
(c) $a=\frac{5}{6}, b=\frac{1}{2}, c=\frac{1}{3}$ (d) $a=\frac{-5}{6}, b=\frac{1}{2}, c=\frac{1}{3}$

17. A system has basic dimensions as density $[D]$, velocity $[V]$ and area $[A]$. The dimensional representation of force in this system is
(a) $[AV^2D]$ (b) $[A^2VD]$
(c) $[AVD^2]$ (d) $[A^0VD]$

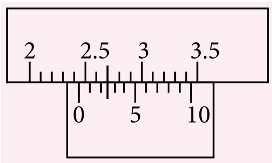
18. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°), then the least count of the instrument is
(a) one minute (b) half minute
(c) one degree (d) half degree

19. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of 0.01 mm. Take $g = 9.8$ m s^{-2} (exact). The Young's modulus obtained from the reading is
(a) $(2.0 \pm 0.3)\ 10^{11}$ N m^{-2}
(b) $(2.0 \pm 0.2) \times 10^{11}$ N m^{-2}
(c) $(2.0 \pm 0.1) \times 10^{11}$ N m^{-2}
(d) $(2.0 \pm 0.05) \times 10^{11}$ N m^{-2}

More than One Options Correct Type

20. If p, ρ, g and h denote pressure, density, acceleration due to gravity and height, respectively. In the case of flow of fluid, $\frac{p}{\rho g}+\frac{v^2}{2g}+h=u_0$ where v denotes velocity.
Mark the correct options.
(a) The equation is dimensionally correct.
(b) If dimensions of u_0 are $[M^0LT^0]$, the equation is dimensionally correct.
(c) Dimensions of $\frac{p}{\rho g}$ are same as that of $\frac{v^2}{2g}$.
(d) None of the above.

21. In the given figure, main scale is graduated in millimeter. Mark the correct options, if least count is 0.1 mm.



(a) The main scale reading is 2.4 cm.
(b) The vernier scale reading is 0.3 mm.
(c) The total reading is 2.43 cm.
(d) The total reading is 2.4 cm.

22. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stopwatch with the least count of

1 s for this and records 40 s for 20 oscillations. For the observation, which of the following statement(s) is/are correct?
(a) Error ΔT in measuring T, the time period is 0.05 s.
(b) Error ΔT in measuring T, the time period is 1 s.
(c) Percentage error in the determination of g is 5%.
(d) Percentage error in the determination of g is 2.5%.

23. If $\left(A+\frac{B}{C^2}\right)(D-x)=y$, then
(a) dimensions of A and B must be same
(b) dimensions of A and B may be same
(c) dimensions of D and x must be same
(d) dimensions of AD and y must be same

Integer Answer Type

24. The time period of oscillation of a body is given by

$$T=2\pi\sqrt{\frac{mgA}{K}}$$

K : Represents the kinetic energy, m mass, g acceleration due to gravity and A is unknown.
If $[A]=M^x L^y T^z$
Then what is the value of $x+y+z$?

25. If $P=\frac{x^3y^3}{z\sqrt{w}}$, the percentage error in x, y, z and w are 1%, 1%, 1% and 4% respectively. Find percentage error in P.

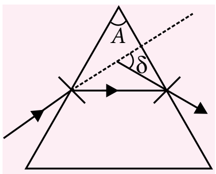
26. The plate current in a triod can be written as

$$I=K\left(V_g+\frac{V_p}{\mu}\right)^{3/2}.$$

Here, V_g and V_p denote potential. In the dimensional formula of K, find the ratio of dimensions of length and mass.

Comprehension Type

The refractive index μ of the material of glass prism is given by $\mu=\dfrac{\sin\left(\dfrac{A+\delta_{min}}{2}\right)}{\sin\dfrac{A}{2}}$



Glass prism

where, A = Angle of prism, δ_{min} = Angle of minimum deviation.

A glass prism of 60° gives angle of minimum deviation as 36° with the maximum error of 1.05° when a beam of parallel light passed through the prism during experiment. (Given, cot 48° = 0.900404 and sin 48° = 0.743).

27. Find the percentage error in the measurement of refractive index of the material of the prism.
(a) 1% (b) 1.65%
(c) 4% (d) 3.3%

28. Find the range of experimental value of refractive index μ.
(a) $1.46\le\mu\le1.51$ (b) $2\ge\mu\ge1.51$
(c) $0<\mu<\infty$ (d) $1.46\le\mu\le2.51$

Matrix Match Type

29.

Column I	Column II
(A) Inductance	(P) $[K^{-1}ML^2T^{-2}]$
(B) Electric resistivity	(Q) $[M^{-1}T^{-1}]$
(C) Specific activity	(R) $[A^{-2}ML^3T^{-3}]$
(D) Boltzmann's constant	(S) $[A^{-2}ML^2T^{-2}]$

	A	B	C	D
(a)	P	Q	R	S
(b)	Q	P	S	R
(c)	R	S	P	Q
(d)	S	R	Q	P

30.

Column I	Column II
(A) Same dimension in mass	(P) electric field
(B) same dimension in time	(Q) magnetic field
(C) (–1) dimension in current	(R) inductive reactance
(D) zero dimension in length	(S) resistivity

	A	B	C	D
(a)	P,S	P,Q,R	R,S	P
(b)	P,Q,R	P,R	R,S	Q
(c)	P,Q,R,S	P,Q,R	P,Q	Q
(d)	P,Q,R,S	P,R	P,Q	P

◈◈

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CLASS XII Series 2

ACE YOUR WAY CBSE

Current Electricity

Time Allowed : 3 hours
Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.

(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.

(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.

(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.

(v) Q. no. 23 is a value based question and carries 4 marks.

(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.

(vii) Use log tables if necessary, use of calculators is not allowed.

Previous Years Analysis

	2016		2015		2014	
	Delhi	AI	Delhi	AI	Delhi	AI
VSA	–	1	1	1	1	–
SA-I	1	1	1	1	1	2
SA-II	–	1	1	1	1	1
VBQ	–	–	–	–	–	–
LA	1	–	–	–	–	–

SECTION-A

1. A wire of resistivity ρ is stretched to three times its length. What will be its new resistivity?

2. If 3.2×10^{17} electrons pass through a wire in 0.5 s, calculate the current through it.

3. Distinguish between static electricity and current electricity.

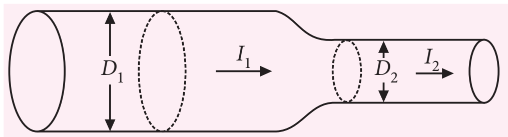
4. What is the colour code for a resistor of resistance 3.5 kΩ with 5% tolerance ?

5. State the condition in which terminal voltage across a cell or battery is equal to its emf.

SECTION-B

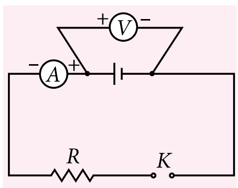
6. Two heater wires of the same dimensions are first connected in series and then in parallel to a source of supply. What will be the ratio of heat produced in the two cases?

7. Current flows through a constricted conductor, as shown in figure. The diameter D_1 = 2.0 mm and the current density to the left of the constriction is $j_1 = 1.27 \times 10^6$ A m^{-2}. (i) What current flows into the constriction? (ii) If the current density is doubled as it emerges from the right side of the constriction, what is diameter D_2?



8. A 10 C of charge flows through a wire in 5 minutes. The radius of the wire is 1 mm. It contains 5×10^{22} electrons per centimetre^3. Calculate the drift velocity.

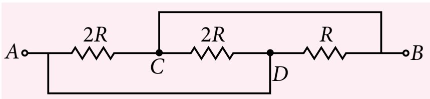
9 In the circuit shown in figure, the voltmeter reads 1.5 V, when the key is open. When the key is closed, the voltmeter reads 1.35 V and ammeter reads 1.5 A. Find the emf and the internal resistance of the cell.



OR

A battery of emf ε, and internal resistance r, gives a current of 0.5 A with an external resistor of 12 Ω and a current of 0.25 A with an external resistor of 25 Ω. Calculate (i) internal resistance of the cell and (ii) emf of the cell.

10. What is the equivalent resistance between points A and B of the circuit shown in figure.



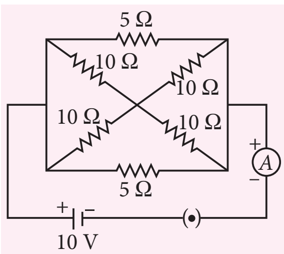
SECTION-C

11. (a) At what temperature, would the resistance of a copper conductor be double of its resistance at 0°C? Given that α for copper = 6.8×10^{-3} °C^{-1}.

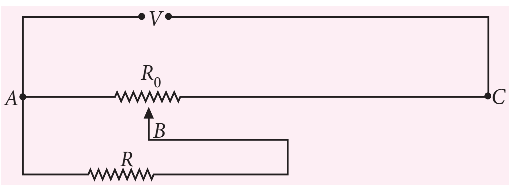
(b) Does this temperature hold for all copper conductors regardless of shape or size?

12. Calculate the relaxation time and mean free path in copper at room temperature of 27°C if the number density of free electrons per unit volume is 8.5×10^{28} m^{-3} and resistivity of copper is 1.7×10^{-8} Ω m. [Given that mass of an electron, $m = 9.1 \times 10^{-31}$ kg; electronic charge, $e = 1.60 \times 10^{-19}$ C and Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J K^{-1}

13. Calculate the current shown by the ammeter A in the circuit arrangement of figure.



14. A resistance of R Ω draws current from a potentiometer. The potentiometer has a total resistance R_0 Ω. A voltage V is supplied to the potentiometer. Derive an expression for the voltage across R when the sliding contact is in the middle of the potentiometer.



15. Find the minimum number of cells required to produce an electric current of 1.5 A through a resistance of 30 Ω. Given that the emf and internal resistance of each cell are 1.5 V and 1.0 Ω respectively.

OR

A set of 4 cells, each of emf 2 V and internal resistance 1.5 Ω, are connected across an external load of 10 Ω with 2 rows, 2 cells in each branch. Calculate current in each branch and potential difference across 10 Ω.

16. A wire of resistance 16 Ω is cut into 4 identical parts. Each part is stretched to twice its original length. If all the four wires, after stretching, are connected in parallel, calculate the new resistance of the combination.

17. A potential difference V is applied to a conductor of length L, diameter D. How are the electric field E, the drift velocity v_d and the resistance R affected when (i) V is doubled (ii) L is doubled (iii) D is doubled?

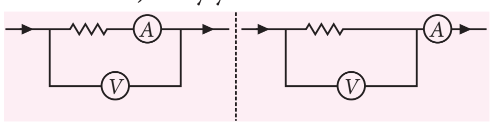
18. (a) Why do we prefer a potentiometer to measure emf of a cell rather than a voltmeter ?

(b) What do you understand by sensitivity of a potentiometer and how can you increase the sensitivity of a potentiometer?

19. A series battery of 6 cells each of emf 2 V and internal resistance 0.5 Ω is charged by a 100 V d.c. supply. What resistance should be used in the charging circuit in order to limit the charging current to 8 A? Using this, obtain (a) the power supplied by the d.c. source, (b) the power dissipated as heat and (c) the chemical energy stored in the battery in 15 minutes.

20. At the temperature 0°C, the electric resistance of conductor B is n times that of conductor A. Their temperature coefficients of resistance are equal to α_2 and α_1 respectively. Find the resistance and temperature coefficients of resistance of a circuit segment consisting of these two conductors when they are connected in series.

21. You are given two resistors X and Y whose resistances are to be determined using an ammeter of resistance 0.5 Ω and a voltmeter of resistance 20 kΩ. It is known that X is in the range of a few ohms while Y is in the range of several thousand ohms. In each case, which of the following two connections figure would you choose for resistance measurement ? Justify your answer.



22. A cylindrical aluminium shell is made of two coaxial cylinders of inner radius 1.0 cm and outer radius 2.0 cm and length 3.0 m. Estimate the resistance of conductor between the cylinders, given that resistivity of aluminium is $2.63 \times 10^{-8}\ \Omega$ m at room temperature.

SECTION-D

23. On a cold morning while going for office, Mrs. Siddiqui observed that her car was not starting. She realised that the parking lights of her car were left on by mistake and that is why the car battery was discharged. Shyam who is a neighbour and also a class XII student observed her problem and came running and adviced not to apply the self for long time. In order to solve the problem he took out another battery from his own car and connected its terminals to the terminals of the car battery to get the engine started. Once the engine was on , he disconnected his car battery . This is known as 'Jump starting'. Mrs. Siddiqui appreciated his efforts and thanked him.

(a) What values did Shyam have?

(b) Why did he advise not to apply the self for longer duration?

(c) Why did he remove his battery after jump start , the same problem may have occurred again and again as the battery was discharged?

(d) A storage battery of emf 8 V and internal resistance 0.5 Ω is being charged by a 120 V d.c. supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?

SECTION-E

24. Deduce the condition for balance in a Wheatstone Bridge. Using the principle of Wheatstone Bridge, describe the method to determine the resistance of a wire in the laboratory. Draw the circuit diagram and write the formula used.

OR

State the working principle of a potentiometer. With the help of the circuit diagram, explain how a potentiometer is used to compare the emf's of two primary cells. Obtain the required expression used for comparing the emfs.

25. A cell of unknown emf ε and internal resistance r, two unknown resistances R_1 and $R_2 (R_2 > R_1)$ and a perfect ammeter are given. The current in the circuit is measured in five different situations:

(i) Without any external resistance in the circuit,

(ii) With resistance R_1 only,

(iii) With resistance R_2 only,

(iv) With both R_1 and R_2 used in series combination, and

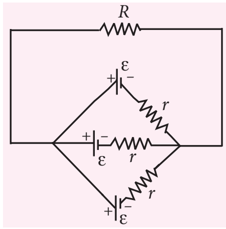
(v) With R_1 and R_2 used in parallel combination.

The current obtained in the five cases are 0.42 A, 0.6 A, 1.05 A, 1.4 A and 4.2 A, but not necessarily in that order. Identify the currents in the given five cases and calculate ε, R_1 and R_2 in terms of r.

OR

(a) State Kirchhoff's rules for an electrical network. Explain their use by drawing a simple circuit diagram.

(b) Find the total current I, supplied to an external resistance R, connected as shown across a parallel combination of three cells of equal emf ε and same internal resistance r.



26. (i) Define the term drift velocity.

(ii) On the basis of electron drift, derive an expression for resistivity of a conductor in terms of number density of free electrons and relaxation time. On what factors does resistivity of a conductor depend ?

(iii) Why alloys like constantan and manganin are used for making standard resistors ?

OR

Define electrical resistivity of a material. Explain how electrical resistivity vary in conductors, alloys and semiconductors with temperature. Also plot the variation graphs.

SOLUTIONS

1. There will be no change in the resistivity of the wire as it depends only upon the material of the wire and not on any other parameter (length, area of cross-section) of the wire.

2. Here, $n = 3.2 \times 10^{17}$, $t = 0.5$ s, $e = 1.6 \times 10^{-19}$C

As $I = \frac{Q}{t} = \frac{ne}{t} = \frac{3.2 \times 10^{17} \times 1.6 \times 10^{-19}}{0.5} = 0.1024$ A

3. Static electricity deals with the study of charges at rest and current electricity deals with charges in motion.

4. Given, resistance = $3.5\ k\Omega \pm 5\% = 35 \times 10^2\ \Omega \pm 5\%$

$\therefore$ Colour code of given resistor is orange, green, red and gold.

5. The terminal potential difference across a cell or battery or source of emf is equal to its emf when no current is being drawn from the cell, *i.e.*, the cell is in an open circuit.

6. As heater wires are of the same dimensions, their resistances are equal, say R each. In series, their resultant resistance $R_s = 2R$. Now, if connected to supply source of voltage V for time t, the heat produced, $H_s = \frac{V^2 t}{R_s} = \frac{V^2 t}{2R}$

In parallel, the resultant resistance, $R_p = \frac{R}{2}$

For same supply voltage V and for same time t, the heat produced,

$$H_p = \frac{V^2 t}{R_p} = \frac{V^2 t}{(R/2)} = \frac{2V^2 t}{R} \Rightarrow \frac{H_p}{H_s} = 4$$

7. Here $D_1 = 2.0$ mm, $j_1 = 1.27 \times 10^6$ A m^{-2}

(i) Current flowing into the constriction,

$$I_1 = j_1 \ A = j_1 \times \pi \left(\frac{D_1}{2}\right)^2$$

$$= 1.27 \times 10^6 \times 3.14 \times (1 \times 10^{-3})^2 = 3.988 \text{ A}$$

(ii) For a steady flow of current, $I_1 = I_2$

or $j_1 A_1 = j_2 A_2$

or $$j_1 \times \pi \left(\frac{D_1}{2}\right)^2 = j_2 \times \pi \left(\frac{D_2}{2}\right)^2$$

or $$j_1 \times \pi \left(\frac{D_1}{2}\right)^2 = 2j_1 \times \pi \left(\frac{D_2}{2}\right)^2 \qquad [\because j_2 = 2j_1]$$

or $$D_2 = \frac{1}{\sqrt{2}} D_1 = \frac{1}{\sqrt{2}} \times 2.0 \text{ mm} = 1.414 \text{ mm}$$

8. $$I = \frac{q}{t} = \frac{10 \text{ C}}{5 \times 60 \text{ s}} = 3.33 \times 10^{-2} \text{ A}$$

$$v_d = \frac{I}{enA} = \frac{I}{en(\pi r^2)}$$

$$= \frac{3.33 \times 10^{-2}}{1.6 \times 10^{-19} \times 5 \times 10^{22} \times 10^6 \times 3.14 \times (10^{-3})^2}$$

$$= 1.326 \times 10^{-6} \text{ m s}^{-1}.$$

9. When the key is open, the voltmeter reads almost the emf of the cell.

$\therefore \quad \varepsilon = 1.5$ V

When the key is closed, voltmeter reads the terminal potential difference V.

$V = 1.35$ V, $I = 1.5$ A, $r = ?$

$$r = \frac{\varepsilon - V}{I} = \frac{1.5 - 1.35}{1.5} = 0.1 \ \Omega$$

OR

In first case, $\varepsilon = 0.5\ (12 + r)$

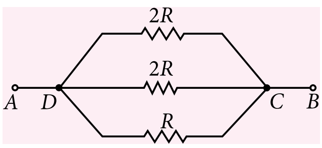
in second case, $\varepsilon = 0.25\ (25 + r)$

$\therefore \quad 0.5\ (12 + r) = 0.25\ (25 + r)$

On solving, we get $r = 1\ \Omega$

Hence, $\varepsilon = 0.5\ (12 + 1) = 6.5$ V

10. The points A and D are equipotential points. Also, the points B and C are equipotential points.



So the given network of resistances reduces to the equivalent circuit shown in figure.

The three resistances form a parallel combination. Their equivalent resistance R_{eq} is given by

$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R} = \frac{1+1+2}{2R} = \frac{2}{R} \text{ or } R_{eq} = R/2$$

11. (a) As per question, $R_T = 2R_0$

and for copper, $\alpha = 6.8 \times 10^{-3}$ °C^{-1}

As $R_T = R_0[1 + \alpha(T - T_0)]$

$\therefore \quad 2R_0 = R_0[1 + 6.8 \times 10^{-3}\ (T - T_0)]$

$$\Rightarrow \quad (T - T_0) = \frac{2-1}{6.8 \times 10^{-3}} = 147°\text{C}$$

$\therefore \quad T = T_0 + 147$°C $= (273 + 147)$ K $= 420$ K

(b) This temperature holds good for all copper conductors regardless of shape or size because resistivity of a substance is independent of its dimensions.

12. As per question, $n = 8.5 \times 10^{28}$ m^{-3}; resistivity, $\rho = 1.7 \times 10^{-8}\ \Omega$ m; $T = 27$°C $= 300$ K; $m = 9.1 \times 10^{-31}$ kg; $e = 1.60 \times 10^{-19}$ C and $k_B = 1.38 \times 10^{-23}$ J K^{-1}

As resistivity, $\rho = \frac{m}{n e^2 \tau}$

$\Rightarrow$ Relaxation time, $\tau = \frac{m}{n e^2 \rho}$

$$= \frac{9.1 \times 10^{-31}}{8.5 \times 10^{28} \times (1.60 \times 10^{-19})^2 \times (1.7 \times 10^{-8})}$$

$= 2.5 \times 10^{-14}$ s

The rms speed of an electron at room temperature,

$$\bar{u} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{9.1 \times 10^{-31}}} = 1.2 \times 10^5 \text{ m s}^{-1}$$

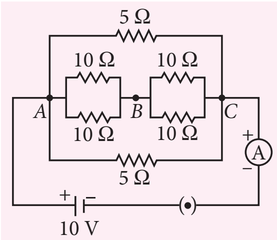
MPP-1 CLASS XI ANSWER KEY

1.	(b)	2.	(a)	3.	(a)	4.	(a)	5.	(b)
6.	(b)	7.	(a)	8.	(c)	9.	(d)	10.	(b)
11.	(a)	12.	(a)	13.	(b)	14.	(d)	15.	(a)
16.	(d)	17.	(a)	18.	(a)	19.	(b)	20.	(b,c)
21.	(a,b,c)	22.	(a,c)	23.	(b,c,d)	24.	(3)	25.	(9)
26.	(2)	27.	(b)	28.	(a)	29.	(d)	30.	(c)

$\therefore$ Mean free path, $\lambda = \bar{u}\tau = 1.2 \times 10^5 \times 2.5 \times 10^{-14}$
$= 3 \times 10^{-9}$ m or 3 nm

13. The circuit arrangement may be redrawn as shown in figure.

Here, resistances of 10 Ω and 10 Ω are joined in parallel between points A and B and form an effective resistance,



$$R_1 = \frac{10 \times 10}{10 + 10} = 5\ \Omega$$

Similarly, effective resistance between points B and C,

$R_2 = 5\ \Omega$

Thus, net resistance of branch $ABC = 5 + 5 = 10\ \Omega$

Now, it is obvious that between points A and C, we have three resistances of 5 Ω, 10 Ω and 5 Ω joined in parallel.

Hence, equivalent resistance of network,

$$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{10} + \frac{1}{5} \Rightarrow R_{eq} = 2\ \Omega$$

$\therefore$ Current drawn from the cell (or ammeter reading)

$$I = \frac{\varepsilon}{R_{eq}} = \frac{10\ \text{V}}{2\ \Omega} = 5\ \text{A}$$

14. When the slide is in the middle of the potentiometer, only half of its resistance, *i.e.*, $R_0/2$ will be between the points A and B.

Hence, total resistance (say, R_1) between points A and B will be

$$R_1 = \frac{\left(\frac{R_0}{2}\right) \cdot R}{\left(\frac{R_0}{2} + R\right)} = \frac{R_0 R}{R_0 + 2R} \quad \text{...(i)}$$

$\therefore$ Total resistance between points A and $C = R_1 + \dfrac{R_0}{2}$

$\therefore$ The current flowing through potentiometer,

$$I = \frac{V}{R_1 + \frac{R_0}{2}} = \frac{2V}{2R_1 + R_0}$$

Voltage across R,

$$V_1 = IR_1 = \frac{2VR_1}{(2R_1 + R_0)} \quad \text{...(ii)}$$

On substituting value of R_1 from (i) in (ii), we have

$$V_1 = \frac{2V}{\left[2\left(\frac{R_0 R}{R_0 + 2R}\right) + R_0\right]} \times \left[\frac{R_0 R}{R_0 + 2R}\right]$$

$$= \frac{2V R_0 R}{2R_0 R + R_0(R_0 + 2R)} \Rightarrow V_1 = \frac{2VR}{R_0 + 4R}$$

15. Here, $\varepsilon = 1.5$ V, $r = 1.0\ \Omega$, $R = 30\ \Omega$, $I = 1.5$ A

As r is comparable to R, so cells are to be connected in mixed grouping for maximum current. Let there be n cells in series in each row and m such rows in parallel. Then, $\varepsilon_{eq} = n\varepsilon$ and $R_{eq} = R + \dfrac{nr}{m}$

$\therefore$ Current in the mixed grouping of cells is

$$I = \frac{mn\varepsilon}{mR + nr} \quad \therefore 1.5 = \frac{mn \times 1.5}{m \times 30 + n \times 1}$$

or $\quad 45\,m + 1.5\,n = 1.5\,m\,n$ ...(i)

For maximum current, $R = \dfrac{nr}{m}$

so, $\quad 30 = \dfrac{n \times 1}{m}$

or $\quad n = 30\,m$ ...(ii)

Putting this value in (i), we have

$45\,m + 1.5 \times 30\,m = 1.5\,m \times 30\,m$

or $45\,m + 45\,m = 45\,m^2$

or $\quad 90 = 45\,m$ or $m = 2$

From (ii), $\quad n = 30 \times 2 = 60$

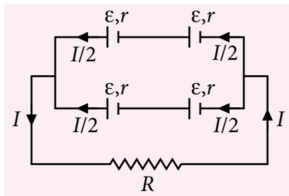
$\therefore$ Total number of cells required

$= m\,n = 2 \times 60 = 120$

OR

Let ε, r be the emf and internal resistance of each cell respectively.

The cells are connected as shown in figure.



Here, $\varepsilon = 2$ V, $r = 1.5\ \Omega$, $R = 10\ \Omega$

Effective emf of the two cells in a row

$= 2\,\varepsilon = 2 \times 2 = 4$ V

Effective emf of the 4 cells in two rows

ε' = emf of two cells in a row = 4 V

The effective internal resistance of two cells in a row

$= 1.5 + 1.5 = 3.0\ \Omega$

Total internal resistance r' of the 4 cells is

$$r' = \frac{3 \times 3}{3 + 3} = \frac{3}{2} = 1.5\ \Omega$$

Total resistance of circuit

$= R + r' = 10 + 1.5 = 11.5\ \Omega$

Current in circuit,

$$I = \frac{\varepsilon'}{R + r'} = \frac{4}{11.5} = 0.348\ \text{A}$$

As two branches are identical, current in each branch $= \dfrac{I}{2} = \dfrac{0.348}{2} = 0.174$ A

Potential difference across the external resistance

$= IR = 0.348 \times 10 = 3.48$ V

16. As per question, a wire of resistance $R = 16\ \Omega$ is cut into 4 equal parts, so the resistance of each part,

$$R_1 = \frac{16}{4}\ \Omega = 4\ \Omega$$

Now, each part of a wire, say having length l is stretched to twice its length, *i.e.*, $l' = 2\,l$. Correspondingly, the cross-section area of wire changes from A to A' such that $A'l' = Al \Rightarrow A' = \frac{Al}{l'} = \frac{Al}{2l} = \frac{A}{2}$

$\therefore$ New resistance of each part,

$$R_1' = \frac{\rho l'}{A'} = \frac{\rho 2l}{(A/2)} = 4\frac{\rho l}{A} = 4\,R_1 = 4\times 4 = 16\ \Omega$$

Now, all these four stretched wires, each of resistance $R_1' = 16\ \Omega$ are connected in parallel, hence new resistance of combination,

$$R_2 = \frac{R_1'}{4} = \frac{16}{4} = 4\ \Omega$$

17. We know, Electric field, $E = \frac{V}{L}$

Drift velocity, $v_d = \frac{eE}{m}\tau = \frac{eV}{mL}\tau$

Resistance, $R = \rho\frac{L}{A} = \frac{4\rho L}{\pi D^2}$

(i) When V is doubled, E becomes double, v_d becomes double and R remains unchanged.

(ii) When L is doubled, E becomes half, v_d becomes half and R becomes double.

(iii) When D is doubled, E remains unchanged, v_d also remains unchanged and R becomes one-fourth.

18. (a) A potentiometer does not draw any current from the cell whose emf is to be determined, whereas a voltmeter always draws some little current. Therefore, emf measured by voltmeter is slightly less than actual value of emf of the cell.

(b) The sensitivity of a potentiometer means the smallest potential difference that can be measured with its help. The sensitivity of a potentiometer can be increased by decreasing its potential gradient. This can be achieved by increasing the length of potentiometer wire. If the potentiometer wire is of fixed length, the potential gradient can be decreased by reducing the current in the potentiometer wire circuit with the help of rheostat.

19. Given, number of cells $n = 6$

EMF of each cell, $\varepsilon = 2$ V

internal resistance of each cell, $r = 0.5\ \Omega$; charging voltage $V = 100$ V. Let R be the resistance used in series with the circuit while charging the cells. Then current in the circuit will be

$$I = \frac{V - n\varepsilon}{nr + R} \text{ or } R = \frac{V - n\varepsilon}{I} - nr = \frac{100 - 6\times 2}{8} - 6\times 0.5$$
$$= 11 - 3 = 8\ \Omega$$

(a) Power supplied by d.c. source

$= V \times I = 100 \times 8 = 800$ W

(b) Power dissipated as heat

$= I^2\,(R + nr) = 8^2\,(8 + 6\times 0.5) = 704$ W

(c) Rate at which the chemical energy is stored

$= 800 - 704 = 96$ W

$\therefore$ Chemical energy stored in 15 min

$= 96 \times 15 \times 60 = 86400$ J

20. Let R_0 be the resistance of conductor A at 0°C. Resistance of conductor B at 0°C $= nR_0$.

If R_{t_1}, R_{t_2} are the resistances of conductors A and B at t°C, then

$R_{t_1} = R_0\,(1 + \alpha_1 t)$ and $R_{t_2} = n\,R_0\,(1 + \alpha_2 t)$

If R_s is the resistance at t°C when they are connected in series then

$$R_s = R_{t_1} + R_{t_2} = R_0(1 + \alpha_1 t) + n\,R_0(1 + \alpha_2 t)$$
$$= R_0\,(1 + n) + R_0\,(\alpha_1 + n\alpha_2)t$$
$$= R_0(1+n)\left[1 + \frac{(\alpha_1 + n\alpha_2)}{(1+n)}t\right] \quad \text{...(i)}$$

If R_0' is the resistance of combination of two conductors at 0°C and α is the temperature coefficient of these two conductors in series, then

$$R_s = R_0'(1 + \alpha t) \quad \text{...(ii)}$$

Comparing (i) and (ii), we have

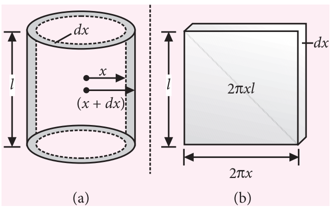
$$R_0' = R_0\,(1 + n) \text{ and } \alpha = \left(\frac{\alpha_1 + n\alpha_2}{1 + n}\right)$$

21. (i) For the determination of X : Let $X = 10\ \Omega$. In the arrangement (a), potential difference determined is across $(10 + 0.5)\ \Omega$ instead of $10\ \Omega$. The error involved is approximately $(0.5/10) \times 100 = 5\%$. In arrangement (b), potential difference is determined across $10\ \Omega$. Thus the arrangement (b) is better.

(ii) For the determination of Y : Let $Y = 10,000\ \Omega$. In the arrangement (a) potential difference determined is across $(10,000 + 0.5)\ \Omega$ where in the arrangement (b), potential difference is determined across $10,000\ \Omega$. Since both the potential difference are almost equal, the arrangement (a) or the arrangement (b) can be used. Theoretically speaking, arrangement (b) is better.

22. Let l be the length of the cylinder and r_1, r_2 be its internal and external radii.

Consider a coaxial cylindrical shell of radius x and thickness dx as shown in figure. If the shell is cut parallel to its axis and spread out, it forms a rectangular sheet of surface area $2\pi xl$ and thickness dx as shown in figure. The resistance (dR) of this imaginary cylindrical shell (which is now in the form of a conductor of area $2\pi xl$ and length dx is given by



$$dR = \rho\frac{dx}{2\pi xl} = \frac{\rho}{2\pi l}\left(\frac{dx}{x}\right) \qquad \left(\text{as } R = \rho\frac{l}{A}\right)$$

If R is the resistance of the entire cylindrical shell (between the two radii r_1 and r_2),

$$R = \frac{\rho}{2\pi l}\int_{r_1}^{r_2}\frac{dx}{x} = \frac{\rho}{2\pi l}|\ln x|_{r_1}^{r_2}$$

$$= \frac{\rho}{2\pi l}\ln\frac{r_2}{r_1} = 2.3026\frac{\rho}{2\pi l}\log\frac{r_2}{r_1}$$

Here; $r_1 = 1$ cm $= 1 \times 10^{-2}$ m, $r_2 = 2$ cm $= 2 \times 10^{-2}$ m, $l = 3$ m and $\rho = 2.63 \times 10^{-8}\ \Omega$ m

Thus, $R = 2.3026 \times \frac{2.63\times10^{-8}}{2\times3.14\times3}\left(\log\frac{2}{1}\right)\Omega$

or $R = 2.3026 \times \frac{2.63\times10^{-8}}{2\times3.14\times3}\times 0.3010\ \Omega = 9.7\times10^{-10}\ \Omega$

23. (a) Shyam showed the values of being helpful, decision making ability, concern for the problems of others, application of theoretical knowledge and alertness.

(b) When the self is applied, a very large current is drawn from the battery of the order of 100 A, the battery cannot provide such high current for long duration. Hence self should be applied for short durations only; otherwise the battery may get discharged completely.

(c) Once the engine is started, the alternator connected to engine produce sufficient current and charges the battery, hence the discharged battery will again get charged soon.

(d) Current in the circuit

$I = (120 - 8)/(15.5 + 0.5) = 7$ A

Terminal voltage of the battery being charged

$V = E + Ir = 8 + 7 \times 0.5 = 11.5$ V

24. **Refer to point 2.5 (4, 6) page no. 100, 101 (MTG Excel in Physics)**

OR

Refer to point 2.5 (7) page no. 101, 102 (MTG Excel in Physics)

25. The electric current flowing in the circuit from the cell is inversely proportional to the resistance in the circuit for same applied emf, as $I = \frac{\varepsilon}{R + r}$

As resistances of circuit are in following order

$$(R_s = R_1 + R_2) > R_2 > R_1 > \left(R_p = \frac{R_1R_2}{R_1 + R_2}\right) > (R = 0)$$

So, currents in the circuit are in following order

(I in series combination) < (I through R_2)
< (I through R_1) < (I in parallel combination)
< (I in absence of R)

So,

(i) I without external resistance = 4.2 A
(ii) I with R_1 and R_2 in parallel = 1.4 A
(iii) I with R_1 alone = 1.05 A
(iv) I with R_2 alone = 0.6 A
(v) I with R_1 and R_2 in series = 0.42 A

Without any resistance in circuit

$I = \frac{\varepsilon}{r} = 4.2$ A. So, $\varepsilon = 4.2\,r$ ...(i)

With resistance R_1 in circuit

$I = \frac{\varepsilon}{R_1 + r} = 1.05$ A

or $4.2r = 1.05\,(R_1 + r)$, or $4r = R_1 + r$

or $R_1 = 3r$ ...(ii)

With resistance R_2 in circuit

$I = \frac{\varepsilon}{R_2 + r} = 0.6$ A

or $4.2r = 0.6\,(R_2 + r)$, or $7r = R_2 + r$

or $R_2 = 6r$ ...(iii)

OR

(a) **Refer to point 2.5 (1,2,3) page no. 99, 100 (MTG Excel in Physics)**

(b) Three cells are connected in parallel, so

Net emf = ε

Net internal resistance = $r/3$

Net external resistance = R

So, total electric current I supplied to external resistance R is

$$I = \frac{\varepsilon}{R + r/3} = \frac{3\varepsilon}{3R + r}$$

26. (i) **Refer to point 2.1(3) page no. 92 (MTG Excel in Physics)**

(ii) **Refer to points 2.1(4) and 2.2(4, 9(a)) page no. 92-94 (MTG Excel in Physics)**

(iii) **Refer to point 2.2(9(b)), page no. 94 (MTG Excel in Physics)**

OR

Refer to point 2.2 (3,9) page no. 93, 94 (MTG Excel in Physics)

BRAIN MAP

CLASS XI

UNITS AND MEASUREMENTS

MEASUREMENT

defined by the

- **Units of measurement** — Internationally accepted — **SI units**
  - that can be related in terms of fundamental base quantities → **Dimension**
  - SI units which consist of
    - seven Base SI units
    - Derived units
    - Supplementary units
    - Practical units in different scales
      - Small scale
        - Length → 1Å (angstrom) = 10^{-10} m; 1 fermi = 10^{-15} m
        - Mass → mass of atoms in a.m.u.
      - Large scale
        - Mass → 1 metric ton = 10^3 kg; 1 solar mass ≈ 10^{30} kg
        - Intersteller distance → 1 AU = 1.49×10^{11} m; 1 ly = 9.46×10^{15} m; 1 parsec = 3.08×10^{16} m — Method to measure such intersteller distances from earth → **Parallex Method**
- **Dimension**
  - using five base quantities: mass M, temperature K, electric current A, length L, time T
  - which can be deduced by → **Dimensional analysis**, which is useful to
    - Show the relationship between different system of units
    - Implicitly tell how to derive a relation
    - Provide a check on relation between quantities
- **Magnitude of measurement** — accurately known digits plus first uncertain digit in a measurement → **Significant figures**

ERROR

- Uncertainty in the measured values → Combination of errors — for basic operations
  - $Z = A \pm B$; $\Delta Z = \Delta A + \Delta B$
  - $Z = A^2$; $\frac{\Delta Z}{Z} = 2\frac{\Delta A}{A}$
  - $Z = A{\cdot}B$ or A/B; $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
- arising from
  - **Systematic error** — causes → range of observed values — determined by → **Precision** of measurement
  - **Random error**
    - results in → scatter of repeated measurement — characterized by
      - range of observed values
      - deviation of mean from "true value" — determined by → **Accuracy** of measurement
      - which can be calculated by
        - **Absolute error** $\Delta a_n = a_n - a_{mean}$
        - **Relative error** Rel $\Delta a = \frac{\Delta a_m}{a_m}$
        - **Percentage error** $\delta_a = \frac{\Delta a_m}{a_m} \times 100\%$
    - caused by → noise (fluctuation in values)
  - **Least count error** — caused by → error in resolution of the instrument

CLASS XII

ELECTRIC CHARGES AND FIELDS

Basic Properties of Charges

- **Quantization of charge :** Total charge on a body is always an integral multiple of a basic unit of charge denoted by e and is given by $q = ne$.
- **Conservation of charge :** Total charge of an isolated system remains unchanged with time.
- **Additivity of charge :** Total charge of a system is the algebraic sum (*i.e.* sum taking into account with proper signs) of all individual charges in the system.

Coulomb's Law

$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\vec{r}_{12}$ for like charges

$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^3}\vec{r}_{21}$ for unlike charges

using Superposition principle

- The vector sum of forces would give us the total force.
- Force between two charges is unaffected by the presence of the other charges.

Electric Field

Electric Field : Electric field intensity at a point distant r from a point charge q in air is $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$.

Basic characteristics

- Field lines start from positive charges and end at negative charges.
- Electrostatic field lines do not form any closed loops.
- Two field lines can never cross each other.

Electric Dipole

Electric Dipole : Every dipole is associated with a dipole moment $\vec{p}$ whose magnitude is equal to the product of the magnitude of either charge (q) and the distance $2a$ between the charges, *i.e.*, $\vec{p} = q \times (2\vec{a})$.

- Electric field due to dipole
  - On **Equatorial line** (Broad on position) $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$
  - On **Axial line** (End on position) $E = \frac{2p}{4\pi\varepsilon_0 r^3}$
- Dipole in an external field experiences → **Torque** $\vec{\tau} = \vec{p} \times \vec{E}$ or $\tau = pE\sin\theta$.

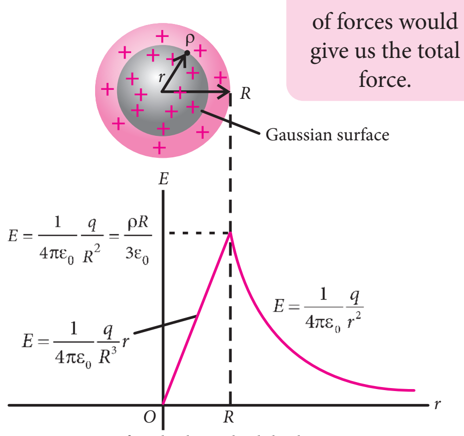
Electric Flux

Electric Flux : Electric flux over an area in an electric field represents the total number of electric field lines crossing this area.

Gauss's Theorem : Total normal electric flux over a closed surface S in vacuum is $1/\varepsilon_0$ times the charge (Q) contained inside the surface. $\phi_E = \oint_S \vec{E}.d\vec{S} = \frac{Q}{\varepsilon_0}$.

Applications

- **volume charge density, $\rho = \frac{q}{V}$** → Electric field due to **a uniformly charged thin spherical shell**
  - At a point outside the shell *i.e.*, $r > R$, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2}$
  - On the surface $r = R$, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$
  - Inside the shell *i.e.*, $r < R$, $E = 0$
- **surface charge density, $\sigma = \frac{q}{A}$** → Electric field due to **a uniformly charged infinite thin plane sheet**, $E = \frac{\sigma}{2\varepsilon_0}$.
- **linear charge density, $\lambda = \frac{q}{l}$** → Electric field due to **an infinitely long thin uniformly charged straight wire**, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$.



For uniformly charged solid sphere

NEET | JEE ESSENTIALS

Class XII

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Unit 1 ELECTROSTATICS

Electric Charge and its Properties

- Charge is a basic property associated with the elementary particles. It is scalar.
- Elementary charge (e) is one of the fundamental constants with the value, $e = -1.602 \times 10^{-19}$ C.
- There are two kinds of charges, positive and negative.
- Mass, length and time depend on state of rest or motion while charge is invariant (theory of relativity).
- Electric charge is quantised
  $q = ne$; $n = 0, \pm1, \pm2, \pm3, \ldots\ldots$
- The charge of an isolated system is conserved.
- Amount of induced charge ≤ inducing charge, sign of equality holds only for metals.

Coulomb's Law and Factors Affecting it

- Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}; \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{ N m}^2\text{ C}^{-2}$$

- Electric force between two charges does not depend on neighbouring charges.
- If a dielectric slab (ε_r) of thickness t is placed between two charges kept at a distance d then the electric force decreases. This reduced force is given by

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \text{ where } r = d - t + t\sqrt{\varepsilon_r}$$

- If $F_g = F_e$ for two identical charges, then

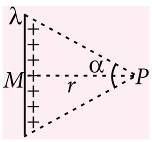
$$\frac{q}{m} = \sqrt{4\pi\varepsilon_0 G}$$

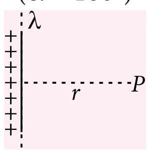
- When two charges q_1 and q_2 are placed some distance apart, neutral point is nearer to smaller charge and in between q_1 and q_2 if charges are alike.

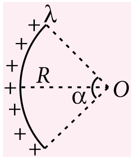
Electric Field

- Charge $\rightleftarrows$ Field $\rightleftarrows$ Charge
  First charge sets up an electric field and the second charge interacts with the electric field of the first charge.
- $$\vec{E} = \frac{\vec{F}}{q_0} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$
  The direction of $\vec{E}$ is the same as the direction of $\vec{F}$ because q_0 (test charge) is a positive scalar.
- Electric force on a charge in a uniform electric field (E) is constant hence acceleration is constant so equation of motion can be used (acceleration $a = qE/m$).
- Electric field due to linear charge distribution

- Finite wire
- Infinite wire ($\alpha = 180°$)
- Charged arc

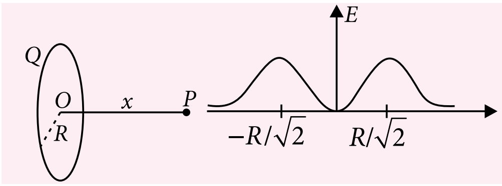






$E_P = \dfrac{2k\lambda}{r}\sin\dfrac{\alpha}{2}$ $\quad E_P = \dfrac{2k\lambda}{r}$ $\quad E_O = \dfrac{2k\lambda}{r}\sin\dfrac{\alpha}{2}$

- Charged ring of radius R



$$E_P = \frac{kQx}{(R^2+x^2)^{3/2}};\ \text{if}\begin{cases} x >> R \Rightarrow E_P = \dfrac{kQ}{x^2} \\ x << R \Rightarrow E_P = \dfrac{kQx}{R^3} \end{cases}$$

  - As x increases : $\vec{E}$ due to ring first increases then decreases and at $x = \dfrac{\pm R}{\sqrt{2}}$ it is maximum.
- Force between two parallel wires having linear charge density λ_1 and λ_2 is

  $F(\text{on unit length}) = \dfrac{2k\lambda_1\lambda_2}{d}$
- A small metal ball is suspended in a uniform $\vec{E}$ with the help of insulated thread. If X-ray beam falls on it, the ball will be deflected in the direction of $\vec{E}$.
- If both $\vec{E}$ and $\vec{g}$ (gravity) are present, then use resultant acceleration.
  - $\vec{F}_E$ and $\vec{F}_g$ are in same direction : $a = g + \dfrac{qE}{m}$
  - $\vec{F}_E$ and $\vec{F}_g$ are in opposite direction :

    $a = g - \dfrac{qE}{m}$
  - $\vec{F}_E$ and $\vec{F}_g$ are perpendicular to each other :

    $a^2 = g^2 + \left[\dfrac{qE}{m}\right]^2$

  e.g. Time period of simple pendulum, $T = 2\pi\sqrt{\dfrac{L}{a}}$ (a = net acceleration)
- If identical charges are placed on each vertix of a regular polygon, then $\vec{E}$ at the centre = zero.

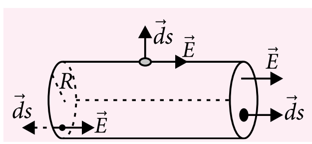
Lines of Electric Force

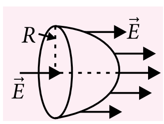
- Lines of force are drawn in such a way that the tangent to a line of force at a point gives the direction of the resultant electric field there.
- **Properties :**
  - They can never cross each other.
  - They can never form a closed loop.
  - They end or start normally at the surface of a conductor.
  - Lines of force per unit normal area at a point represent magnitude of intensity, crowded lines represent strong field while distant weak field.
  - The number of lines originating or terminating on a charge is proportional to the magnitude of charge.

Electric Flux (ϕ)

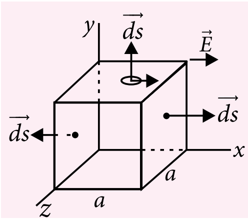
- $\phi = \vec{E}\cdot\vec{S} = ES\cos\theta$

  $\vec{S}$ is always normal to surface and pointed outwards.
- For open surface, $\phi_o = \int \vec{E}\cdot\vec{ds}$

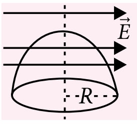
  For closed surface, $\phi_c = \oint \vec{E}\cdot\vec{ds}$
- The value of ϕ does not depend upon the distribution of charges and distance between them inside the closed surface.
- The value of ϕ is zero in the following circumstances :
  - If a dipole is (or many dipoles are) enclosed by a closed surface.
  - Magnitude of positive and negative charges are equal inside a closed surface.
  - If no charge is enclosed by the closed surface.
  - Incoming flux (–ve) = outgoing flux (+ve)
- Electric flux through some known structures
  - 

    $\phi_{in} = -\pi R^2 E$ and $\phi_{out} = \pi R^2 E$ $\Rightarrow \phi_{total} = 0$
  - 

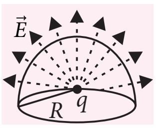
    $\phi_{in} = \phi_{circular} = -\pi R^2 E$ and $\phi_{out} = \phi_{curved} = \pi R^2 E$ $\Rightarrow \phi_{total} = 0$



$$\phi_{in} = -a^2E$$
and $\phi_{out} = a^2E$
$\Rightarrow \phi_{total} = 0$



$$\phi_{in} = -\frac{1}{2}\pi R^2 E$$
and $\phi_{out} = \frac{1}{2}\pi R^2 E$
$\Rightarrow \phi_{total} = 0$



$$E = \frac{kq}{R^2}$$
$$\phi = 2\pi R^2 \times \frac{q}{4\pi\varepsilon_0 R^2} = \frac{q}{2\varepsilon_0}$$

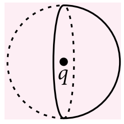
Note : Here electric field is radial.

☞ Gauss's Theorem and its Applications

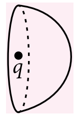
- Gauss's theorem $\oint \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$
  - This law is suitable for symmetrical charge distribution and valid for all vector fields obeying inverse square law.
  - Number of electric lines related to 1C (unit charge) $=1/\varepsilon_0$.
- **Gaussian surface**
  - It is imaginary surface.
  - It is spherical for a point charge, conducting and non-conducting spheres.
  - It is cylindrical for infinite sheet of charges, conducting charge surfaces, infinite line of charges, charged cylindrical conductors etc.
- **Applications of Gauss's Theorem :**
  - For an imaginary cube

Position of charge	Main centre	Face centre	Centre of side	Corner
Flux from cube	$\frac{q}{\varepsilon_0}$	$\frac{q}{2\varepsilon_0}$	$\frac{q}{4\varepsilon_0}$	$\frac{q}{8\varepsilon_0}$

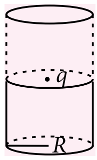
  - Electric field due to a plane sheet of charge, $E = \frac{\sigma}{2\varepsilon_0}$
  - Electric field near a charged conducting surface, $E = \frac{\sigma}{\varepsilon_0}$



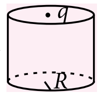
$\phi_{Total} = \frac{q}{\varepsilon_0} \Rightarrow$



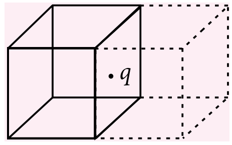
$\phi_{hemisphere} = \frac{q}{2\varepsilon_0}$



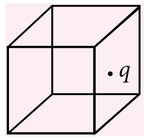
$\phi_{Total} = \frac{q}{\varepsilon_0} \Rightarrow$



$\phi_{cylinder} = \frac{q}{2\varepsilon_0}$



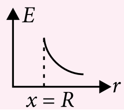
$\phi_{Total} = \frac{q}{\varepsilon_0}$

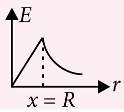


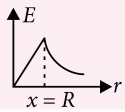
$\phi_{cube} = \frac{q}{2\varepsilon_0}$

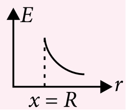
$\vec{E}$ due to non-conducting

Sphere: Shell	Sphere: Solid	Cylinder: Solid	Cylinder: Hollow
$E_{in} = 0$	$E_{in} = \frac{kQr}{R^3} \rightarrow E_{in} \propto r$	$E_{in} \propto r \leftarrow E_{in} = \frac{2k\lambda r}{R^2}$	$E_{in} = 0$
$E_s = \frac{kQ}{R^2}$	$E_s = \frac{kQ}{R^2}$	$E_s = \frac{2k\lambda}{R}$	$E_s = \frac{2k\lambda}{R}$
$E_{out} = \frac{kQ}{r^2}$	$E_{out} = \frac{kQ}{r^2}$	$E_{out} = \frac{2k\lambda}{r}$	$E_{out} = \frac{2k\lambda}{r}$





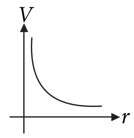
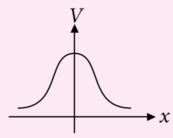
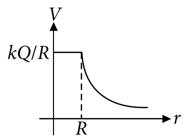
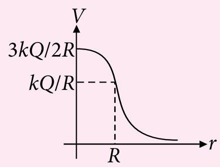




☞ Potential and Potential Difference

- Potential is a relative parameter, while potential difference is absolute *i.e.* potential depends on the choosen reference. That's why potential of same object may have positive or negative or zero potential.
- Only one relation by which work done in moving a charge is calculated
  $W_{external} = -W_{electrical} = q[V_f - V_i]$
  V_f = Potential at final point, V_i = Potential at initial point.
- Electric potential at a point is defined as the amount of work done in bringing a unit positive charge from infinity to that point. It is denoted by symbol V.
  $$V = \frac{W}{q}$$

Electric potential due to various charge distributions are given in table below :

Name /Type	Formula	Note	Graph
Point charge	$V = \dfrac{kq}{r}$	▸ q is source charge. ▸ r is the distance of the point from the point charge.	
Ring (uniform/ non-uniform charge distribution)	$V = \dfrac{kQ}{R}$, at centre $V = \dfrac{kQ}{\sqrt{R^2 + x^2}}$, along the axis	▸ Q is charge on the ring. ▸ x is the distance of the point on the axis.	
Uniformly charged hollow conducting/ hollow non-conducting/solid conducting sphere	For $r \geq R$, $V = \dfrac{kQ}{r}$ For $r \leq R$, $V = \dfrac{kQ}{R}$	▸ R is radius of sphere ▸ r is the distance from centre of sphere to the point ▸ Q is total charge = $\sigma 4\pi R^2$	
Uniformly charged solid non-conducting sphere	For $r \geq R$, $V = \dfrac{kQ}{r}$ For $r \leq R, V = \dfrac{kQ(3R^2 - r^2)}{2R^3}$ $= \dfrac{\rho}{6\varepsilon_0}(3R^2 - r^2)$	▸ R is radius of sphere ▸ r is distance from centre to the point ▸ $V_{\text{centre}} = \dfrac{3}{2} V_{\text{surface}}$ ▸ Q is total charge $= \rho \dfrac{4}{3}\pi R^3$ ▸ Inside sphere potential varies parabolically with r. ▸ Outside potential varies hyperbolically with r.	
Infinite line charge	Not defined	▸ Absolute potential is not defined. ▸ Potential difference between two points is given by formula $V_B - V_A = -2k\lambda \ln (r_B/r_A)$	
Infinite non-conducting thin sheet	Not defined	▸ Absolute potential is not defined ▸ Potential difference between two points is given by formula $V_B - V_A = -\dfrac{\sigma}{2\varepsilon_0}(r_B - r_A)$	
Infinite charged conducting thin sheet	Not defined	▸ Absolute potential is not defined ▸ Potential difference between two points is given by formula $V_B - V_A = -\dfrac{\sigma}{\varepsilon_0}(r_B - r_A)$	

☞ Relation between Electric Field and Potential

$\vec{E} = -\vec{\nabla} V$

where $\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$

–ve sign shows that the direction of $\vec{E}$ is the direction of decreasing potential.

☞ Electric Potential Energy

- Electric potential energy of a system of charges is the total amount of work done in bringing the various charges to their respective positions from infinitely large mutual separations. The SI unit of electrical potential energy is joule.
- Electric potential energy of a system of two charges is

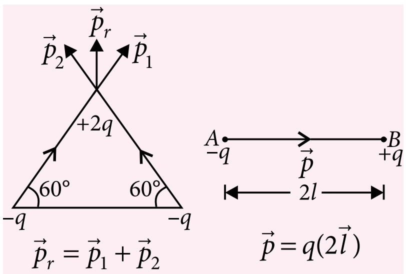
  $$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

  where r_{12} is the distance between q_1 and q_2.
- Electric potential energy of a system of n point charges

  $$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}; \; j < k$$

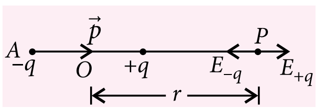
☞ Electric Dipole

- Electric dipole moment, $\vec{p} = q(2\vec{l})$



- Electric field at any point on the axial line

$\vec{E}_{\text{axial}} = \vec{E}_{+q} + \vec{E}_{-q}$



$$\vec{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2 - l^2)^2}\hat{p} \approx \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}\hat{p} \text{ (For } r >> l)$$

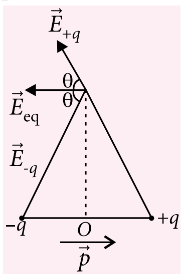
- Electric field at any point on the equatorial line

$$\vec{E}_{\text{eq}} = -(E_{-q}\cos\theta + E_{+q}\cos\theta)\hat{p}$$

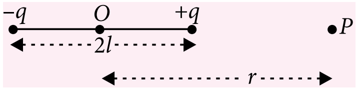
$$= -2E_{-q}\cos\theta\,\hat{p}$$

$$\approx -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3} \quad \text{(For } r >> l)$$

$E_{\text{axial}} = 2E_{\text{eq}}$



- Net electric potential at axial point

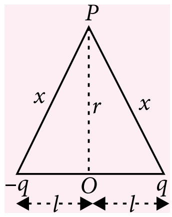


$$V = V_1 + V_2 = \frac{kq}{(r-l)} + \frac{-kq}{(r+l)} = \frac{kq \times 2l}{(r^2 - l^2)} = \frac{kp}{r^2 - l^2}$$

If $r >> l$, $V = \frac{kp}{r^2}$

- Net electric potential at equatorial line

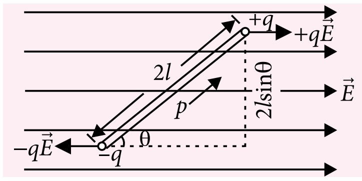
$V = V_1 + V_2 = \frac{kq}{x} - \frac{kq}{x} = 0$



- In general,

$V = \frac{kp\cos\theta}{r^2} = k\frac{\vec{p}\cdot\hat{r}}{r^2}$

- Torque, $\vec{\tau} = \vec{p} \times \vec{E} = pE\sin\theta\,\hat{n}$



- Potential energy of a dipole is work done in rotating a dipole from a direction perpendicular to the field to the given direction

  $$W = \int_{90°}^{\theta}\tau\, d\theta = \int_{90°}^{\theta} pE\sin\theta\, d\theta = -pE\cos\theta$$

  This work is stored in the form of potential energy.

  So, $U = W = -\vec{p}\cdot\vec{E}$
- Work done in rotating an electric dipole in an electric field, $W = \int_{\theta_1}^{\theta_2}\tau\, d\theta = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$

  $= pE(\cos\theta_1 - \cos\theta_2) = U_2 - U_1$

☞ Basic Electrostatic Properties of a Conductor

- Inside a conductor, electric field is zero.
- At the surface of a charged conductor, electric field must be normal to the surface at every point.
- The interior of a conductor can have no excess charge in the static situation.
- Electric potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.
- Electric field at the surface of a charged conductor

  $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$ where σ is the surface charge density and

$\hat{n}$ is a unit vector normal to the surface in the outward direction.

Electrostatic shielding : It is the phenomenon of protecting a certain region of space from external electric field.

Capacitors and Capacitance

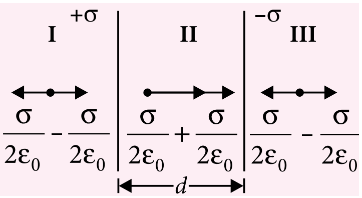
- A capacitor is a device that stores electrical energy. It consists of conductors of any shape and size carrying charges of equal magnitude and opposite sign and separated by an insulating medium.
- Capacitance (C) of a capacitor is the ratio of charge(Q) given and the potential (V) to which it is raised.

$$C = \frac{Q}{V}$$

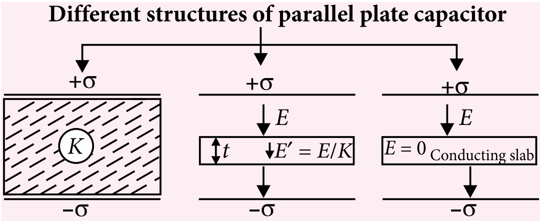
- **Parallel plate capacitor**

$$V = Ed = \frac{\sigma}{\varepsilon_0} d$$

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

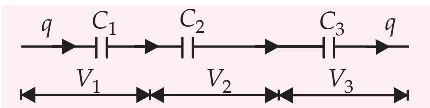


Different structures of parallel plate capacitor



$$C = \frac{K\varepsilon_0 A}{d} \qquad C = \frac{\varepsilon_0 A}{\left[d - t + \frac{t}{K}\right]} \qquad C = \frac{\varepsilon_0 A}{(d-t)}$$

- **Capacitors in series**

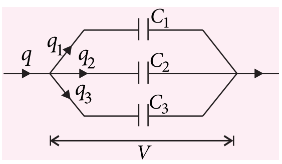


  - Same charge flows through each capacitor.
  - Different potential difference exist across each capacitor if $C_1 \neq C_2 \neq C_3$.

$$V = V_1 + V_2 + V_3$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- **Capacitors in parallel**



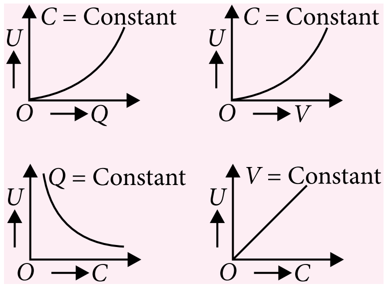
  - Across each capacitor, potential difference is same.
  - Different charges flow through each capacitor if $C_1 \neq C_2 \neq C_3$.

$$q = q_1 + q_2 + q_3$$
$$C = C_1 + C_2 + C_3$$

- **Energy stored in the capacitor**

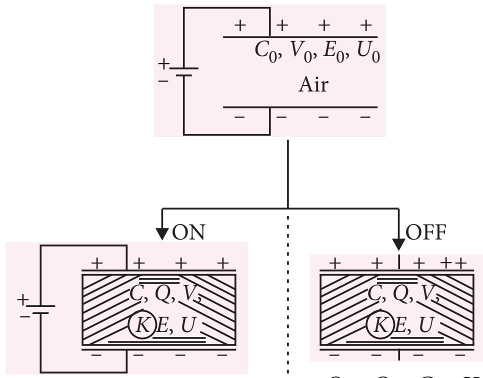
$$U = W = \int_O^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

  - Electric energy density $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{\sigma^2}{2\varepsilon_0}$
  - For a parallel plate capacitor, variation of electrostatic potential energy stored with change in Q, V, C etc., has been shown in the following graphs.



  - When a capacitor is charged with the help of a battery, the potential energy of a capacitor is obtained at the cost of chemical energy stored in the battery.

Effect of dielectric on capacitor when charging battery is



ON:

$$V = V_0,\ Q = CV_0 = KC_0V_0$$

$$E = \frac{V}{d} = \frac{V_0}{d} = E_0$$

$$U = \frac{1}{2}CV^2 = KU_0$$

OFF:

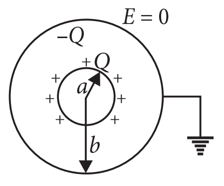
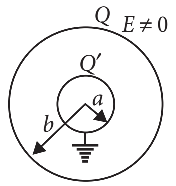
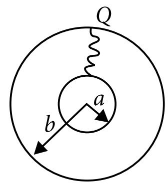
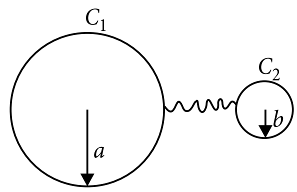
$$Q = Q_0,\ C = KC_0$$

$$V = \frac{Q}{C} = \frac{V_0}{K}$$

$$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K}$$

$$U = \frac{1}{2}CV^2 = \frac{U_0}{K}$$

- **Capacity of spherical systems**

System	Inner is given charge (outer is earthed)	Inner is earthed and outer is given charge	Connected spheres	Connected spheres (far away)
Diagram		 $Q' = \frac{-a}{b}Q$		
Capacity	$C = 4\pi\varepsilon_0 ab/[b - a]$	$C = 4\pi\varepsilon_0 b^2/[b - a]$ = capacity of spherical capacitor + $4\pi\varepsilon_0 b$	$C = 4\pi\varepsilon_0 b$	$C = C_1 + C_2$ $C = 4\pi\varepsilon_0(a + b)$

- Capacity of cylindrical capacitor, $C = 2\pi\varepsilon_0 l/\ln(b/a)$

Van de Graaff generator

A Van de Graaff generator consists of a large spherical conducting shell (a few metres in diameter). By means of a moving belt and suitable brushes, charge is continuously transferred to the shell, and potential difference of the order of several million volts is built up, which can be used for accelerating charged particles.

SPEED PRACTICE

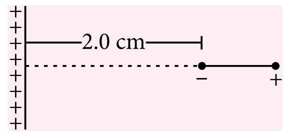
1. Two point charges +3 μC and +8 μC repel each other with a force of 40 N. If a charge of –5 μC is added to each of them, then the force between them will become

(a) –10 N (b) +10 N
(c) +20 N (d) –20 N

2. A charge Q is to be divided on two objects. What should be the values of the charges on the objects so that the force between the objects can be maximum?

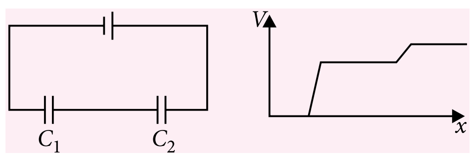
(a) $\frac{Q}{3}, \frac{2Q}{3}$ (b) $\frac{Q}{2}, \frac{Q}{2}$
(c) $\frac{Q}{8}, \frac{7Q}{8}$ (d) $\frac{Q}{4}, \frac{3Q}{4}$

3. An electric dipole consists of charges $\pm 2.0 \times 10^{-8}$ C separated by a distance of 2.0×10^{-3} m. It is placed near a long line charge of linear charge density 4.0×10^{-4} C m^{-1} as shown in the figure, such that the negative charge is at a distance of 2.0 cm from the line charge. The force acting on the dipole will be



(a) 7.2 N towards the line charge
(b) 6.6 N away from the line charge
(c) 0.6 N away from the line charge
(d) 0.6 N towards the line charge

4. Figure shows two capacitors connected in series and joined to a battery. The graph shows the variation in potential as one moves from left to right on the branch containing the capacitors. Then



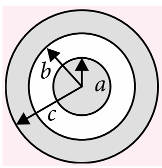
(a) $C_1 > C_2$
(b) $C_1 = C_2$ (c) $C_1 < C_2$
(d) The information is not sufficient to decide the relation between C_1 and C_2.

5. A student originally charges a fixed capacitor to have a potential energy of 1 J. If the student wishes to give the capacitor a potential energy of 4 J, then the student should
   (a) quadruple the potential difference across the capacitor but leave the charge unchanged.
   (b) double the potential difference across the capacitor but leave the charge unchanged.
   (c) double both the potential difference across the capacitor and the charge.
   (d) leave the potential difference across the capacitor unchanged while doubling the charge.

6. The electric field in a region is radially outward with magnitude $E = Ar$. What will be the charge contained in a sphere of radius a centred at the origin? Take $A = 100\ \text{V m}^{-2}$ and $a = 20.0$ cm.
   (a) 8.89×10^{-11} C (b) 4.47×10^{-11} C
   (c) 8.89×10^{-9} C (d) 4.47×10^{-9} C

7. If potential (in volts) in a region is expressed as $V(x, y, z) = 6xy - y + 2yz$, then the electric field (in N C^{-1}) at point (1, 1, 0) is
   (a) $-(2\hat{i}+3\hat{j}+\hat{k})$ (b) $-(6\hat{i}+9\hat{j}+\hat{k})$
   (c) $-(3\hat{i}+5\hat{j}+3\hat{k})$ (d) $-(6\hat{i}+5\hat{j}+2\hat{k})$

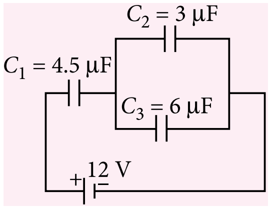
8. A solid conducting sphere of radius a has a net positive charge $+2Q$. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and has a net charge $-Q$. The surface charge density on the surface of solid sphere and on the inner and outer surfaces of the spherical shell will be



   (a) $\frac{2Q}{4\pi a^2}, \frac{-2Q}{4\pi b^2}, \frac{Q}{4\pi c^2}$
   (b) $\frac{Q}{4\pi a^2}, \frac{-Q}{4\pi b^2}, \frac{Q}{4\pi c^2}$
   (c) $0, 0, \frac{Q}{4\pi c^2}$ (d) $\frac{2Q}{4\pi a^2}, \frac{-Q}{4\pi b^2}, \frac{Q}{4\pi c^2}$

9. An alpha particle ($q = +2e$) in a nuclear accelerator moves from one terminal at a potential of $V_a = +6.5 \times 10^6$ V to another at a potential of $V_b = 0$. Assuming that the terminals and their charges do not move and that no external forces act on the system, what is the change in kinetic energy of the particle?
   (a) 2.1×10^{-12} J (b) 4.2×10^{-12} J
   (c) 2.1×10^{-9} J (d) 4.2×10^{-9} J

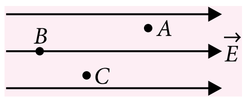
10. In the circuit shown in the figure, the potential difference across the 4.5 μF capacitor is



   (a) 8/3 V (b) 4 V (c) 6 V (d) 8 V

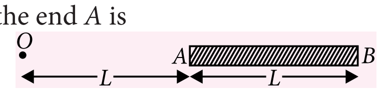
11. Two particles, each having a mass of 5 g and charge 1.0×10^{-7} C, stay in limiting equilibrium on a horizontal table with a separation of 10 cm between them. The coefficient of friction between each particle and the table is the same. The value of this coefficient is
   (a) 0.28 (b) 0.18 (c) 0.11 (d) 0.31

12. A, B and C are three points in a uniform electric field. The electric potential is



   (a) maximum at C
   (b) same at all the three points A, B and C
   (c) maximum at A
   (d) maximum at B **[NEET 2013]**

13. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is

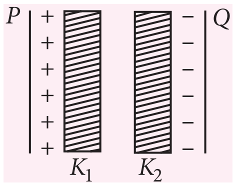


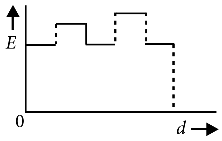
   (a) $\frac{Q}{4\pi\varepsilon_0 L}\ln 2$ (b) $\frac{Q}{8\pi\varepsilon_0 L}\ln 2$
   (c) $\frac{3Q}{4\pi\varepsilon_0 L}$ (d) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$
   [JEE Main 2013]

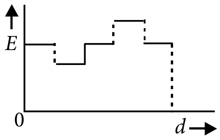
14. A disk of radius $R = 4.8$ cm carries a total charge $q = +2.5$ nC that is uniformly distributed over its surface and held in fixed location (consider the surface to behave like an insulator). An electron is initially at rest at a distance of $d = 3.0$ cm from the disk along its axis. When the electron is released, it is attracted toward the disk. What is the speed of the electron when it strikes the centre of the disk?

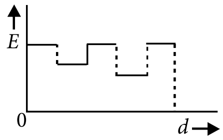
(a) 1.21×10^7 m s^{-1} (b) 2.42×10^7 m s^{-1}
(c) 1.46×10^6 m s^{-1} (d) 2.14×10^7 m s^{-1}

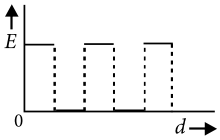
15. Two thin dielectric slabs of dielectric constants K_1 and K_2 ($K_1 < K_2$) are inserted between plates of a parallel plate capacitor, as shown in the figure. The variation of electric field E between the plates with distance d as measured from plate P is correctly shown by



(a) 

(b) 

(c) 

(d) 

[AIPMT 2015]

16. Two identical charged spheres suspended from a common point by two massless strings of lengths l, are initially at a distance d ($d << l$) apart because of their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with a velocity v. Then v varies as a function of the distance x between the spheres, as

(a) $v \propto x^{-1/2}$ (b) $v \propto x^{-1}$
(c) $v \propto x^{1/2}$ (d) $v \propto x$

[NEET Phase I 2016]

17. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V m^{-1}, the charge density of the positive plate will be close to

(a) 6×10^4 C m^{-2} (b) 6×10^{-7} C m^{-2}
(c) 3×10^{-7} C m^{-2} (d) 3×10^4 C m^{-2}

[JEE Main 2014]

18. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then

(a) $9C_1 = 4C_2$ (b) $5C_1 = 3C_2$
(c) $3C_1 = 5C_2$ (d) $3C_1 + 5C_2 = 0$

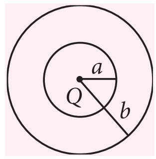
[JEE Main 2013]

19. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y << a$) along the y-axis, the net force acting on the particle is proportional to

(a) $-\frac{1}{y}$ (b) y (c) $-y$ (d) $\frac{1}{y}$

[JEE Main 2013]

20. The region between two concentric spheres of radii a and b, respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre.



At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is

(a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2 - a^2)}$

(c) $\frac{2Q}{\pi(a^2 - b^2)}$ (d) $\frac{2Q}{\pi a^2}$

[JEE Main 2016]

SOLUTIONS

1. **(a):** $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(3\,\mu\text{C})(8\,\mu\text{C})}{r^2} = 40\text{ N}$

and $F' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1' q_2'}{r^2}$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{\{(3-5)\,\mu\text{C}\}\{(8-5)\,\mu\text{C}\}}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{(-2\,\mu\text{C})(3\,\mu\text{C})}{r^2} = -\frac{F}{4} = -\frac{40}{4} = -10\text{ N}$$

2. **(b):** Suppose one object receives a charge q and the other $(Q - q)$. The force between the objects is

$$F = \frac{q(Q-q)}{4\pi\varepsilon_0 d^2},$$

where d is the separation between them. For F to be maximum, the quantity

$y = q(Q - q) = Qq - q^2$

should be maximum. This is the case when,

$$\frac{dy}{dq} = 0 \text{ or, } Q - 2q = 0 \text{ or, } q = \frac{Q}{2}.$$

Thus, the charge should be divided equally on the two objects.

3. (d): The electric field at a distance r from the line charge of linear density λ is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Hence, the field at the negative charge,

$$E_1 = \frac{(4.0\times10^{-4})(2\times9\times10^{9})}{0.02} = 3.6\times10^{8}\ \text{N C}^{-1}$$

The force on the negative charge,

$$F_1 = (3.6\times10^{8})(2.0\times10^{-8})$$
$$= 7.2\ \text{N towards the line charge}$$

Similarly, the field at the positive charge, *i.e.*, at $r = 0.022$ m is

$$E_2 = 3.3\times10^{8}\ \text{N C}^{-1}.$$

The force on the positive charge,

$$F_2 = (3.3\times10^{8})\times(2.0\times10^{-8})$$
$$= 6.6\ \text{N away from the line charge.}$$

Hence, the net force on the dipole = 7.2 N – 6.6 N = 0.6 N towards the line charge

4. (c) **5. (c)**

6. (a): The electric field at the surface of the sphere is Aa and being radial it is along the outward normal. The flux of the electric field is, therefore,

$$\Phi = \oint E dS\cos0^\circ = Aa(4\pi a^2).$$

From Gauss's law, the charge contained in the sphere

$$Q_{\text{inside}} = \varepsilon_0\Phi = 4\pi\varepsilon_0 Aa^3$$
$$= \frac{1}{9\times10^9}\times100\ \text{V m}^{-2}\times(0.20)^3$$
$$= 8.89\times10^{-11}\ \text{C}$$

7. (d): The electric field $\vec{E}$ and potential V in a region are related as

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

Here, $V(x, y, z) = 6xy - y + 2yz$

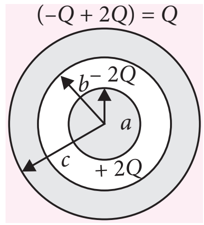
$$\therefore\ \vec{E} = -\left[\frac{\partial}{\partial x}(6xy - y + 2yz)\hat{i} + \frac{\partial}{\partial y}(6xy - y + 2yz)\hat{j} + \frac{\partial}{\partial z}(6xy - y + 2yz)\hat{k}\right]$$

$$= -[(6y)\hat{i} + (6x - 1 + 2z)\hat{j} + (2y)\hat{k}]$$

At point (1, 1, 0),

$$\vec{E} = -[(6(1))\hat{i} + (6(1) - 1 + 2(0))\hat{j} + (2(1))\hat{k}]$$
$$= -(6\hat{i} + 5\hat{j} + 2\hat{k})$$

8. (a): The charge distribution, including the induced charges, is shown in the figure.



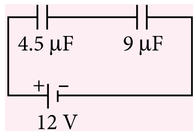
$$\therefore\ \sigma_A = \frac{+2Q}{4\pi a^2},\ \sigma_B = \frac{-2Q}{4\pi b^2}$$

and $\sigma_C = \dfrac{(+2Q - Q)}{4\pi c^2} = \dfrac{Q}{4\pi c^2}$

9. (a): $\Delta U = U_b - U_a = q(V_b - V_a)$
$= (+2)(1.6\times10^{-19}\ \text{C})(0 - 6.5\times10^{6}\ \text{V}) = -2.1\times10^{-12}$ J.
If no external force acts on the system, then its mechanical energy $E = U + K$ must remain constant. That is, $\Delta E = \Delta U + \Delta K = 0$, and so

$$\Delta K = -\Delta U = +2.1\times10^{-12}\ \text{J}$$

10. (d): The given arrangement can be redrawn as



$\therefore$ Potential difference across 4.5 μF capacitor

$$= \frac{9\ \mu\text{F}}{(9\ \mu\text{F} + 4.5\ \mu\text{F})}\times12\ \text{V} = 8\ \text{V}$$

11. (b): The electric force on one of the particles due to the other is

$$F = (9\times10^{9}\ \text{N m}^2\text{C}^{-2})\times(1.0\times10^{-7}\ \text{C})^2\times\frac{1}{(0.10\ \text{m})^2}$$
$$= 0.009\ \text{N}$$

The frictional force in limiting equilibrium

$$f = \mu\,(5\times10^{-3}\text{kg})\times(9.8\ \text{m s}^{-2}) = (0.049\ \mu)\ \text{N}$$

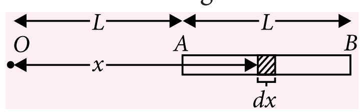
As these two forces balance each other,

$$0.049\ \mu = 0.009\ \text{or}\ \mu = 0.18$$

12. (d): In the direction of electric field, electric potential decreases.

$$\therefore\ V_B > V_C > V_A$$

13. (a): Consider a small length element of charge situated as shown in the figure.



Then, $dV = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{dq}{x}$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{\left(\frac{Q}{L}dx\right)}{x} = \frac{Q}{4\pi\varepsilon_0 L}\cdot\frac{dx}{x}$$

$$\Rightarrow V = \int_L^{2L}\frac{Q}{4\pi\varepsilon_0 L}\cdot\frac{dx}{x} = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{2L}{L}\right) = \frac{Q}{4\pi\varepsilon_0 L}\ln 2$$

14. (a): The charge density on the disk,

$$\sigma = \frac{q}{\pi R^2} = \frac{2.5\times10^{-9}\ \text{C}}{\pi(0.048\ \text{m})^2} = 3.45\times10^{-7}\ \text{C m}^{-2}$$

The difference in potential between the locations with $z = d$ and $z = 0$

$$\Delta V = V(0) - V(d) = \frac{\sigma R}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + d^2} - d\right)$$

$$= \frac{3.45 \times 10^{-7}\,\text{C m}^{-2}}{2(8.85 \times 10^{-12}\,\text{C}^2\,\text{N}^{-1}\,\text{m}^{-2})}[0.048\text{ m}$$

$$-\sqrt{(0.048\text{ m})^2 + (0.030\text{ m})^2} + 0.030\text{ m})] = 417\text{ V}.$$

Change in the electron's potential energy

$$\Delta U = q\Delta V = (-1.6 \times 10^{-19}\text{ C})(417\text{ V})$$
$$= -6.67 \times 10^{-17}\text{ J}$$

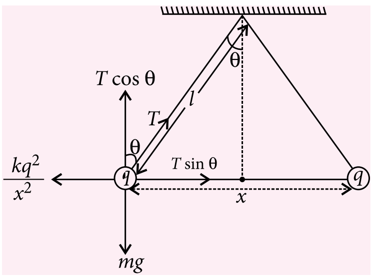
Conservation of energy gives,

$$\Delta K = -\Delta U = 6.67 \times 10^{-17}\text{ J}$$

$$\therefore \quad v = \sqrt{\frac{2\Delta K}{m}} = \sqrt{\frac{2 \times (6.67 \times 10^{-17}\text{ J})}{9.11 \times 10^{-31}\text{ kg}}} = 1.21 \times 10^7\text{ m s}^{-1}$$

15. (c)

16. (a):



From figure, $T\cos\theta = mg$ ...(i)

$$T\sin\theta = \frac{kq^2}{x^2} \quad \text{...(ii)}$$

From eqns. (i) and (ii), $\tan\theta = \dfrac{kq^2}{x^2 mg}$

Since θ is small, $\therefore \tan\theta \approx \sin\theta = \dfrac{x}{2l}$

$$\therefore \quad \frac{x}{2l} = \frac{kq^2}{x^2 mg} \;\Rightarrow\; q^2 = x^3\frac{mg}{2lk} \text{ or } q \propto x^{3/2}$$

$$\Rightarrow \quad \frac{dq}{dt} \propto \frac{3}{2}\sqrt{x}\frac{dx}{dt} = \frac{3}{2}\sqrt{x}v$$

Since, $\dfrac{dq}{dt}$ = constant $\therefore\; v \propto \dfrac{1}{\sqrt{x}}$

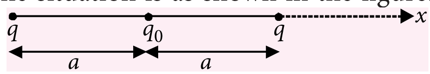
17. (b): Here, $K = 2.2$, $E = 3 \times 10^4$ V m^{-1}

Electric field between the parallel plate capacitor with dielectric, $E = \dfrac{\sigma}{K\varepsilon_0}$

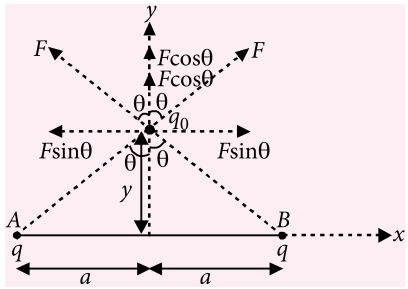
$$\Rightarrow \quad \sigma = K\varepsilon_0 E = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$$
$$\approx 6 \times 10^{-7}\text{ C m}^{-2}$$

18. (c) : For potential to be made zero, after connection $120C_1 = 200C_2 \Rightarrow 3C_1 = 5C_2$

19. (b): The situation is as shown in the figure.



When a particle of mass m and charge $q_0\left(=\dfrac{q}{2}\right)$ placed at the origin is given a small displacement along the y-axis, then the situation is shown in the figure.



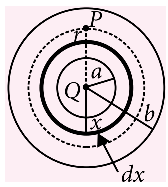
$\therefore$ The net force acting on the particle is

$$F_{\text{net}} = 2F\cos\theta$$

$$= \frac{2}{4\pi\varepsilon_0}\frac{q\left(\frac{q}{2}\right)}{(y^2 + a^2)}\frac{y}{\sqrt{(y^2 + a^2)}} = \frac{1}{4\pi\varepsilon_0}\frac{q^2 y}{(y^2 + a^2)^{3/2}}$$

As $y << a$ so $F_{\text{net}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2 y}{a^3}$ or $F_{\text{net}} \propto y$

20. (a): Using Gauss's theorem for radius r



$$\int \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0}(Q + q)$$

$$\Rightarrow \quad E \times 4\pi r^2 = \frac{1}{\varepsilon_0}(Q + q) \quad \text{...(i)}$$

q = charge enclosed between $x = a$ and $x = r$.

$$q = \int_a^r \frac{A}{x}4\pi x^2 dx = 4\pi A\int_a^r x\,dx$$

$$= 4\pi A\left[\frac{x^2}{2}\right]_a^r = 2\pi A(r^2 - a^2)$$

Putting the value of q in equation (i), we get

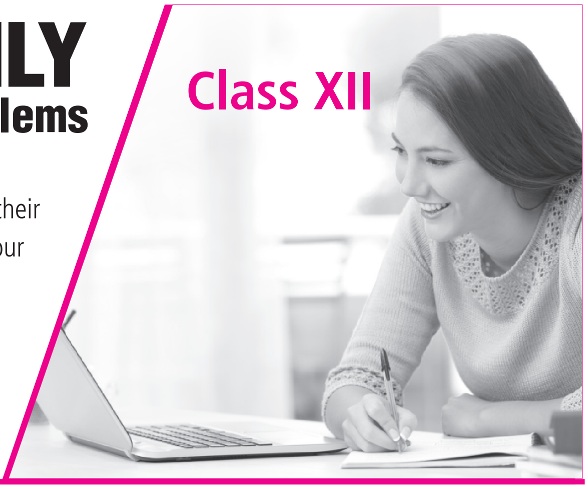
$$E \times 4\pi r^2 = \frac{1}{\varepsilon_0}\left[Q + 2\pi A(r^2 - a^2)\right]$$

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r^2} + 2\pi A - \frac{2\pi A a^2}{r^2}\right]$$

E will be constant if it is independent of r

$$\therefore \quad \frac{Q}{r^2} = \frac{2\pi A a^2}{r^2} \quad \text{or} \quad A = \frac{Q}{2\pi a^2}$$

MPP-1 MONTHLY Practice Problems



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

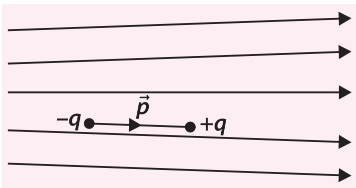
Electric Charges and Fields

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS / PMTs

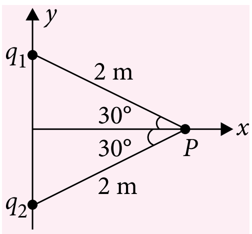
Only One Option Correct Type

1. Electric field lines in which an electric dipole is placed are as shown in the figure. Which of the following statements is correct ?



(a) The dipole will not experience any force.
(b) The dipole will experience a force towards right.
(c) The dipole will experience a force towards left.
(d) The dipole will experience a force upwards.

2. Two point charges $q_1 = -4\ \mu C$ and $q_2 = 8\ \mu C$ are lying on the y-axis. They are equidistant from the point P, which lies on the x-axis. A small object of charge $q_0 = 8\ \mu C$ and mass $m = 12$ g is placed at P. When it is released what is its acceleration in m s^{-2} ?



(a) $3\sqrt{3}\hat{i}+9\hat{j}$ (b) $9\hat{i}+3\sqrt{3}\hat{j}$
(c) $3\hat{i}+3\sqrt{3}\hat{j}$ (d) $3\sqrt{3}\hat{i}+3\hat{j}$

3. What total charge q must a disk of radius 2.50 cm carry so that the electric field on the surface of the disk at its center equals the value at which air breaks down electrically, producing sparks ?
(Electrical break down of air = 3×10^6 N C^{-1})
(a) 208 nC (b) 156 nC
(c) 52 nC (d) 104 nC

4. A proton orbits with a speed v = 294 km s^{-1} just outside a charged sphere of radius r = 1.13 cm. What is the charge on the sphere ?
(a) −1.13 nC (b) +1.13 nC
(c) −2.26 nC (d) + 2.26 nC

5. A long string with a charge of λ per unit length passes through an imaginary cube of edge a. What is the maximum flux of the electric field through the cube ?
(a) $\dfrac{a\lambda}{\varepsilon_0}$ (b) $\dfrac{\sqrt{2}\,a\lambda}{\varepsilon_0}$ (c) $\dfrac{\sqrt{3}\,a\lambda}{\varepsilon_0}$ (d) $\dfrac{2a\lambda}{\varepsilon_0}$

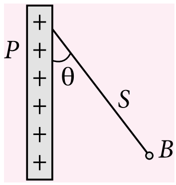
6. A charged particle of mass 5×10^{-6} g is kept over a large horizontal sheet of charge density 4×10^{-6} C m^{-2}. How many electrons should be removed to give some charge to the particle so that if released it does not fall down ?
(a) 1.36×10^6 (b) 2.72×10^6
(c) 1.6×10^7 (d) 3.2×10^6

7. A and B are two identical spherical charged bodies which repel each other with force F, kept at a finite distance. A third uncharged sphere of the same size is brought in contact with sphere B and removed. It is then kept at midpoint of A and B. The magnitude of force on C is
(a) F/2 (b) F/8 (c) F (d) zero

8. An electric dipole is placed at an angle of 60° with an electric field of intensity 10^5 N C^{-1}. It experiences a torque equal to $8\sqrt{3}$ N m. What is the charge on the dipole, if dipole length is 2 cm ?

(a) 16×10^{-3}C (b) 32×10^{-3}C
(c) 4×10^{-3}C (d) 8×10^{-3}C

9. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P as shown in the figure. The surface charge density of the sheet is proportional to



(a) $\cos\theta$ (b) $\cot\theta$ (c) $\sin\theta$ (d) $\tan\theta$

10. An imaginary, closed, spherical surface S of radius R is centered on the origin. A positive charge $+q$ is originally at the origin, and the flux through the surface is Φ_E. Three additional charges are now added along the x-axis : $-3q$ at $x = -R/2$, $+5q$ at $x = R/2$, and $+4q$ at $x = 3R/2$. The flux through S is now

(a) $2\Phi_E$ (b) $3\Phi_E$ (c) $6\Phi_E$ (d) $7\Phi_E$

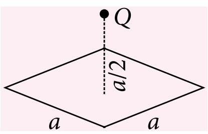
11. Three conducting metal spherical shells of radii R, $2R$ and $3R$ are given charges Q_1, Q_2 and Q_3 respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. The ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$ is

(a) 1 : 2 : 3 (b) 1 : 3 : 5
(c) 1 : 4 : 9 (d) 1 : 8 : 18

12. A charge Q is placed at a distance $a/2$ above the centre of a horizontal, square surface of edge a as shown in the figure. Find the flux of the electric field through the square surface.

(a) $\dfrac{Q}{\varepsilon_0}$ (b) $\dfrac{Q}{2\varepsilon_0}$

(c) $\dfrac{Q}{4\varepsilon_0}$ (d) $\dfrac{Q}{6\varepsilon_0}$



Assertion & Reason Type

Directions : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion
(b) If both assertion and reason are true but reason is not the correct explanation of assertion
(c) If assertion is true but reason is false
(d) If both assertion and reason are false.

13. Assertion : In electrostatics, electric lines of force can never be closed loops, as a line can never start and end on the same charge.

Reason : The number of electric lines of force originating or terminating on a charge is proportional to the magnitude of charge.

14. Assertion : If a point charge q is placed in front of an infinite grounded conducting plane surface, the point charge will experience a force.

Reason : This force is due to the induced charge on the conducting surface which is at zero potential.

15. Assertion : Charge is quantized.

Reason : Charge which is less than 1 C is not possible.

JEE MAIN / JEE ADVANCED / PETs

Only One Option Correct Type

16. In a 1911 paper, Ernest Rutherford said : In order to form some idea of the forces required to deflect an alpha particle through a large angle, consider an atom containing a point positive charge Ze at its center and surrounded by a distribution of negative electricity, $-Ze$ uniformly distributed within a sphere of radius R. The electric field E at a distance r from the center for a point inside the atom is

(a) $\dfrac{Ze}{4\pi\varepsilon_0}\left(\dfrac{1}{r^2} - \dfrac{r}{R^3}\right)$ (b) $\dfrac{Ze}{4\pi\varepsilon_0 r^2}$

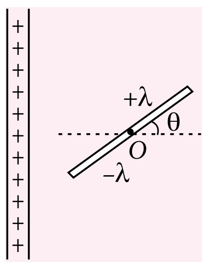
(c) $\dfrac{Zer}{4\pi\varepsilon_0 R^2}$ (d) $\dfrac{Ze}{4\pi\varepsilon_0}\left(\dfrac{1}{r^2} + \dfrac{r}{R^3}\right)$

17. Electric field on the axis of a small electric dipole at a distance r is $\vec{E}_1$ and $\vec{E}_2$ at a distance of $2r$ on a line of perpendicular bisector. Then

(a) $\vec{E}_2 = \dfrac{-\vec{E}_1}{8}$ (b) $\vec{E}_2 = \dfrac{-\vec{E}_1}{16}$

(c) $\vec{E}_2 = \dfrac{-\vec{E}_1}{4}$ (d) $\vec{E}_2 = \dfrac{-\vec{E}_1}{8}$

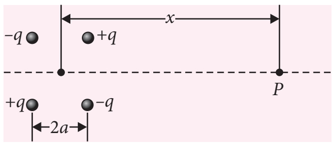
18. A large sheet carries uniform surface charge density σ. A rod of length $2l$ has a linear charge density λ on one half and $-\lambda$ on the second half. The rod is hinged at mid point O and makes an angle θ with the normal to the sheet. The torque experienced by the rod is



(a) zero (b) $\dfrac{\sigma\lambda l^2}{2\varepsilon_0}\sin\theta$

(c) $\dfrac{\sigma\lambda l^2}{\varepsilon_0}\sin\theta$ (d) $\dfrac{\sigma\lambda l^2}{2\varepsilon_0}$

19. One type of electric quadrupole is formed by four charges located at the vertices of a square of side $2a$. Point P lies a distance x from the center of the quadrupole on a line parallel to two sides of the square as shown in the figure. For $x >> a$, the electric field at P is approximately equal to



(a) $\frac{3(2qa^2)}{2\pi\varepsilon_0 x^4}$ (b) $\frac{4(2qa^2)}{2\pi\varepsilon_0 x^4}$

(c) $\frac{3(2qa^2)}{4\pi\varepsilon_0 x^4}$ (d) $\frac{3(2qa)}{2\pi\varepsilon_0 x^3}$

More than One Options Correct Type

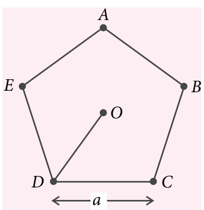
20. Measured values of the electric field E at a distance z along the axis of a charged plastic disk are given here :

z (cm)	E (10^7 N C^{-1})
0	2.043
1	1.732
2	1.442
3	1.187
4	0.972
5	0.797

(a) The radius of the disk is 6.5 cm.
(b) The radius of the disk is 13 cm.
(c) The charge on the disk is 4.8 μC.
(d) The charge on the disk is 9.8 μC.

21. A solid non-conducting sphere of radius R carries a non-uniform charge distribution, with charge density $\rho = \rho_s r/R$, where ρ_s is a constant and r is the distance from the center of the sphere.
(a) The total charge, Q on the sphere is $\pi\rho_s R^3$.
(b) The total charge, Q on the sphere is $\frac{4}{3}\pi\rho_s R^3$.
(c) The electric field inside the sphere is $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^4}r^2$.
(d) The electric field inside the sphere $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$.

22. Five charges, q each are placedat the corners of a regular pentagon $ABCDE$ of side a as shown in the figure.



(a) The electric field at the centre of the pentagon O will be zero.
(b) The electric field at O will be $\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ if the charge from one of the corners (say A) is removed.
(c) The electric field at O will be $\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ if the charge q at A is replaced by $-q$.
(d) The electric field at O will be $\frac{1}{4\pi\varepsilon_0}\cdot\frac{nq}{a^2}$ if pentagon is replaced by n-sided regular polygon with charge q at each of its corners.

23. A particle of charge q and mass m moves rectilinearly under the action of a electric field $E = A - Bx$ where B is a positive constant with appropriate unit and x is distance from the point where the particle was initially at rest.
(a) The distance travelled by the particle till it comes to rest is $2A/B$.
(b) Acceleration of the particle at the instant before coming to rest is $\frac{-qA}{m}$.
(c) The speed of the particle as a function of x is $\sqrt{\frac{2q}{m}\left(Ax - \frac{Bx^2}{2}\right)}$.
(d) The speed of the particle as a function of x is $\sqrt{\frac{2}{m}(Ax - Bx^2)}$.

Integer Answer Type

24. A point charge 0.140 nC is placed on the apex of a cone of semi-vertical angle 30°. Find the electric flux (in SI units) through the base of cone.

25. Point charges, q_1 and q_2, are placed on the X-axis, with q_1, at $x = 0$ and q_2 at $x = d$. A third point charge $+Q$, is placed at $x = 3d/4$. If the net electrostatic force experienced by the charge $+Q$ is zero, then find the ratio q_1/q_2.

26. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k(q^2/\gamma)^{1/N}$, where k is a constant. Then find N.

Comprehension Type

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let N be the number density of free electrons, each of mass m. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency ω_p, which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

27. Taking the electronic charge as e and the permitivity as ε_0, use dimensional analysis to determine the correct expression for ω_p.

(a) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$ (c) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

28. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}\,\text{m}^{-3}$. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.

(a) 800 nm (b) 600 nm
(c) 300 nm (d) 200 nm

Matrix Match Type

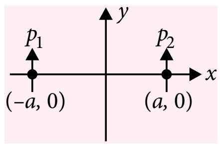
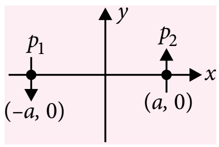
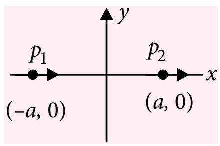
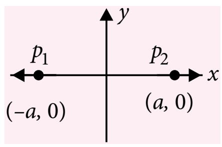
29. Column I gives the dependence of electric field (E) on distance (r) due to certain charged objects and Column II lists these objects. Match the entries of Column I with the entries of Column II.

Column I	Column II
(A) r^0	(P) Point charge
(B) r^{-1}	(Q) Thin infinitely long wire of uniform linear charge density
(C) r^{-3}	(R) Infinite uniformly charged plane sheet
(D) r^{-2}	(S) Electric field inside the region between two uniformly charged parallel planes
	(T) Electric field on the axial line of a short electric dipole.

Codes

	A	B	C	D
(a)	P	Q	R	S, T
(b)	R, S	Q	T	P
(c)	R, S	P, Q	T	P
(d)	P, T	R	Q	S

30. In each situation of column I two electric point dipoles having dipole moments $\vec{p}_1$ and $\vec{p}_2$ of same magnitude (that is, $p_1 = p_2$) are placed on x-axis symmetrically about origin in different orientations as shown. In column II certain inferences are drawn for these two dipoles. Match the different orientations of dipoles in column I with the corresponding results in column II.

Column I	Column II
(A) 	(P) The torque on one dipole due to other is zero
(B) 	(Q) The potential energy of one dipole in electric field of other dipole is negative
(C) 	(R) There is one straight line in x-y plane (not at infinity) which is equipotential
(D) 	(S) Electric field at origin is zero

Codes

	A	B	C	D
(a)	P, R	P, Q, R, S	P, Q, S	P
(b)	Q, R	P, R	P, R	Q, S
(c)	P, Q, S	P, R, S	P, R	Q, S
(d)	R, S	P, R	P, S	P, Q

◆◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIIMS / Other PMTs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / various PMTs. The detailed solutions of these problems will be published in next issue of Physics For You.

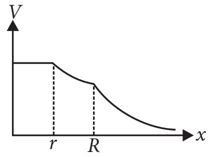
The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.

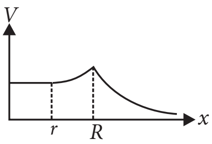
We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

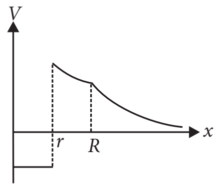
PROBLEM Set 36

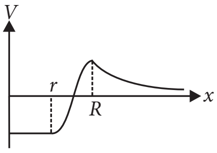
SINGLE OPTION CORRECT TYPE

1. Two concentric spherical shells of radii r and R ($r < R$) have surface charge densities $-\sigma$ and $+\sigma$ respectively. The variation of electric potential V with distance x from the centre O of the shells is plotted. Which of the following graphs best depict the variation qualitatively?

(a) 

(b) 

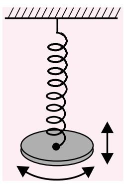
(c) 

(d) 

2. Much of the material making up Saturn's rings is in the form of tiny dust grains having radii of the order of 10^{-6} m. These grains are located in a region containing a dilute ionized gas, and they pick up excess electrons. As an approximation, suppose each grain is spherical, with radius $R = 1.0 \times 10^{-6}$ m. How many electrons would one grain have to pick up to have a potential of – 360 V on its surface ? (Take $V = 0$ at infinity)

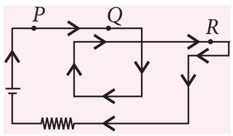
(a) 250000 (b) 2500
(c) 100000 (d) 50000

3. A solid disk of radius R is suspended from a spring of linear spring constant k and torsional constant c, as shown in the figure. In terms of k and c, what value of R will give the same time period for the vertical and torsional oscillations of this system?



(a) $\sqrt{\frac{2c}{k}}$ (b) $\sqrt{\frac{c}{2k}}$ (c) $2\sqrt{\frac{c}{k}}$ (d) $\frac{1}{2}\sqrt{\frac{c}{k}}$

4. A battery establishes a steady current around the circuit shown. A compass needle is placed successively at points P, Q and R, just above the wire (slightly out of the plane of the page). The relative deflection of the needle, in descending order, is



(a) P, Q, R (b) Q, R, P
(c) R, Q, P (d) Q, P, R

5. A spherical capacitor is formed by two metallic and concentric spherical shells. The capacitor is then charged so that the outer shell carries a positive charge and the inner shell carries an equal but negative charge. Even if the capacitor is not connected to any circuit, the charge will eventually leak away due to the small conductivity of the material between the shells. What is the character of the magnetic field induced by this leakage current?

(a) Radially outwards from the inner shell to the outer shell.
(b) Radially inwards from the outer shell to the inner shell.
(c) Circular field lines between the shells and perpendicular to the radial direction
(d) No magnetic field is induced.

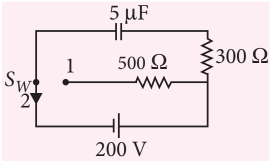
SUBJECTIVE TYPE

6. A cyclist turns her bicycle upside down to tinker with it. After she gets it upside down, she notices the front wheel executing a slow, small-amplitude, back-and-forth rotational motion with a period of 12 s. Considering the wheel to be a thin ring of mass

By Akhil Tewari, Author Foundation of Physics for JEE Main & Advanced, Senior Professor Physics, RAO IIT ACADEMY, Mumbai.

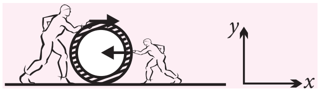
600 g and radius 30 cm, whose only irregularity is the presence of the small tire valve stem, determine the mass of the valve stem(in gm). Take $\pi^2 = g$ and appropriate approximation.

7. A capacitor of capacitance 5 μF is connected to a source of constant emf of 200 V for a long time, then the switch was shifted to contact 1 from contact 2. Find the amount of heat generated in the 500 Ω resistance.

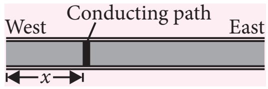


8. An electrometer is charged to 3 kV. Then the electrometer is touched with an initially neutral metal ball, mounted on an insulating rod, and then the metal ball is taken away and earthed. The process is done for 10 times, and finally the electrometer reads 1.5 kV. After this, at least how many times must the above process be repeated in order that the electrometer reads less than 1 kV?

9. A man is trying to roll a barrel along a level street by pushing forward along its top rim. At the same time another man is pushing backward at the middle, with a force of equal magnitude 4 N (see figure), the barrel rolls without slipping. Find the magnitude and direction of the friction force at the point of contact with the street. The barrel is a uniform cylinder of mass 1 kg.



10. A 10 km long underground cable extends east to west and consists of two parallel wires, each of which has resistance 13 Ω km^{-1}. A short develops at distance x from the west end when a conducting path of resistance R connects the wires as shown in the figure.
The resistance of the wires and the short is then 100 Ω when the measurement is made from the east end, 200 Ω when it is made from the west end. What is value of R (in ohm)?



More Girls Qualify for IITs, But Fail to Make it to Top 100

Of the 36,566 who qualified to join IITs from the 1,47,678 appeared, girls comprised 12.49%, better than last year's 11.54%.

More girls qualified this year to join the prestigious Indian Institutes of Technology, but none made it to the top-100 ranks.
Riya Singh, the topper among the girls this year, got an all-India rank of 133, according to the results of the IIT entrance test, JEE-Advanced 2016, declared on 12th June. That was worse than last year, when there was one girl candidate among the first 100, ranked 47. In 2014, there were five girls in the top 100.
Of the 36,566 who qualified to join IITs from the 1,47,678 appeared, girls comprised 12.49%, better than last year's 11.54%. Also, their representation this year had been better: 23.16% of those who appeared for the JEE-Advanced were girls, compared with 17.35% in last year's total of 1,17,237 students. The IIT-Bombay zone had the highest number of successful candidates for the third year running. Most number of top 10 rank holders were from the IIT – Madras zone.

The top rank went to Jaipur's Aman Bansal, with 320 out of the total 372 marks. Yamuna Nagar, Haryana's Bhavesh Dhingra came second, followed by Kunal Goyal of Jaipur. Bansal and Goyal are from the IIT-Bombay zone, which had 8,810 of the total candidates who have qualified.

Tough Competition

While more girls have qualified in the JEE-Advanced this year, not being able to make it to the top ranks means they may find it difficult to get through to coveted streams such as computer science and engineering seats at top IITs

Bansal, whose father is a government employee and mother housewife, said he aspires to do something of his own in the field of education.
While more girls have qualified in the JEE-Advanced this year, not being able to make it to the top ranks means they may find it difficult to get through to coveted streams such as computer science and engineering seats at top IITs such as Bombay, Madras and Delhi. Historically, the highest rank holders prefer to opt for computer science and engineering. At IIT-Bombay, computer science seats are usually filled up by rank 50-60.
"I believe that more women must pursue STEM courses and am glad that this year, the exam has seen more women applying," said Biocon Chairman Kiran Mazumdar-Shaw. "As more and more women get accepted into the job market, their competitive spirit will also improve and they'll fare better in the top ranks as well."

Results difficult to access

The huge volume of traffic took its toll on the JEE Advanced portal, even as IIT-Guwahati, the organising institute for JEE-Advanced this year, did its best to cope. Even five hours after the results were declared, a bulk of the nearly 1.47 lakh students who sat for the exam were unable to access their results.

BOOST your NEET score

Practice paper for phase II

1. A particle is projected with a velocity u so that its horizontal range is twice the greatest height attained. The horizontal range is
(a) $\frac{u^2}{g}$ (b) $\frac{2u^2}{3g}$
(c) $\frac{4u^2}{5g}$ (d) none of these

2. A cannon ball has a range R on a horizontal plane, such that the corresponding possible maximum heights reached are H_1 and H_2. Then, the correct expression for R is
(a) $\frac{(H_1 + H_2)}{2}$ (b) $(H_1H_2)^{1/2}$
(c) $2(H_1H_2)^{1/2}$ (d) $4(H_1H_2)^{1/2}$

3. A spherical shell is uniformly charged with the surface density σ. Using the energy conservation law, the magnitude of the electric force acting on a unit area of the shell is
(a) $\frac{\sigma^2}{2\varepsilon_0}$ (b) $\frac{\sigma}{2\varepsilon_0}$ (c) $\frac{\sigma^2}{\varepsilon_0}$ (d) $\frac{\sigma}{\varepsilon_0}$

4. A ball rolls of the top of a stair way with a horizontal velocity u m s^{-1}. If the steps are h metre high and b metre wide , the time taken by the ball to hit the edge of n^{th} step, is
(a) $\frac{hu}{gb}$ (b) $\frac{2hu}{gb}$ (c) $\frac{2hu^2}{gb}$ (d) $\frac{hu^2}{2gb}$

5. Which of the following types of electromagnetic radiation travels at the greatest speed in vacuum?
(a) Radio waves
(b) Visible light
(c) X-rays
(d) All of these travel at the same speed

6. A body is projected horizontally from a point above the ground and motion of the body is described by the equation $x = 2t$, $y = 5t^2$ where x and y are horizontal and vertical coordinates in metre after time t. The initial velocity of the body will be
(a) $\sqrt{29}$ m s^{-1} horizontal
(b) 5 m s^{-1} horizontal
(c) 2 m s^{-1} vertical (d) 2 m s^{-1} horizontal

7. The velocity of the bullet reduces by 1/20$^{\text{th}}$ of its initial velocity after passing through a plank. The minimum number of plank it can cross is
(a) 11 (b) 10 (c) 19 (d) 20

8. The power and type of lens by which a person can see clearly the distant objects, if the person cannot see objects beyond 40 cm, are
(a) –2.5 D and concave lens
(b) –2.5 D and convex lens
(c) –3.5 D and concave lens
(d) –3.5 D and convex lens

9. In a series LCR circuit, the voltage across the resistance, capacitance and inductance is 10 V each. If the capacitance is short circuited the voltage across the inductance will be
(a) 10 V (b) $10\sqrt{2}$ V (c) $\frac{10}{\sqrt{2}}$ V (d) 20 V

10. The reactance of a capacitance C is X. If both the frequency and capacitance be doubled, then new reactance will be
(a) X (b) $2X$ (c) $4X$ (d) $\frac{X}{4}$

11. Two bulbs consume same power when operated at 200 V and 300 V respectively. When the two bulbs are connected in series across a DC source of 500 V, then ratio of
(a) potential difference across them is 3 : 2

(b) potential difference across them is 2 : 3
(c) power consumed by them is 4 : 9
(d) power consumed by them is 2 : 3

12. Two resistances 3 Ω and 6 Ω are connected in parallel and a 4 Ω resistance is connected in series with this combination. The current through 3 Ω resistance is 0.8 A. Then the potential drop across 4 Ω resistance is
(a) 9.6 V (b) 2.6 V (c) 4.8 V (d) 1.2 V

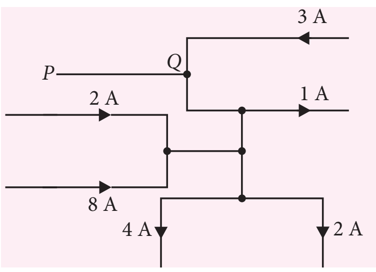
13. Two weights w_1 and w_2 are suspended from the ends of a light string over a smooth fixed pulley. If the pulley is pulled up with acceleration g, the tension in the string will be
(a) $\dfrac{4w_1w_2}{w_1+w_2}$ (b) $\dfrac{2w_1w_2}{w_1+w_2}$
(c) $\dfrac{w_1-w_2}{w_1+w_2}$ (d) $\dfrac{w_1w_2}{2(w_1+w_2)}$

14. A block has been placed on an inclined plane. The slope angle θ of the plane is such that the block slides down the plane at a constant speed. The coefficient of kinetic friction is equal to
(a) sin θ (b) cos θ (c) cot θ (d) tan θ

15. The area of cross-section of one limb of an U tube is twice that of the other. Both the limbs contain mercury at the same level. Water is poured in the wider tube so that mercury level in it goes down by 1 cm. The height of water column is
(Density of water = 10^3 kg m^{-3}, density of mercury = 13.6×10^3 kg m^{-3})
(a) 13.6 cm (b) 40.8 cm
(c) 27.2 cm (d) 54.4 cm

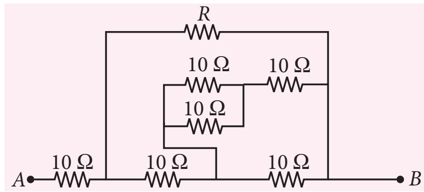
16. A rigid container with thermally insulated walls contains a coil of resistance 100 Ω, carrying current 1 A. Change in internal energy after 5 minute will be
(a) 0 kJ (b) 10 kJ (c) 20 kJ (d) 30 kJ

17. Figure below shows a portion of an electric circuit with the currents in amperes and their directions. The magnitude and direction of the current in the portion PQ is



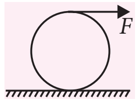
(a) 0 A (b) 3 A from P to Q
(c) 4 A from Q to P (d) 6 A from Q to P

18 For what value of R the net resistance of the circuit will be 18 Ω?



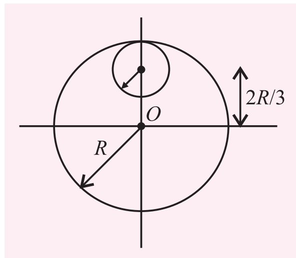
(a) 8 Ω (b) 10 Ω (c) 16 Ω (d) 24 Ω

19. A force F is applied at the top of a ring of mass M and radius R placed on a rough horizontal surface as shown in figure.



Friction is sufficient to prevent slipping. The friction force acting on the ring is
(a) $\dfrac{F}{2}$ towards right (b) $\dfrac{F}{3}$ towards left
(c) $\dfrac{2F}{3}$ towards right (d) zero

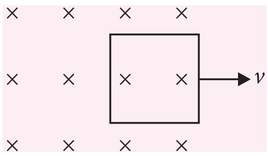
20. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is



(a) $4MR^2$ (b) $\dfrac{40}{9}MR^2$
(c) $10MR^2$ (d) $\dfrac{37}{9}MR^2$

21. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 s for this and records 40 s for 20 oscillations. For this observation, which of the following statements is true?
(a) Error ΔT in measuring T, the time period, is 0.02 second
(b) Error ΔT in measuring T, the time period, is 1 second.
(c) Percentage error in the determination of g is 5%.
(d) Percentage error in the determination of g is 2.5%.

22. Figure shows a square loop of side 0.5 m and resistance 10 Ω. The magnetic field has a magnitude B = 1.0 T. The work done in pulling the loop out of the field slowly and uniformly in 2.0 s is



(a) 3.125×10^{-3} J (b) 6.25×10^{-4} J
(c) 1.25×10^{-2} J (d) 5.0×10^{-4} J

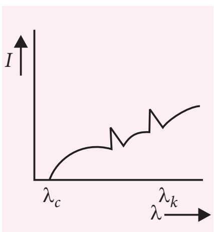
23. Two concentric coils each of radius equal to 2π cm are placed at right angle to each other. 3 A and 4 A are the currents flowing in each coil respectively. The magnetic induction (in Wb m^{-2}) at the centre of the coils will be
(a) 12×10^{-5} (b) 10^{-5}
(c) 5×10^{-5} (d) 7×10^{-5}

24. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions mA^{-1} and voltage sensitivity is 2 divisions mV^{-1}. In order that each division reads 1 V, the resistance (in ohm) needed to be connected in series with the coil will be
(a) 10^3 (b) 10^5 (c) 99995 (d) 9995

25. The mass of the moon is 1/81 of earth's mass and its radius 1/4th that of the earth. If the escape velocity from the earth's surface is 11.2 km s^{-1}, its value for the moon will be
(a) 0.15 km s^{-1} (b) 5 km s^{-1}
(c) 2.5 km s^{-1} (d) 0.5 km s^{-1}

26. The escape velocity of a planet is v_e. A particle starts from rest at a large distance from the planet, reaches the planet only under gravitational attraction, and passes through a smooth tunnel through its centre. Its speed at the centre of the planet will be
(a) v_e (b) $2v_e$ (c) $1.5\, v_e$ (d) $\sqrt{1.5}v_e$

27. The intensity of X-rays from a Coolidge tube is plotted against wavelength as shown in the figure. The minimum wavelength found is λ_c and the wavelength of the k_α line is λ_k. As the accelerating voltage is increased



(a) $(\lambda_k - \lambda_c)$ increases (b) $(\lambda_k - \lambda_c)$ decreases
(c) λ_k increases (d) λ_k decreases

28. A body cools from 80°C to 64°C in 5 min and same body cools from 80°C to 52°C in 10 min, what is the temperature of the surrounding?
(a) 24°C (b) 28°C (c) 22°C (d) 25°C

29. In an experiment on photoelectric emission from a metallic surface, wavelength of incident light is 2×10^{-7} m and stopping potential is 2.5 V. The threshold frequency of the metal is approximately (Charge of electron e = 1.6×10^{-19} C, Planck's constant $h = 6.6 \times 10^{-34}$ J s)
(a) 12×10^{15} Hz (b) 9×10^{15} Hz
(c) 9×10^{14} Hz (d) 12×10^{13} Hz

30. A radioactive element X converts into another stable element Y. Half-life of X is 2 h. Initially only X is present. After time t, the ratio of atoms of X and Y is found to be 1 : 4, then t in hour is
(a) 2 (b) 4
(c) 6 (d) between 4 and 6

31. A ball is projected vertically upward with a certain initial speed. Another ball of the same mass is projected at an angle of 60° with the vertical with the same initial speed. At highest point of their journeys, the ratio of their potential energies will be
(a) 1 : 1 (b) 2 : 1 (c) 3 : 2 (d) 4 : 1

32. Unpolarized light is travelling from a medium of refractive index 2 to a medium of index 3. The angle of incidence is 60°. Then
(a) reflected light will be partially polarized.
(b) reflected light will be plane polarized in a plane perpendicular to plane of incidence.
(c) refracted light will be plane polarized in a plane perpendicular to plane of incidence.
(d) refracted light will be plane polarized in a plane parallel to plane of incidence.

33. Li nucleus has three protons and four neutrons. Mass of Li nucleus is 7.016005 amu, mass of proton is 1.007277 amu and mass of neutron is 1.008665 amu. Mass defect of lithium nucleus in amu is

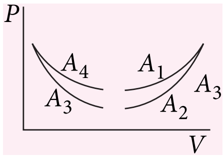
MPP-1 CLASS XII ANSWER KEY

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(a)	**5.**	(c)
6.	(a)	**7.**	(c)	**8.**	(d)	**9.**	(d)	**10.**	(b)
11.	(b)	**12.**	(d)	**13.**	(b)	**14.**	(a)	**15.**	(c)
16.	(a)	**17.**	(b)	**18.**	(b)	**19.**	(a)	**20.**	(a,c)
21.	(a,c)	**22.**	(a,b)	**23.**	(a,b,c)	**24.**	(1)	**25.**	(9)
26.	(3)	**27.**	(c)	**28.**	(b)	**29.**	(b)	**30.**	(a)

(a) 0.040486 amu (b) 0.040500 amu
(c) 0.040524 amu (d) 0.040555 amu

34. A transmitting antenna of height h and the receiving antenna of height 45 m are separated by a distance of 40 km for satisfactory communication in line-of-sight mode. Then the value of h is (Take, radius of the earth = 6400 km)
(a) 15 m (b) 20 m (c) 30 m (d) 25 m

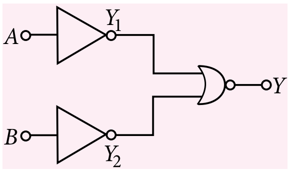
35. The four curves A_1, A_2, A_3 and A_4 are shown on PV diagram. Which of the curves represents adiabatic process?



(a) A_3 (b) A_4 (c) A_1 (d) A_2

36. The rms value of the electric field of the light coming from the sun is 720 N C^{-1}. The average total energy density of the electromagnetic wave is
(a) 3.33×10^{-3} J m^{-3} (b) 4.58×10^{-6} J m^{-3}
(c) 6.37×10^{-9} J m^{-3} (d) 81.35×10^{-12} J m^{-3}.

37. Which logic gate is represented by the following combination of logic gates?



(a) AND (b) NOR (c) OR (d) NAND

38. A man measures the period of a simple pendulum inside a stationary lift and finds it to be T second. If the lift accelerates upwards with an acceleration $\frac{g}{4}$, then the period of the pendulum will be

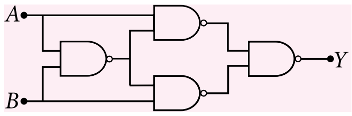
(a) T (b) $\frac{T}{4}$ (c) $\frac{2T}{\sqrt{5}}$ (d) $2\sqrt{5}T$

39. A pendulum bob of mass 10^{-2} kg is raised to a height 5×10^{-2} m and then released. At the bottom of its swing, it picks up a mass 10^{-3} kg. To what height will the combined mass rise?
(a) 4.1×10^{-2} m (b) 2.5×10^{-2} m
(c) 1.3×10^{-2} m (d) 0.7×10^{-2} m

40. Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1\cos\pi t$. The initial phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is

(a) $\frac{\pi}{6}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $-\frac{\pi}{6}$

41. The following configuration of four NAND gates is equivalent to

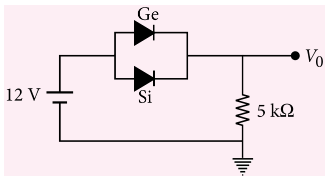


(a) NAND gate (b) AND gate
(c) OR gate (d) XOR gate

42. In Young's double slit experiment, the spacing between the slits is d and wavelength of light used is 6000 Å. If the angular width of a fringe formed on a distant screen is 1°, then value of d is
(a) 1 mm (b) 0.05 mm
(c) 0.03 mm (d) 0.01 mm

43. A resistance of 2 Ω is connected across the gap of a metre-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2 Ω, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any correction, the unknown resistance is
(a) 3 Ω (b) 4 Ω (c) 5 Ω (d) 6 Ω

44. Ge and Si diodes conduct at 0.3 V and 0.7 V respectively. In the figure, if Ge diode connection is reversed, then the value of output voltage V_0 changes by



(a) 0.2 V (b) 0.4 V (c) 0.6 V (d) 0.8

45. A radioactive nucleus of mass M emits a photon of frequency υ and the nucleus recoils. The recoil energy will be
(a) $Mc^2 - h\upsilon$ (b) $h^2\upsilon^2/2Mc^2$
(c) zero (d) $h\upsilon$

ANSWER KEYS

6.	(d)	**7.**	(a)	**8.**	(a)	**9.**	(c)	**10.**	(d)
11.	(c)	**12.**	(c)	**13.**	(a)	**14.**	(d)	**15.**	(b)
16.	(d)	**17.**	(d)	**18.**	(c)	**19.**	(d)	**20.**	(a)
21.	(c)	**22.**	(a)	**23.**	(c)	**24.**	(d)	**25.**	(c)
26.	(d)	**27.**	(a)	**28.**	(a)	**29.**	(c)	**30.**	(d)
31.	(d)	**32.**	(a)	**33.**	(a)	**34.**	(b)	**35.**	(a)
36.	(b)	**37.**	(a)	**38.**	(c)	**39.**	(a)	**40.**	(d)
41.	(d)	**42.**	(c)	**43.**	(a)	**44.**	(b)	**45.**	(b)

PAPER-I

SECTION 1 (Maximum Marks : 15)

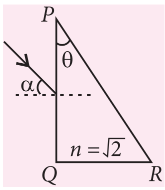
- *This section contains FIVE questions.*
- *Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.*
- *For each question, darken the bubble corresponding to the correct option in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : *+3 If only the bubble corresponding to the correct option is darkened.*

Zero Marks : *0 If none of the bubbles is darkened.*

Negative Marks : *–1 In all other cases.*

1. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is



(a) 15° (b) 22.5°
(c) 30° (d) 45°

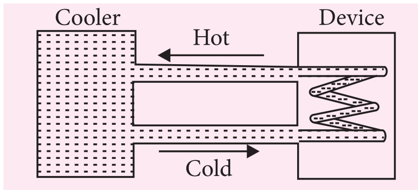
2. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ m s^{-1} and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of J s) found from such an experiment is

(a) 6.0×10^{-34} (b) 6.4×10^{-34}
(c) 6.6×10^{-34} (d) 6.8×10^{-34}

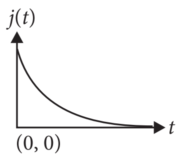
3. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is

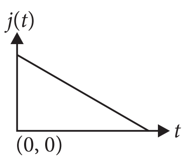


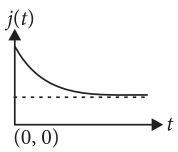
(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

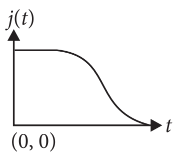
(a) 1600 (b) 2067
(c) 2533 (d) 3933

4. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time $t = 0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density $j(t)$ at any point in the material?

(a)


(b)


(c) 

(d) 

5. A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth, vertical wall of height $h(< l)$ such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/l and the frictional force f at the bottom of the stick are ($g = 10$ m s^{-2})

(a) $\frac{h}{l}=\frac{\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}$ N (b) $\frac{h}{l}=\frac{3}{16}, f=\frac{16\sqrt{3}}{3}$ N

(c) $\frac{h}{l}=\frac{3\sqrt{3}}{16}, f=\frac{8\sqrt{3}}{3}$ N (d) $\frac{h}{l}=\frac{3\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}$ N

SECTION 2 (Maximum Marks : 32)

- *This section contains EIGHT questions.*
- *Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct.*
- *For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : *+4 If only the bubble(s) corresponding to the correct option(s) is(are) darkened.*

Partial Marks : *+1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.*

Zero Marks : *0 If none of the bubbles is darkened.*

Negative Marks : *–2 In all other cases.*

- *For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.*

6. The position vector $\vec{r}$ of a particle of mass m is given by the following equation

$$\vec{r}(t)=\alpha t^3\,\hat{i}+\beta t^2\,\hat{j},$$

where $\alpha = 10/3$ m s^{-3}, $\beta = 5$ m s^{-2} and $m = 0.1$ kg. At $t = 1$ s, which of the following statement(s) is (are) true about the particle?

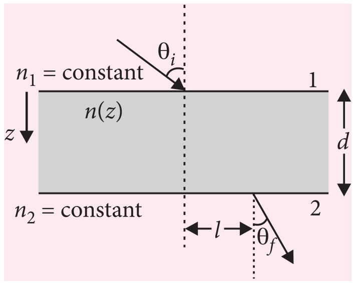
(a) The velocity $\vec{v}$ is given by $\vec{v}=(10\hat{i}+10\hat{j})$ m s^{-1}

(b) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L}=-(5/3)\hat{k}$ N m s

(c) The force $\vec{F}$ is given by $\vec{F}=(\hat{i}+2\hat{j})$ N

(d) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau}=-(20/3)\hat{k}$ N m

7. A transparent slab of thickness d has a refractive index $n(z)$ that increases with z. Hence z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and $n_2\,(> n_1)$, as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l.



Which of the following statement(s) is (are) true?

(a) l is independent of n_2 (b) $n_1\sin\theta_i = n_2\sin\theta_f$

(c) l is dependent on $n(z)$

(d) $n_1\sin\theta_i = (n_2 - n_1)\sin\theta_f$

8. A plano-convex lens is made of material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true?

(a) The refractive index of the lens is 2.5

(b) The radius of curvature of the convex surface is 45 cm

(c) The faint image is erect and real

(d) The focal length of the lens is 20 cm

9. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where $n >> 1$. Which of the following statement(s) is(are) true?

(a) Relative change in the radii of two consecutive orbitals does not depend on Z

(b) Relative change in the radii of two consecutive orbitals varies as $1/n$

(c) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(d) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

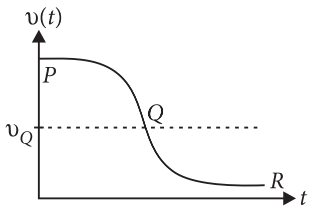
10. A length-scale (l) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is(are) dimensionally correct?

(a) $l = \sqrt{\left(\dfrac{nq^2}{\varepsilon k_B T}\right)}$ (b) $l = \sqrt{\left(\dfrac{\varepsilon k_B T}{nq^2}\right)}$

(c) $l = \sqrt{\left(\dfrac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (d) $l = \sqrt{\left(\dfrac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

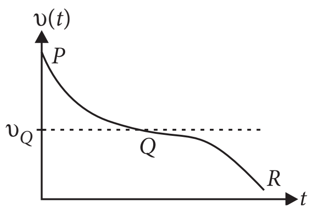
11. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $\upsilon(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let υ_P, υ_Q and υ_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 m s^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person?

(a) The plot below represents schematically the variation of beat frequency with time



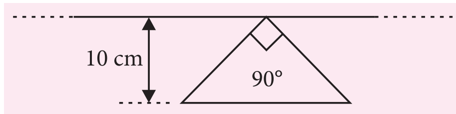
(b) $\upsilon_P + \upsilon_R = 2\upsilon_Q$

(c) The plot below represents schematically the variation of beat frequency with time



(d) The rate of change in beat frequency is maximum when the car passes through Q

12. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 A s^{-1}. Which of the following statement(s) is(are) true?



(a) The induced current in the wire is in opposite direction to the current along the hypotenuse

(b) The magnitude of induced emf in the wire is $\left(\dfrac{\mu_0}{\pi}\right)$ volt

(c) There is a repulsive force between the wire and the loop

(d) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\dfrac{\mu_0}{\pi}\right)$ volt is induced in the wire

13. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true?

(a) The temperature distribution over the filament is uniform

(b) The resistance over small sections of the filament decreases with time

(c) The filament emits more light at higher band of frequencies before it breaks up

(d) The filament consumes less electrical power towards the end of the life of the bulb

SECTION 3 (Maximum Marks : 15)

- *This section contains FIVE questions.*
- *The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.*
- *For each question, darken the bubble corresponding to the correct integer in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : *+3 If only the bubble corresponding to the correct answer is darkened.*

Zero Marks : *0 In all other cases.*

14. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistance R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time $t = 0$. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is

15. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2 (P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C?

16. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking $hc/e = 1.237 \times 10^{-6}$ V m and the ground state energy of hydrogen atom as −13.6 eV, the number of lines present in the emission spectrum is

17. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is

18. The isotope $^{12}_{5}B$ having a mass 12.014 u undergoes β-decay to $^{12}_{6}C$ · $^{12}_{6}C$ has an excited state of the nucleus ($^{12}_{6}C^*$) at 4.041 MeV above its ground state. If $^{12}_{5}B$ decays to $^{12}_{6}C^*$, the maximum kinetic energy of the β-particle in units of MeV is
(1 u = 931.5 MeV/c^2, where c is the speed of light in vacuum).

PAPER-II

SECTION 1 (Maximum Marks : 18)

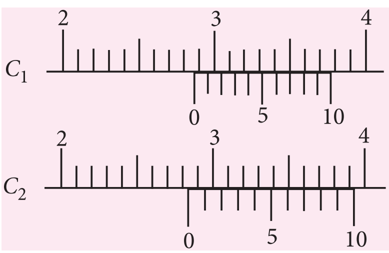
- *This section contains SIX questions.*
- *Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.*
- *For each question, darken the bubble corresponding to the correct option in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : *+3 If only the bubble corresponding to the correct option is darkened.*

Zero Marks : *0 If none of the bubbles is darkened.*

Negative Marks : *−1 In all other cases.*

1. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2, respectively, are



(a) 2.87and 2.86 (b) 2.85 and 2.82
(c) 2.87 and 2.87 (d) 2.87 and 2.83

2. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

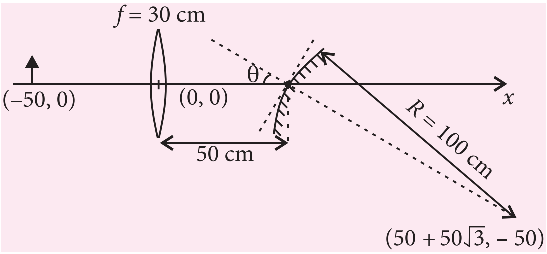
The measured masses of the neutron, $^{1}_{1}H$, $^{15}_{7}N$ and $^{15}_{8}O$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the $^{15}_{7}N$ and $^{15}_{8}O$ nuclei are same, 1 u = 931.5 MeV/c^2 (c is the speed of light) and $e^2/(4\pi\varepsilon_0) = 1.44$ MeV fm. Assuming that the difference between the binding energies of $^{15}_{7}N$ and $^{15}_{8}O$ is purely due to the electrostatic energy, the radius of either of the nuclei is (1 fm = 10^{-15} m)

(a) 2.85 fm (b) 3.03 fm
(c) 3.42 fm (d) 3.80 fm

3. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is 1.2×10^{-5} K^{-1}, the change in length of the wire PQ is

(a) 0.78 mm (b) 0.90 mm
(c) 1.56 mm (d) 2.34 mm

4. A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure.



If the origin of the coordinate system is taken to be at the centre of lens, the coordinates (in cm) of the point (x, y) at which the image is formed are

(a) $(25, 25\sqrt{3})$ (b) $(0, 0)$

(c) $(125/3, 25/\sqrt{3})$ (d) $(50-25\sqrt{3}, 25)$

5. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = (1/32) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps : an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately

(a) 112 J (b) 294 J (c) 588 J (d) 813 J

6. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?

(a) 64 (b) 90 (c) 108 (d) 120

SECTION 2 (Maximum Marks : 32)

- *This section contains EIGHT questions.*
- *Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct.*
- *For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : +4 *If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.*

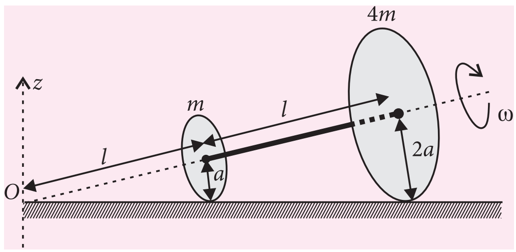
Partial Marks : +1 *For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.*

Zero Marks : 0 *If none of the bubbles is darkened.*

Negative Marks : –2 *In all other cases.*

- *For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.*

7. Two thin circular discs of mass m and $4m$, having radii of a and $2a$, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}a$ through their centers. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point O is $\vec{L}$ (see the figure). Which of the following statement(s) is (are) true?



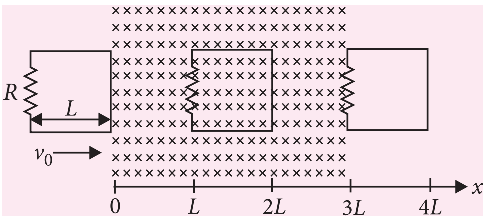
(a) The magnitude of angular momentum of the assembly about its center of mass is $17ma^2\omega/2$

(b) The center of mass of the assembly rotates about the z-axis with an angular speed of $\omega/5$

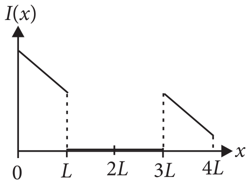
(c) The magnitude of the z-component of $\vec{L}$ is $55ma^2\omega$

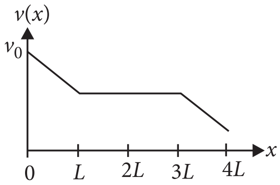
(d) The magnitude of angular momentum of center of mass of the assembly about the point O is $81ma^2\omega$

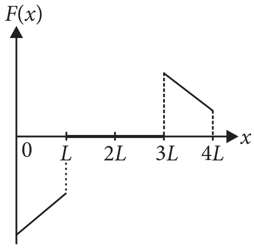
8. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters a region of length $3L$ where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive.

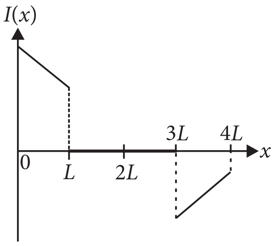


Which of the following schematic plot(s) is(are) correct? (Ignore gravity)

(a)


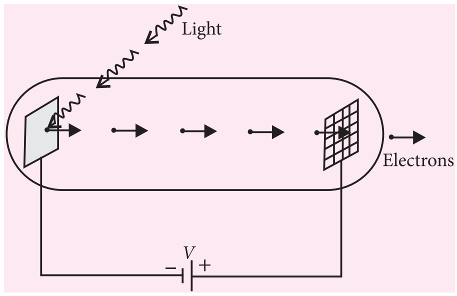
(b)


(c)


(d)


9. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\dfrac{7(R-r)}{5g}}$. The value of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is(are) true?
(a) The error in the measurement of r is 10%
(b) The error in the measurement of T is 3.57%
(c) The error in the measurement of T is 2%
(d) The error in the determined value of g is 11%

10. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statement(s) is(are) true?

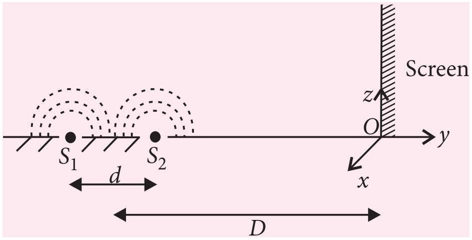


(a) For large potential difference $(V >> \phi/e)$, λ_e is approximately halved if V is made four times
(b) λ_e decreases with increase in ϕ and λ_{ph}
(c) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < hc/\phi$
(d) λ_e is approximately halved, if d is doubled

11. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_C < R/2$, which of the following statement(s) about any one of the galvanometers is(are) true?
(a) The maximum voltage range is obtained when all the components are connected in series
(b) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer
(c) The maximum current range is obtained when all the components are connected in parallel
(d) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors

12. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting

light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for $z > 0$) at a distance $D = 3$ m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the sources $d = 0.6003$ mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is (are) true of the intensity pattern on the screen?



(a) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

(b) Straight bright and dark bands parallel to the x-axis

(c) Semi circular bright and dark bands centered at point O

(d) The region very close to the point O will be dark

13. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass $m(<M)$ is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M?

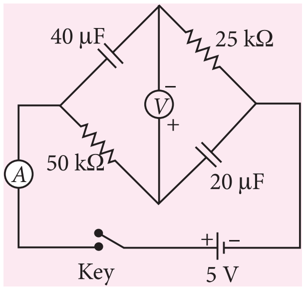
(a) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(b) The final time period of oscillation in both the cases is same

(c) The total energy decreases in both the cases

(d) The instantaneous speed at x_0 of the combined masses decreases in both the cases

14. In the circuit shown below, the key is pressed at time $t = 0$. Which of the following statement(s) is(are) true?



(a) The voltmeter displays –5 V as soon as the key is pressed, and displays +5 V after a long time

(b) The voltmeter will display 0 V at time $t = \ln 2$ seconds

(c) The current in the ammeter becomes $1/e$ of the initial value after 1 second

(d) The current in the ammeter becomes zero after a long time

SECTION 3 (Maximum Marks : 12)

- *This section contains TWO paragraphs.*
- *Based on each paragraph, there are TWO questions.*
- *Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.*
- *For each question, darken the bubble corresponding to the correct option in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : +3 *If only the bubble corresponding to the correct option is darkened.*

Zero Marks : 0 *In all other cases.*

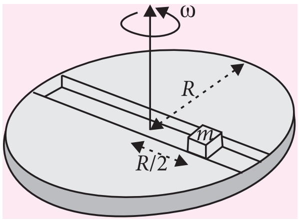
PARAGRAPH 1

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{v}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega},$$

where $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis ($\vec{\omega} = \omega\hat{k}$). A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at t = 0 and is constrained to move only along the slot.



15. The distance r of the block at time t is

(a) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (b) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

(c) $\frac{R}{2}\cos 2\omega t$ (d) $\frac{R}{2}\cos \omega t$

16. The net reaction of the disc on the block is

(a) $-m\omega^2 R\cos\omega t\,\hat{j} - mg\,\hat{k}$

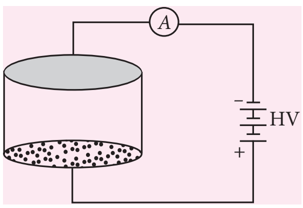
(b) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\,\hat{k}$

(c) $m\omega^2 R\sin\omega t\,\hat{j} - mg\,\hat{k}$

(d) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\,\hat{k}$

PARAGRAPH 2

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r << h$. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)



17. Which one of the following statements is correct?

(a) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(b) The balls will stick to the top plate and remain there

(c) The balls will execute simple harmonic motion between the two plates

(d) The balls will bounce back to the bottom plate carrying the same charge they went up with

18. The average current in the steady state registered by the ammeter in the circuit will be

(a) proportional to $V_0^{1/2}$

(b) zero

(c) proportional to V_0^2

(d) proportional to the potential V_0

SOLUTIONS

PAPER-I

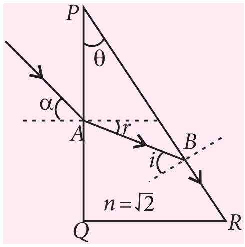
1. (a): Using Snell's law at point A,

$1 \sin\alpha = n\sin r$

$\sin 45° = \sqrt{2}\sin r$

$\sin r = \frac{1}{2} = \sin 30°$

$\therefore \quad r = 30°$



At point B, minimum value of i for total internal reflection to take place,

$$i = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

In triangle ABP, (90° + r) + (90° – i) + θ = 180°

90° + 30° + 90° – 45° + θ = 180°

$\therefore \quad \theta = 15°$

2. (b): According to Einstein's photoelectric equation,

$$\text{KE} = eV_0 = \frac{hc}{\lambda} - \phi_0$$

For two different values of λ, this equation becomes

$$eV_{01} = \frac{hc}{\lambda_1} - \phi_0 \quad ...(i)$$

$$eV_{02} = \frac{hc}{\lambda_2} - \phi_0 \quad ...(ii)$$

Subtracting equation (ii) from equation (i), we get

$$e(V_{01} - V_{02}) = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$h = \frac{e(V_{01} - V_{02})(\lambda_1\lambda_2)}{c(\lambda_2 - \lambda_1)}$$

Here, $c = 3 \times 10^8$ m s^{-1}, $V_{01} = 2$ V, $V_{02} = 1$ V

$\lambda_1 = 0.3\ \mu\text{m} = 3 \times 10^{-7}$ m, $\lambda_2 = 0.4\ \mu\text{m} = 4 \times 10^{-7}$ m

$e = 1.6 \times 10^{-19}$ C, $h = ?$

$$\therefore\ h = \frac{1.6\times10^{-19}(2-1)(3\times10^{-7}\times4\times10^{-7})}{3\times10^{8}(4\times10^{-7}-3\times10^{-7})}$$

$= 6.4 \times 10^{-34}$ J s

3. **(b) :** Heat generated by device (thermal load) in 3 h

$= 3 \times 1000 \times 3 \times 3600 = 324 \times 10^5$ J

Heat used to raise temperature of water $= ms\Delta T$

$= 120 \times 4.2 \times 10^3 \times (30 - 10) = 100.8 \times 10^5$ J

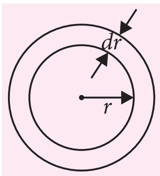
Required heat absorbed by coollant $= P \times t$

$\Rightarrow\ Pt = 324 \times 10^5 - 100.8 \times 10^5$

$$\Rightarrow\ P = \frac{(324-100.8)\times10^5}{3\times3600} = \frac{223.2}{3\times36}\times10^3 \approx 2067\ \text{W}$$

4. **(a) :** Suppose linear charge density on the wire be λ at time t.

Then electric field at a distance r from line charge



$$E = \frac{\lambda}{2\pi\varepsilon r} = -\frac{dV}{dr}$$

$$dV = -\frac{\lambda}{2\pi\varepsilon r}dr$$

Current through the elemental shell

$$I = \frac{|dV|}{dR} = \frac{\frac{\lambda}{2\pi\varepsilon r}dr}{\frac{1}{\sigma}\times\frac{dr}{2\pi rl}} = \frac{\lambda\sigma l}{\varepsilon}$$

This current is radially outwards, so

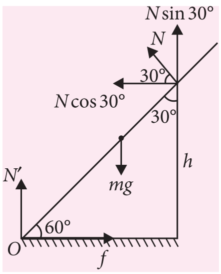
$$\frac{d}{dt}(\lambda l) = \frac{-\lambda\sigma l}{\varepsilon} \Rightarrow \int_{\lambda_0}^{\lambda}\frac{d\lambda}{\lambda} = -\left(\frac{\sigma}{\varepsilon}\right)\int_0^t dt$$

$\therefore\quad \lambda = \lambda_0 e^{-(\sigma/\varepsilon)t}$

$$\text{So,}\quad J = \frac{I}{2\pi rl} = \frac{\lambda\sigma l}{2\pi\varepsilon rl} = \frac{\lambda\sigma}{2\pi\varepsilon r}$$

$$J = \left(\frac{\lambda_0\sigma}{2\pi\varepsilon r}\right)e^{-(\sigma/\varepsilon)t} \Rightarrow J = J_0\,e^{-(\sigma/\varepsilon)t}$$

5. **(d) :** Given situation is shown in the figure.



Here $m = 1.6$ kg, $g = 10$ m s^{-2}

Also, $N' = N$

For translation equilibrium,

Horizontal, $f = N\cos30°$ ...(i)

Vertical, $N' + N\sin30° = mg$

$$N + \frac{N}{2} = 1.6\times10 = 16$$

$$\frac{3N}{2} = 16 \ \Rightarrow\ N = \frac{32}{3}\ \text{N}$$

From equation (i),

$$f = \frac{32}{3}\times\cos30° = \frac{16\sqrt{3}}{3}\ \text{N}$$

Stick is also in rotational equilibrium, so net torque about point O will be zero.

$\tau_o = 0$

$$\Rightarrow\ -(mg)\left(\frac{l}{2}\cos60°\right) + (N)\left(\frac{h}{\sin60°}\right) = 0$$

$$\Rightarrow\ mg\frac{l}{4} = \frac{2Nh}{\sqrt{3}} \ \Rightarrow\ \frac{h}{l} = \frac{\sqrt{3}mg}{8N}$$

$$\Rightarrow\ \frac{h}{l} = \frac{\sqrt{3}\times1.6\times10}{8\times\frac{32}{3}} = \frac{3\sqrt{3}}{16}$$

6. **(a, b, d) :** Position of the particle is

$\vec{r}(t) = \alpha t^3\,\hat{i} + \beta t^2\,\hat{j}$

$$\text{Velocity,}\ \vec{v} = \frac{d\vec{r}}{dt} = 3\alpha t^2\,\hat{i} + 2\beta t\,\hat{j}$$

$$\text{Acceleration,}\ \vec{a} = \frac{d\vec{v}}{dt} = 6\alpha t\,\hat{i} + 2\beta\,\hat{j}$$

Here, $\alpha = \frac{10}{3}$ m s^{-3}, $\beta = 5$ m s^{-2}, $m = 0.1$ kg, $t = 1$ s

Putting these values to get $\vec{r}, \vec{v}$ and $\vec{a}$ at $t = 1$ s

$$\vec{r} = \left(\frac{10}{3}\hat{i} + 5\hat{j}\right)\text{m};\ \vec{v} = (10\hat{i} + 10\hat{j})\ \text{m s}^{-1}$$

$$\vec{a} = (20\hat{i} + 10\hat{j})\ \text{m s}^{-2}$$

Required force on the particle $\vec{F} = m\vec{a}$

$\vec{F} = 0.1(20\hat{i} + 10\hat{j}) = (2\hat{i} + \hat{j})$ N

Required angular momentum, $\vec{L} = \vec{r}\times\vec{p} = m(\vec{r}\times\vec{v})$

$$\vec{L} = 0.1\left(\frac{10}{3}\hat{i} + 5\hat{j}\right)\times(10\hat{i} + 10\hat{j})$$

$$= \left(\frac{10}{3}\hat{i} + 5\hat{j}\right)\times(\hat{i} + \hat{j}) = \frac{10}{3}\hat{k} - 5\hat{k} = -\left(\frac{5}{3}\right)\hat{k}\ \text{N m s}$$

Required torque, $\vec{\tau} = \vec{r}\times\vec{F}$

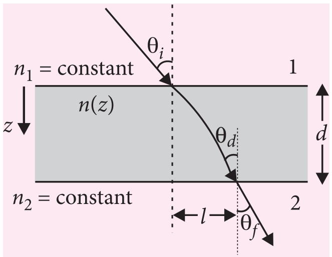
$$\vec{\tau} = \left(\frac{10}{3}\hat{i} + 5\hat{j}\right)\times(2\hat{i} + \hat{j})$$

$$= \frac{10}{3}\hat{k} - 10\hat{k} = -\left(\frac{20}{3}\right)\hat{k}\ \text{N m}$$

7. **(a, b, c) :** From Snell's law,

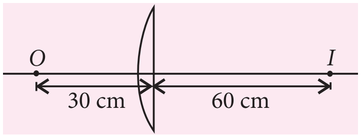
$n_1\sin\theta_i = n(z_1)\sin\theta_{z_1} = n(z_2)\sin\theta_{z_2} = \ldots\ldots$

$\ldots = n(d)\sin\theta_d = n_2\sin\theta_f$



Lateral displacement l is possible due to refractive index of the transparent slab and angle of incidence. Hence l is dependent on $n(z)$ and independent of n_2.

8. **(a, d) :** Case I : For refraction through plano-convex lens,



$m = -2 = \frac{v}{u}$

$v = -2u$

As, $u = -30$ cm

$\therefore \quad v = 60$ cm

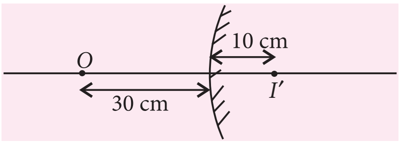
Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{60} - \frac{1}{-30} = \frac{1}{f} \Rightarrow f = 20 \text{ cm}$$

By lens maker's formula, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \frac{1}{20} = (n-1)\left(\frac{1}{R} + \frac{1}{\infty}\right) = \frac{n-1}{R} \Rightarrow n = 1 + \frac{R}{20} \quad \text{...(i)}$$

Case II : For poor reflection from convex surface,



$u = -30$ cm

$v = 10$ cm

$f = R/2$

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}$

$$\frac{1}{10} + \frac{1}{-30} = \frac{2}{R} \Rightarrow \frac{2}{R} = \frac{2}{30}$$

$\therefore \quad R = 30$ cm $\qquad$ ...(ii)

Faint image is erect and virtual.

From equations (i) and (ii)

$$n = 1 + \frac{30}{20} = 2.5$$

9. **(a, b, d):** For hydrogen-like atoms,

Radius of orbit, $r \propto \frac{n^2}{Z}$

$$\therefore \quad \frac{r_{n+1} - r_n}{r_n} = \frac{\frac{(n+1)^2}{Z} - \frac{n^2}{Z}}{\frac{n^2}{Z}} = \frac{2n+1}{n^2} \approx \frac{2}{n} (\because n >> 1)$$

Energy of the electron in the orbit, $E_n \propto \frac{-Z^2}{n^2}$

$$\therefore \quad \frac{E_{n+1} - E_n}{E_n} = \frac{\frac{-Z^2}{(n+1)^2} + \frac{Z^2}{n^2}}{\frac{-Z^2}{n^2}} = -\frac{(2n+1)}{(n+1)^2}$$

$$\approx -\frac{2n}{n^2} = -\frac{2}{n} \quad (n >> 1)$$

Angular momentum, $L = \frac{nh}{2\pi}$

$$\therefore \quad \frac{L_{n+1} - L_n}{L_n} = \frac{(n+1) - n}{n} = \frac{1}{n}$$

10. **(b, d) :** Dimensions of various given physical quantities :

$[n] = [L^{-3}]$, $[q] = [AT]$

$$\left[\frac{q^2}{\varepsilon}\right] = [Fr^2] = [U \times r] = [ML^3T^{-2}]$$

$[k_BT] = [U] = [ML^2T^{-2}]$

(a) $$\text{R.H.S.} = \sqrt{\left(\frac{q^2}{\varepsilon}\right) \times \frac{1}{(k_BT)} \times n}$$

$$= \sqrt{[U \times r] \times \frac{1}{[U]} \times n} = \sqrt{[n] \times [r]}$$

$$= \sqrt{[L^{-3}][L]} = [L^{-1}] = \frac{1}{l} \neq \text{L.H.S.}$$

(b) $$\text{R.H.S.} = \sqrt{\frac{\varepsilon(k_BT)}{n q^2}} = \sqrt{\frac{(k_BT)}{n(q^2/\varepsilon)}} = \sqrt{\frac{[U]}{[n][U \times r]}}$$

$$= \sqrt{\frac{1}{[n][r]}} = \sqrt{\frac{1}{[L^{-3}][L]}} = [L] = l = \text{L.H.S.}$$

(c) $$\text{R.H.S.} = \sqrt{\left(\frac{q^2}{\varepsilon}\right) \frac{1}{(k_BT)} \times \frac{1}{n^{2/3}}}$$

$$= \sqrt{[U \times r] \frac{1}{[U]} \frac{1}{[L^{-2}]}} = [L^{3/2}] = l^{3/2} \neq \text{L.H.S.}$$

(d) $$\text{R.H.S.} = \sqrt{\left(\frac{q^2}{\varepsilon}\right) \times \frac{1}{(k_BT)} \times \frac{1}{n^{1/3}}}$$

$$= \sqrt{[U \times r] \times \frac{1}{[U]} \times \frac{1}{[n^{1/3}]}} = \sqrt{[r] \times \frac{1}{[n^{1/3}]}} = \sqrt{[L] \frac{1}{[L^{-1}]}}$$

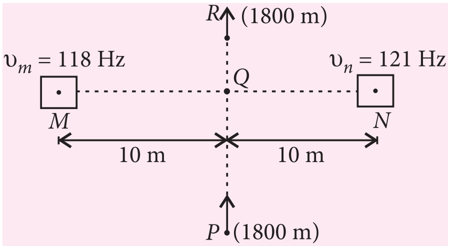
$$= \sqrt{[L^2]} = l = \text{L.H.S.}$$

11. (a, b, d): Here $v = 330\ \text{m s}^{-1}$

$$v_0 = 60\ \text{km h}^{-1} = \frac{50}{3}\ \text{m s}^{-1}$$

As, 1800 m > > 10 m

Apparent frequency of M and N at P are,



$$\upsilon'_m = \left(\frac{v+v_0}{v}\right)\upsilon_m = \left(\frac{330+\frac{50}{3}}{330}\right)\times 118$$

$$= \left(\frac{990+50}{990}\right)\times 118$$

$$\upsilon'_n = \left(\frac{v+v_0}{v}\right)\upsilon_n = \left(\frac{330+\frac{50}{3}}{330}\right)\times 121$$

$$= \left(\frac{990+50}{990}\right)\times 121$$

Beat frequency, $\upsilon_P = \upsilon'_n - \upsilon'_m$

$$= \frac{990+50}{990}(121-118) = \frac{1040}{330}\ \text{Hz}$$

Apparent frequency of M and N at R are

$$\upsilon''_m = \left(\frac{v-v_0}{v}\right)\upsilon_m = \left(\frac{330-\frac{50}{3}}{330}\right)\times 118$$

$$= \left(\frac{990-50}{990}\right)\times 118\ \text{Hz}$$

$$\upsilon''_n = \left(\frac{v-v_0}{v}\right)\upsilon_n = \left(\frac{990-50}{990}\right)\times 121\ \text{Hz}$$

Beat frequency, $\upsilon_R = \upsilon''_n - \upsilon''_m = \dfrac{(990-50)}{990}\times 3$

$$= \frac{940}{990}\times 3 = \frac{940}{330}\ \text{Hz}$$

$$\upsilon_P + \upsilon_R = \frac{1040+940}{330} = 6\ \text{Hz}$$

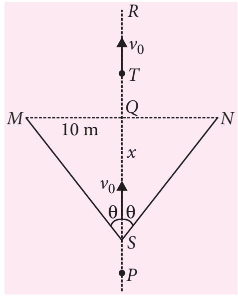
Beat frequency at Q, $\upsilon_Q = 121 - 118 = 3$ Hz

$\therefore\quad \upsilon_P + \upsilon_R = 2\upsilon_Q$

In real world, at point S between P and Q, apparent frequency of M and N,

$$\upsilon'_m = \upsilon_m\left(\frac{v+v_0\cos\theta}{v}\right)$$

$$\upsilon'_n = \upsilon_n\left(\frac{v+v_0\cos\theta}{v}\right)$$



Beat frequency, $\upsilon_S = \upsilon'_n - \upsilon'_m$

$$\upsilon_S = (\upsilon_n - \upsilon_m)\left(\frac{v+v_0\cos\theta}{v}\right)$$

$$\frac{d\upsilon_S}{dt} = (\upsilon_m - \upsilon_n)\frac{v_0}{v}\sin\theta\frac{d\theta}{dt} \quad \text{...(i)}$$

Also, $\tan\theta = \dfrac{10}{x} \Rightarrow \sec^2\theta\dfrac{d\theta}{dt} = -\dfrac{10}{x^2}\cdot\dfrac{dx}{dt}$

$$\frac{d\theta}{dt} = \frac{-10v}{x^2\sec^2\theta} = \frac{-v\tan^2\theta}{10\sec^2\theta} = -\frac{v}{10}\sin^2\theta \quad \text{...(ii)}$$

From eqns. (i) and (ii) we get, $\left|\dfrac{d\upsilon_S}{dt}\right| \propto \sin^3\theta$

Hence, $\left|\dfrac{d\upsilon_S}{dt}\right|$ will be maximum when sin θ is maximum *i.e.*, θ = 90° *i.e.*, at point Q.

12. **(b, c)** **13.** **(c, d)** **14.** **(8)**
15. **(9)** **16.** **(6)** **17.** **(3)**
18. **(9)**

PAPER-II

1. **(d)** **2.** **(c)** **3.** **(a)**
4. **(a)** **5.** **(c)** **6.** **(c)**
7. **(a, b)** **8.** **(b, d)** **9.** **(a, b, d)**
10. **(a)** **11.** **(a, c)** **12.** **(c, d)**
13. **(a, b, d)** **14.** **(a, b, c, d)** **15.** **(b)**
16. **(d)** **17.** **(a)** **18.** **(c)**

For detail solutions refer to :
MTG JEE Advanced Explorer,
MTG 39 years JEE Advanced Chapterwise Solutions.

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

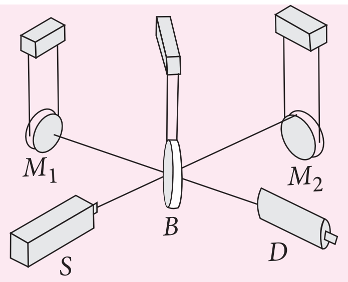
Q1. Can you briefly explain the working of the detector that LIGO used for detection of gravitational waves?

–Wrichik Basu, Kolkata (W.B.)

Ans. Black holes orbiting around each other lose energy through the emission of gravitational waves, causing them to gradually approach each other over billions of years, and then much more quickly in the final minutes. During the final fraction of a second, the two black holes collide into each other at nearly one-half the speed of light and form a single more massive black hole, converting a portion of the combined black holes, mass to energy, according to Einstein's formula $E = mc^2$. This energy is emitted as a final strong burst of gravitational waves. It is these gravitational waves that LIGO has observed.

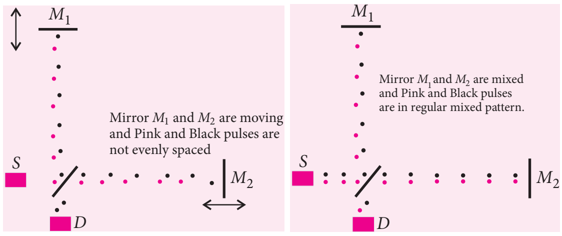
The new LIGO discovery is the first observation of gravitational waves themselves, made by measuring the tiny disturbances the waves make to space and time as they pass through the earth.

Here is the basic set-up: Two mirrors (M_1 and M_2), a receiver (D) (or light detector), a light source (S) and a beamsplitter (B).



This setup, by the way, is called a Michelson interferometer. It is a good setup for gravitational wave detectors.

At each observatory, the two-and-a-half-mile (4-km) long L-shaped LIGO interferometer uses laser light split into two beams that travel back and forth down the arms (four-foot diameter tubes kept under a near-perfect vacuum). The beams are used to monitor the distance between mirros precisely positioned at the ends of the arms. According to Einstein's theory, the distance between the mirrors will change by an infinitesimal amount when a gravitational wave passes by the detector. A change in the lengths of the arms smaller than one-ten-thousandth the diameter of a proton (10^{-19} meter) can be detected.



A *B*

Observe at the way the pulses arrive at our light detector; sometimes Pink and black are almost evenly spaced, sometimes they close together (*A*). That is caused by the gravitational wave. Without the wave, we had strict regularity (*B*).

Q2. Two particles execute SHM of the same amplitude and frequency along the same straight line. They pass one another when going in opposite directions each time their displacement is half their amplitude. What is phase difference between them?

–Soham Das, Kolkata (W.B.)

Ans. Let $y_1 = a \sin\omega t$, $y_2 = a \sin(\omega t + \phi)$

Let at $t = t_1$, $y_1 = a/2$ or $y_2 = -a/2$ (Given)

Now, $a/2 = a \sin \omega t_1$

$\Rightarrow \quad \omega t_1 = \pi/6 = 30°$ ...(i)

Also, $\omega t_1 + \phi = \pi + \pi/6 = 210°$ ...(ii)

from (i) and (ii), the phase difference $\phi = \pi$ or 180°

i.e. going in opposite directions the phase difference is +180°.

Q3. What is ripple factor? Explain it

–Kishan Mani, (U.P.)

Ans. If the DC part is filtered out from an AC signal whose mean value is zero. Now find the rms of this resulting signal. This is called the ripple voltage and expressing this as a fraction of the DC component gives the ripple factor.

A perfect rectified signal has a ripple factor zero. Mathematically it can be defined as, the ratio of rms value of the ripple voltage to the absolute value of the DC component of the output voltage, usually expressed as a percentage.

PHYSICS MUSING

SOLUTION SET-35

1. In the centre of mass frame

$$\frac{1}{2}kx^2 = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(\vec{u}_1 - \vec{u}_2)^2$$

$$\therefore \quad 200x^2 = \left(\frac{3\times 6}{3+6}\right)(2+1)^2$$

$$\Rightarrow \quad x = \frac{3}{10} = 0.3\text{ m} = 30\text{ cm}$$

2. Velocity of the system just after the collision

$$mv_0 = (m+M)v \quad \Rightarrow \quad v = \frac{mv_0}{(m+M)}$$

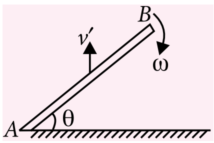
Using work energy theorem,

$$\Delta K = W_g + W_N + W_S \qquad \text{...(i)}$$

$$0 - \frac{1}{2}(m+M)v^2 = 0 + 0 - \frac{1}{2}kx_{max}^2$$

$$\frac{m^2v_o^2}{(m+M)} = kx_{max}^2 \quad \Rightarrow \quad x_{max} = \sqrt{\frac{m^2v_0^2}{k(M+m)}}$$

3. Velocity of end A at the moment it strikes ground $= \sqrt{2gh}$



If velocity of COM of rod just after collision v' and angular velocity acquired by the rod is ω clockwise, then using equation for coefficient of restitution (applied at point A)

velocity of approach = velocity of separation

$$\sqrt{2gh} = v' + \frac{L}{2}\omega\cos\theta \qquad \text{...(i)}$$

Angular momentum can be conserved about A just before collision and after collision as only impulsive force will be acting at A only.

$$\sqrt{2gh}\, M\frac{L}{2}\cos\theta = I_{cm}\omega - Mv'\frac{L}{2}\cos\theta \qquad \text{...(ii)}$$

Putting value of $\omega = (\sqrt{2gh} - v')\dfrac{2}{L\cos\theta}$ (from (i))

$$\sqrt{2gh}M\cdot\frac{L}{2}\cos\theta = \frac{ML^2}{12}(\sqrt{2gh} - v')\frac{2}{L\cos\theta} - Mv'\frac{L}{2}\cos\theta$$

$$\frac{L}{6\cos\theta}v' + \frac{L\cos\theta v'}{2} = \frac{L\sqrt{2gh}}{6\cos\theta} - \frac{\sqrt{2gh}}{2}L\cos\theta$$

$$\Rightarrow \quad v'\left[\frac{1+3\cos^2\theta}{6\cos\theta}\right] = \frac{(1-3\cos^2\theta)}{6\cos\theta}\sqrt{2gh}$$

$$v' = \left(\frac{1-3\cos^2\theta}{1+3\cos^2\theta}\right)\sqrt{2gh}$$

COM will be at maximum height when its velocity becomes zero during upward motion.

$$0 = v'^2 - 2gH$$

$$H = \frac{v'^2}{2g} = \left(\frac{1-3\cos^2\theta}{1+3\cos^2\theta}\right)^2 h = \frac{h}{49}$$

4. Let distance of the particle from the Sun be r and its radius r_0.

Intensity of light at particle, $I = \dfrac{P}{4\pi r^2}$

Force due to radiation = Force due to gravitational attraction

$$\therefore \quad \frac{I\left(\pi r_0^2\right)}{c} = \frac{GMm}{r^2}$$

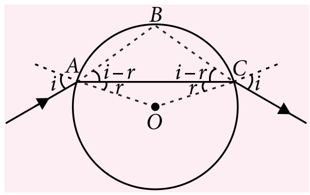
$$\Rightarrow \quad \frac{P}{4\pi r^2}\left(\frac{1}{c}\right)(\pi r_0^2) = \frac{GM\left(\frac{4}{3}\pi r_0^3\right)\rho}{r^2}$$

$$\Rightarrow \quad r_0 = \frac{3P}{16\pi cGM\rho}$$

$$= \frac{3\times 4\times 10^{26}}{16\times\frac{22}{7}\times 3\times 10^8\times\frac{20}{3}\times 10^{-11}\times 2\times 10^{30}\times 1\times 10^3}$$

$= 6\times 10^{-7}$ m $= 0.6$ μm

5. From the figure it is clear that $\angle ABC = \pi - 2(i - r)$



$\angle AOC$ (external) $= 2\pi - \{\pi - 2r\} = \pi + 2r$

From the property of circle, $2\angle ABC = \angle AOC$

$$\therefore \quad 2\{\pi - 2(i - r)\} = \pi + 2r$$

$$\Rightarrow \quad 2i - r = \frac{\pi}{2} \Rightarrow r = \left(2i - \frac{\pi}{2}\right)$$

$$\therefore \quad \mu = \frac{\sin i}{\sin r} = \sqrt{3}\text{ (given)} \Rightarrow \sqrt{3} = \frac{\sin i}{\sin\left[2i - \frac{\pi}{2}\right]}$$

Solution Senders of Physics Musing

SET-35

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SET-34

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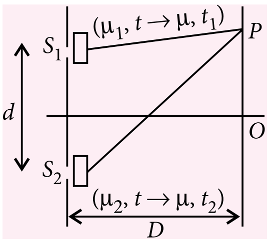
or $\sqrt{3} = \dfrac{\sin i}{-\cos 2i}$ or $\sqrt{3} = \dfrac{\sin i}{2\sin^2 i - 1}$

$2\sqrt{3}\sin^2 i - \sqrt{3} = \sin i$

Solving the equation, we get $\sin i = \dfrac{\sqrt{3}}{2} \Rightarrow i = 60°$

6. Path difference ,

$\Delta x = \{(S_2P - t_2) + \mu_2 t_2\} - \{(S_1P - t_1) + \mu_1 t_1\}$
$= (S_2P - S_1P) + \{(\mu_2 - 1)t_2 - (\mu_1 - 1)t_1\}$



Case (i)

$$y_1 = \frac{D}{d}\{t(\mu_1 - 1) - t(\mu_2 - 1)\} = \frac{D}{d}\{t(\mu_1 - \mu_2)\}$$

or $t = \dfrac{5\times10^{-3}\times1\times10^{-3}}{1\times(1.6-1.4)} = 2.5\times10^{-5}\,\text{m}$

Case (ii)

$$t_1 + t_2 = 5\times10^{-5}\,\text{m} \quad \left(\because\ t = \frac{t_1+t_2}{2}\right) \qquad \text{...(i)}$$

When both sheets have same refractive index

$$\mu = \frac{1.6+1.4}{2} = 1.5$$

$$y_2 = \frac{D}{d}\{t_1(\mu-1) - t_2(\mu-1)\} = \frac{D}{d}\{(\mu-1)(t_1-t_2)\}$$

$\therefore\ t_1 - t_2 = \dfrac{8\times10^{-3}\times1\times10^{-3}}{1\times(1.5-1)} = \dfrac{8\times10^{-6}}{0.5}$

$= 1.6\times10^{-5}\,\text{m}$ ...(ii)

On solving eqns. (i) and (ii), we get

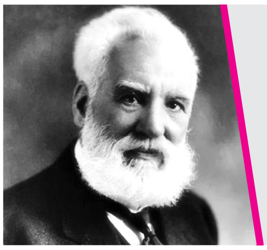
$t_1 = 33\ \mu\text{m},\ t_2 = 17\ \mu\text{m}$

7. Conserving angular momentum

$$m(v_1\cos60°).4R = mv_2R \Rightarrow \frac{v_2}{v_1} = 2 \qquad \text{...(i)}$$

Conserving energy of the system

$$-\frac{GMm}{4R} + \frac{1}{2}mv_1^2 = -\frac{GMm}{R} + \frac{1}{2}mv_2^2$$



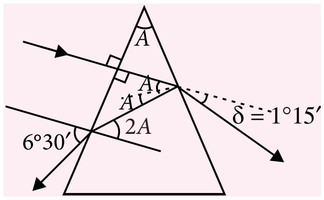
“Concentrate all your thoughts upon the work at hand. The sun's rays do not burn until brought to a focus”

– Alexander Graham Bell

$$\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 = \frac{3}{4}\frac{GM}{R} \text{ or } v_1^2 = \frac{1}{2}\frac{GM}{R} \text{ (Using (i))}$$

$$\Rightarrow\ v_1 = \frac{1}{\sqrt{2}}\sqrt{64\times10^6} = \frac{8000}{\sqrt{2}}\,\text{m s}^{-1}$$

8.



As $\delta = (\mu - 1)A$

$1.25\times\dfrac{\pi}{180} = (\mu-1)A$ ...(i) $\quad (\because\ 1°15' = 1.25°)$

Also, $\dfrac{\sin 2A}{\sin 6.5°} = \dfrac{1}{\mu}$

For small θ, $\sin\theta \approx \theta$

$\therefore\ 2A\mu = 6.5\times\dfrac{\pi}{180}$ ...(ii)

For eqns. (i) and (ii),

$\dfrac{\mu-1}{2\mu} = \dfrac{1.25}{6.5}$ or $6.5(\mu - 1) = 2.5\,\mu \Rightarrow \mu = 13/8$

$A = \dfrac{6.5°}{2\mu} = \dfrac{6.5°}{2\times13}\times8 = 2°$

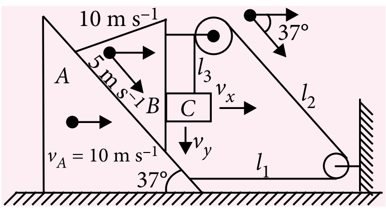
9. Let n moles of gas follows the cycle $ABCDA$.

$$Q_{\text{isochoric}} = nC_V\Delta T = \frac{nR}{\gamma-1}(T_B - T_C)$$

$$W_{\text{isobaric}} = nR\Delta T = nR(T_A - T_D) = nR(T_B - T_C)$$

$\therefore$ Required ratio $= \dfrac{Q_{\text{isochoric}}}{W_{\text{isobaric}}} = \dfrac{1}{\gamma-1} = 4$

10.



Let v_x and v_y be the horizontal and vertical component of velocity of block C.

The component of relative velocity of B and C normal to the surface of contact is zero.

$\therefore\ 10 + 5\cos37° - v_x = 0 \Rightarrow v_x = 14\ \text{m s}^{-1}$

From the figure, $l_1 + l_2 + l_3$ = constant

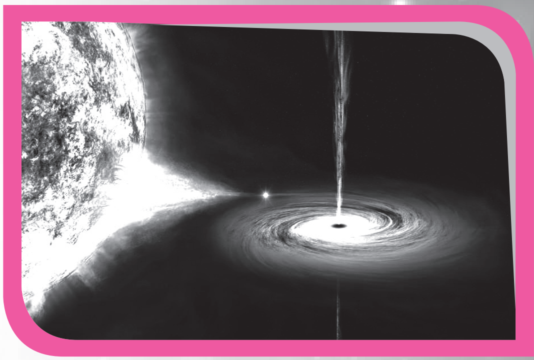
$$\therefore\ \frac{dl_1}{dt} + \frac{dl_2}{dt} + \frac{dl_3}{dt} = 0$$

$(-10) + (-5 - 10\cos37°) + (-5\sin37° + v_y) = 0$

$\therefore\ v_y = 26\ \text{m s}^{-1}$.

◈◈

HAWKING: BLACK HOLES MAY LEAD TO ANOTHER UNIVERSE



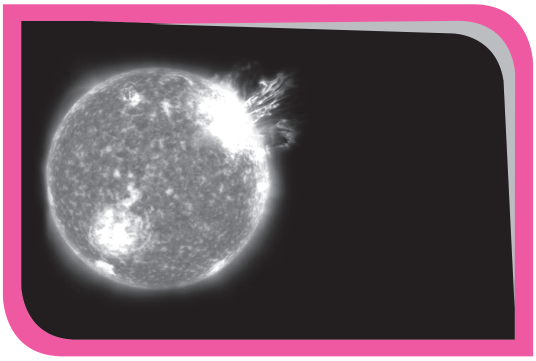
Things might fall through black holes into alternate universe, says a newly-published paper by Stephen Hawking. The professor has laid out a theory that suggests that the holes aren't quite as black as previously thought.

If the work is correct--and the theory , only suggested, has received approval from other experts -then it could solve a central paradox of black holes. Professor Hawking's paper addresses a fundamental assumption about black holes - that they have "no hair".

It has previously been assumed that anything that falls into black hole would be destroyed and lost forever.

That caused problems because the "information" about the object has to be preserved, even if the object itself is entirely swallowed up -and it has remained unclear how those two things could both happen. Hawking has since last year been implying that anything that falls into a black hole shouldn't give up hope of coming back out -somewhere. They would re-appear but somewhere else -perhaps in an alternative universe, he said.

PLANET 9 MAY HAVE BEEN `STOLEN' BY OUR SUN: STUDY



The mysterious Planet 9 may have been stolen from its original star by our sun some 4.5 billion years ago, possibly making it the first exoplanet to be discovered inside the solar system, astronomers have claimed.

An extrasolar planet, or exoplanet, is by definition a planet located outside our solar system. Now it appears that this definition is no longer viable.

According to astronomers at Lund University in Sweden, there is a lot to indicate that Planet 9 was captured by the young sun and has been a part of our solar system completely undetected ever since.

Stars are born in clusters and often pass by one another. It is during these close encounters that a star can "steal" one or more planets in orbit around another star.This is probably what happened when our own Sun captured Planet 9, the researchers behind the study said, when it came in close contact while orbiting another star.

SPACE PROBE FINDS BUILDING BLOCKS OF LIFE ON COMET

ESA's (European Space Agency) Rosetta comet hunting spacecraft has attained a major breakthrough by discovering that Comet 67P Churyumov-Gerasimenko contains ingredients regarded as crucial for the origin of life, the space agency said on Friday .

According to ESA, the ingredients include amino acid glycine which is commonly found in protein, a key component of DNA and cell membranes. "This is the first unambiguous detection of glycine at a comet,'' said Kathrin Alwegg, principal investigator of the ROSINA instrument on Rosetta which made the measurements and lead author of the paper published in `Science Advances'. Rosina was designed and developed at the University of Bern in Switzerland.

Hints of the simplest amino acid glycine were found in samples returned to earth in 2006 from Comet Wild-2 by Nasa's Stardust mission. However, possible terrestrial contamination of the dust samples made the analysis extremely difficult, says ESA.

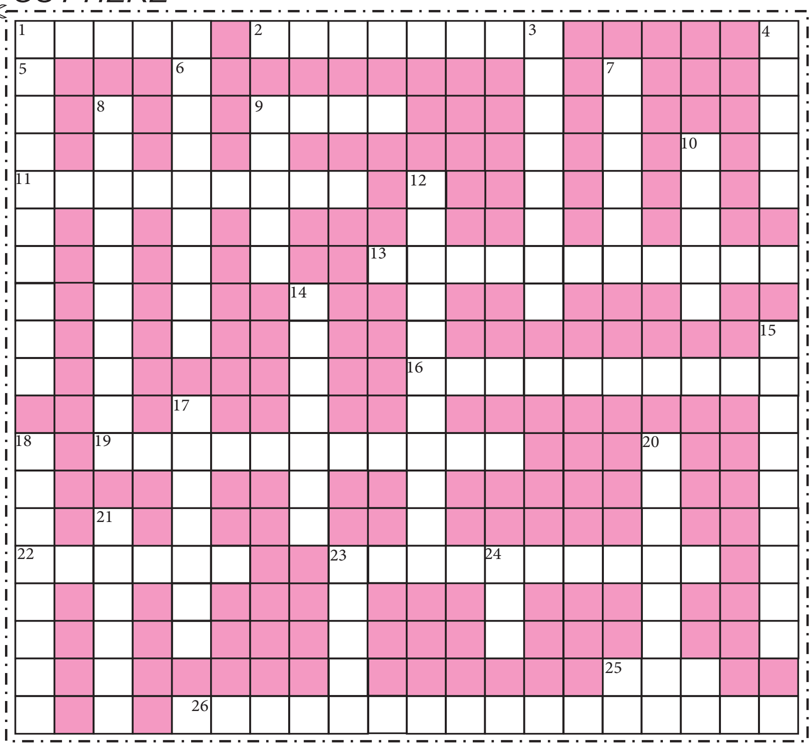
"Now, Rosetta has made direct repeated detections of glycine in the fuzzy atmosphere or `coma' of its comet," according to the announcement.

Courtesy : The Times of India

CROSS WORD

Readers can send their responses at editor@mtg.in or post us with complete address by 25th of every month to win exciting prizes. Winners' name with their valuable feedback will be published in next issue.

CUT HERE



ACROSS

1. A defect in any apparatus that interferes with or prevents normal operation. [5]
2. Method to coat the lenses to reduce back reflection from their surfaces. [8]
9. A prefix denoting 10^{15}. [4]
11. An electrically controlled switch used for switching an electrical power circuit. [9]
13. A primary electric cell in which two electrolytes are kept apart by their different densities. [7, 4]
16. The removal of one or more electrons from an atom. [10]
19. A single force capable of balancing a given system of forces. [11]
22. The another name for eye-piece.[6]
23. The branch of physics dealing with dynamic properties of gases. [10]
25. A unit of intensity of sound. [3]
26. The ratio of luminous flux emitted to the power. [8,8]

DOWN

3. The path with minimum or maximum length between two points in a mathematically defined space. [8]
4. The slow permanent deformation of specimen under sustained stress. [5]
5. Degree of exactness in measurement. [9]
6. A device which can accelerate protons to very high energies.[8]
7. Building block of the universe, massive system of stars. [6]
8. Two or more sounds that, when heard together, sound pleasant. [10]
9. Perceived sound characteristic equivalent to frequency. [5]
10. The SI unit of length. [5]
12. A low pressure gauge in which electrically heated wire loses heat by conduction through gas. [6,5]
14. Dying away of amplitude with time of free oscillations due to resistive forces. [7]
15. An instrument that produces a graph of an alternating voltage. [9]
17. The brightest objects in the universe. [7]
18. A constant pressure device that compensates for the changes of atmospheric pressure. [8]
20. An early form of motion picture projector. [7]
21. An electrical conductor. [6]
23. The point-like particles, conceived to be subcomponents of quarks and leptons. [5]
24. One thousandth of an inch. [3]